# 2019 Mini-Mathletes
# Ciphering Solutions

## Solutions:

1. A rectangle has sides of length 16 and 9. What is the sum of its area and perimeter?

   *Solution:* The area of the rectangle is $16 \times 9 = 144$ and its perimeter is $2 \times (16 + 9) = 50$ so the answer is $144 + 50 = \boxed{194}$.

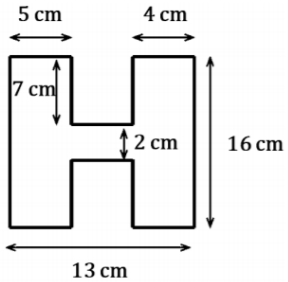
2. At Starbucks, I pay \$3.30 for a tall cup of coffee that has 7 ounces. I also pay \$4.65 for a grande cup that has 11 ounces. What is the difference in the unit rates of each cup per ounce, rounded to the nearest cent? Specify units in your answer.

   *Solution:* The unit rate for the tall cup is $\frac{3.30}{7} \approx 0.47$ dollars and the unit rate for the grande cup is $\frac{4.65}{11} \approx 0.42$ dollars so the difference in the unit rates is $0.47 - 0.42 = \boxed{0.05}$ dollars or $\boxed{5}$ cents.

3. The symbols $\heartsuit$ and $\blacktriangledown$ are whole numbers and satisfy the equation $\heartsuit \times \blacktriangledown = 36$. What is the largest possible value of $(2\,\heartsuit) + (3\,\blacktriangledown)$ ?

   *Solution:* Since $\blacktriangledown$ is being multiplied by a larger number, we want $\blacktriangledown$ to be as large as possible. Therefore the expression is maximized when $\blacktriangledown = 36$ and $\heartsuit = 1$ so the answer is $2 \times 1 + 3 \times 36 = \boxed{110}$.

4. Ava and her sister's rooms are connected with a hallway, and the made a model of their floor plans, where 1 centimeter (cm) represents 2 feet. What is the total area in square feet of their model below? The floor plan shown is in centimeters, and all angles are right.



*Solution:* We first find the floor area in the model. The room on the right has a width of 5 cm and a length of 16 cm so its area is $5 \times 16 = 80$ cm squared. The room on the right has a width of 4 cm and a length of 16 cm so its area is $4 \times 16 = 64$ cm squared. The hallway has a width of $13 - 5 - 4 = 4$ cm and a length of 2 cm so its area is $4 \times 2 = 8$ cm squared. Thus the area of the entire floor is $80 + 64 + 8 = 152$ cm squared. Since 1 cm on the model represents 2 feet, we know that 1 cm squared represents $2^2 = 4$ square feet. Thus the total floor area is $152 \times 4 = \boxed{608}$ square feet.

5. A tree has 3 branches. On each branch there are 7 leaves. Every year it has twice as many branches. For example, after 1 year, the tree will have 6 branches and 42 leaves. After how many whole years will the tree have at least 2019 leaves?

   *Solution:* Since the tree starts with 3 branches and the amount of branches doubles each year, after $n$ years, the tree will have $3 \times 2^n$ branches. Since each branch has 7 leaves, the tree will have $7 \times 3 \times 2^n = 21 \times 2^n$ leaves after $n$ years. Therefore we want to find the smallest whole number $n$ such that $21 \times 2^n \geq 2019$. This is the same as $2^n \geq \frac{2019}{21} \approx 100$. If $n = 6$, then $2^n = 64$ and if $n = 7$, then $2^n = 128$ so the answer is $n = \boxed{7}$ years.

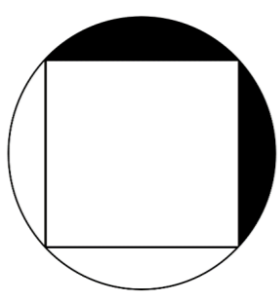
6. Caitlin spends $\frac{2}{5}$ of her money at one store in the mall. She then spends $\frac{1}{9}$ of the remainder at a second store. If Caitlin has \$40 left after purchasing from both stores, how much money did she start with?

   *Solution:* Suppose that Caitlin starts with $x$ dollars. After she spends $\frac{2}{5}$ of her money, she has $\frac{3}{5}x$ dollars left. After she spends $\frac{1}{9}$ of that amount, she has $\frac{8}{9} \times \frac{3}{5}x = \frac{8}{15}x$ dollars left. Therefore $\frac{8}{15}x = 40$ so $x = \boxed{75}$ dollars.

7. A telephone pole 10 meters tall casts a shadow 8 meters long. At the same time, a nearby tree casts a shadow 14 meters long. How tall is the tree? Write your answer as an exact decimal.

   *Solution:* The length of the shadow must be proportional to the height of the object so the ratio $\frac{\text{height}}{\text{shadow}}$ must be constant. So if the tree is $x$ meters tall, then $\frac{x}{14} = \frac{10}{8}$ so $x = 14 \times \frac{10}{8} = \frac{35}{2} = \boxed{17.5}$.

8. A square with side length 8 is inscribed in a circle. What is the area of shaded area in terms of $\pi$?



*Solution:* Since each of the four "wedges" on the side of the square have the same area, the shaded area is equal to half of the difference in the areas of the circle and the square. Since the square has a side length of 8, the radius of the circle is half of the length of the square's diagonal which has length $\sqrt{8^2+8^2} = 8\sqrt{2}$. Therefore the radius of the circle is $4\sqrt{2}$ so the area of the circle is $(4\sqrt{2})^2\pi = 32\pi$. Since the area of the area of the square is $8^2 = 64$, the area of the shaded region is $\frac{1}{2}(32\pi - 64) = \boxed{16\pi - 32}$.

9. A box contains gold coins. If the coins are equally divided among four people, 1 coin is left over. When equally divided among, six people, 3 coins are left over. When equally divided among seven people, 4 coins are left over. What is the smallest possible number of coins in the box?

   *Solution:* Since 4 coins remain when they are divided among 7 people, the total number of coins must be in the list

   $$4, 11, 18, 25, 32, 39, 46, 53, 60, 67, 74, 81, 88, \ldots$$

   Since 3 coins area left over when they are divided among 6 people, the total number of coins must be in the list

   $$3, 9, 15, 21, 27, 33, 39, 45, 51, 57, 63, 69, 75, 81, 87, 95, \ldots$$

   The smallest numbers in both lists are 39 and 81. Since only 81 leaves a remainder of 1 when divided by 4, the smallest number of coins that can be in the box is $\boxed{81}$.

10. The state of Georgia now issues license plates with 4 letters and 3 numbers, but ten years ago, it made plates with 3 letters and 4 numbers. How many more times as many license plates can be issued now than ten years ago? Write your answer as a simplified common fraction.

    *Solution:* There are 26 different letters and 10 different digits. With 4 letters and 3 numbers, $26^4 \times 10^3$ different license plates can be made. With 3 letters and 4 numbers, $26^3 \times 10^4$ different license plates can be made. Therefore now, there are $\frac{26^4 \times 10^3}{26^3 \times 10^4} = \frac{26}{10} = \boxed{\frac{13}{5}}$ times more possible license plates.