

Solutions:

1. A school store bought 575 pencils and 168 erasers to sell for its students. This morning, students bought 119 pencils and 73 erasers. How many total pencils and erasers are left?

(A) 551 (B) 552 (C) 562 (D) 583 (E) 661

Solution: The store starts off with 575 + 168 = 743 and the students buy 119 + 73 = 192 of them so the number of items remaining is 743 - 192 = 551

2. I have 2 quarters, 7 dimes, 5 nickels, and 12 pennies. How many cents do I have in total?

(A) 138 (B) 147 (C) 157 (D) 252 (E) 263

Solution: Using the face value of coins, the total number of cents present is $2 \times 25 + 7 \times 10 + 5 \times 5 + 12 = 157$

3. Peter walked $3\frac{1}{2}$ kilometers. Eric walked $\frac{1}{4}$ kilometer more than Peter. Andy walked $2\frac{2}{5}$ kilometer less than Eric. How far did Andy walk?

(A) $1 \frac{7}{20}$ (B) $3 \frac{7}{20}$ (C) $4 \frac{3}{20}$ (D) $6 \frac{3}{20}$

Solution: Because Peter walked 3 $\frac{1}{2}$ kilometers, Eric walked 3 $\frac{1}{2}$ + $\frac{1}{4}$ = 3 $\frac{3}{4}$ kilometers. Therefore Andy walked 3 $\frac{3}{4}$ - 2 $\frac{2}{5}$ = $\boxed{1\frac{7}{20}}$ kilometers.

4. Joseph runs at a constant speed of 8 miles per hour for 30 minutes, and takes a break afterwards. Then, he runs at a constant speed of 10 miles per hour for 15 minutes. How many total miles has he traveled?

(A) 5 (B) 5.5 (C) 6.5 (D) 7 (E) 7.5

Solution: If Joseph runs at 8 miles per hour for $\frac{1}{2}$ of an hour, the distance he travels is $8 \times \frac{1}{2} = 4$ miles. Then if he runs at 10 miles per hour for $\frac{1}{4}$ of an hour, the distance he travels is $10 \times \frac{1}{4} = 2.5$. Therefore the total distance Joseph travels is 4 + 2.5 = 6.5.

5. I have a picture that is 10 inches wide and 6 inches tall. I put this picture in a frame that is 1 inch thicker on each side. What is the area of the shaded frame?



(A) 16 **(B)** 18 **(C)** 36 **(D)** 60 **(E)** 64

Solution: Because the frame is one inch thick on each side, the dimensions of the frame is 12 in \times 8 in. Then the area of the frame is the area of the big rectangle minus the area of the small rectangle, or $12 \times 8 - 10 \times 6 = 36$.

6. Melissa purchased \$39.46 in groceries at a store. The cashier gave her \$1.48 in change from a \$50 bill. How much more change should the cashier have given Melissa?

(A) \$8.96 (B) \$9.06 (C) \$9.72 (D) \$10.54 (E) \$11.68

Solution: If Melissa used a \$ 50 dollar bill to pay for \$ 39.46 in groceries, the store owes her 50 - 39.46 = 10.54. Therefore the cashier should give Melissa 10.54 - 1.48 = 9.06 more dollars.

7. Daniel has 12 lollipops, Emma has 21 lollipops, and Jack has 29 lollipops. Jack gives Emma some lollipops. Then Emma gives Daniel some lollipops. Now, Jack has 2

more lollipops than both Daniel and Emma. How many lollipops did Emma give to Daniel?

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

Solution: Suppose that Jack ends up with *x* lollipops. Then Daniel and Emma will end up with x - 2 lollipops so the total number of lollipops among them will be x + (x - 2) + (x - 2) = 3x - 4. Since they begin with a total of 12 + 21 + 29 = 62 lollipops, we know that 3x - 4 = 62 so x = 22. Since Jack starts with 29 lollipops and ends up with 22 lollipops, he must have given 7 lollipops to Emma, leaving Emma with 21 + 7 = 28 lollipops. Since Emma ends up with 22 - 2 = 20 lollipops, she must give 8 lollipops to Daniel.

8. Alice, Bob, Carl, Dave, and Evan want to play enough games of chess to be sure every one plays everyone else exactly once. What is the least number of total games they need to play?

(A) 10 **(B)** 15 **(C)** 20 **(D)** 25 **(E)** 30

Solution: If each of the five players plays four other players once, there are $5 \times 4 = 20$ games. However each game twice in this way (for example, Alice playing Bob is the same as Bob) so the total number of games played is 10.

9. How many ways can 35 cents be split into change using nickles, dimes, and quarters?

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

Solution: Suppose we use a quarter. Then for the remaining 10 cents, we can either have a dime or two nickels giving as 2 possibilities.

If a quarter is not used, we can have 3 dimes and a nickel, 2 dimes and 3 nickels, 1 dime and 5 nickels, or 7 nickels, giving us 4 more possibilities.

Therefore there are 6 ways to split the 35 cents.

10. What is the area of a triangle with vertices at the points (2,0), (3,1), and (5,6). **(A)** 1 **(B)** 1.5 **(C)** 2 **(D)** 2.5 **(E)** 3

Solution: Consider the right triangle with vertices at (2,0), (5,6), and (5,0). This right triangle has area $\frac{1}{2} \times 3 \times 6 = 9$. To get the area of the triangle in the problem, we need to subtract the area of two smaller right triangle and a rectangle. The first smaller right triangle has vertices at (2,0), (3,1), and (3,0), and has area $\frac{1}{2} \times 1 \times 1 = 0.5$. The

second right triangle has vertices at (3,1), (5,6), and (5,1), and has area $\frac{1}{2} \times 2 \times 5 = 5$. The rectangle has vertices at (3,0), (3,1), (5,1), and (5,0) and has area $1 \times 2 = 2$. Therefore the area of the desired triangle is 9 - 0.5 - 5 - 2 = 1.5.

11. Given the concave quadrilateral *ABCD* and interior angles at *A*, *B*, and *C*, find $\angle CDA$.



(A) 188° (B) 190° (C) 192° (D) 202° (E) 210°

Solution: Remember that the sum of the interior angles in ANY quadrilateral is 360° so the answer is simply $360^{\circ} - 36^{\circ} - 22^{\circ} - 110^{\circ} = \boxed{192^{\circ}}$.

12. On a circus bike with two circular wheels, the front wheel has a radius of 2.5 feet and the back wheel has a radius of 4 inches. While the front wheel makes 100 revolutions, how many will the the back wheel make?

(A) 600 (B) 650 (C) 700 (D) 750 (E) 800

Solution: The circumference of the front wheel is $2 \times 2.5 \times \pi = 5\pi$ feet so after 100 revolutions, the bike travels $5\pi \times 100 = 500\pi$ feet. The back wheel has a circumference of $2 \times \frac{1}{3} \times \pi = \frac{2\pi}{3}$ feet (since 4 inches is $\frac{1}{3}$ of a foot). Therefore the back wheel will make $500\pi \div \frac{2\pi}{3} = \boxed{750}$ total revolutions.

13. Annie and Allie are building a toy dollhouse (which consists of a right rectangular prism and a right triangular prism on top) together. If the volume of only the rectangular prism 720 cubic units, what is the volume of the entire dollhouse?



Solution: Since the base of the rectangular prism has area $6 \times 8 = 48$ units squared, the prism has width of the prism is $\frac{720}{48} = 15$ units.

Now look at the triangular part of the dollhouse. The height of the triangle is 12 - 8 = 4 units and its base is 6 units so its area is $\frac{1}{2} \times 6 \times 4 = 12$ units squared. Therefore, the volume of the triangular prism portion is $12 \times 15 = 180$ units cubed so the total volume of the dollhouse is $720 + 180 = 900 \text{ u}^3$.

14. Rafael is tiling the floor of his 12-foot by 16-foot living room. He wants to place 1foot by 1-foot square tiles to form a border along the edges of the room and to fill in the rest of the floor with 2-foot by 2-foot square tiles. How many total tiles will he use?

(A) 72 (B) 78 (C) 81 (D) 87 (E) 96

Solution: The total number of 1×1 tiles along the perimeter of the floor is 12 + 16 + 12 + 16 - 4 = 52 so $52 \ 1 \times 1$ tiles will be needed. For the remaining 10×14 interior rectangle, each of the 2×2 tiles will occupy four 1×1 tiles so the total number of 2×2 tiles needed is $(10 \times 14) \div 4 = 35$. Thus a total of $52 + 35 = \boxed{87}$.

15. Mason's teacher told him that his average test score in his math class is 83. He has taken seven tests and received scores of 88, 73, 81, 83, 79, 94, and *x* (an unknown score). If Mason calculates correctly, what is his last test score?

(A) 73 (B) 83 (C) 84 (D) 85 (E) 89

Solution: Since average is the sum of the test scores divided by the number of tests taken, we know that

$$\frac{88+73+81+83+79+94+x}{7} = 83$$

or $498 + x = 7 \times 83$ so x = 83.

16. On Mars, I go to a shopping mall. I have 2 Oompas to spend. 1 Oompa is equal to 18 Loompas. 3 Loompas is equal to 16 Lampas. 4 Lampas is equal to 14 Mampas. If I buy a pair of shoes for 500 Mampas, how many Mampas should I get back in change?

(A) 172 (B) 198 (C) 210 (D) 224 (E) 248

Solution: Using the exchange rates, we see that two Oompas are equivalent to $2 \times 18 = 36$ Loompas, 36 Loompas are equivalent to $36 \times \frac{16}{3} = 192$ Lampas, and 192 Lampas are equivalent to $192 \times \frac{14}{4} = 672$ Mampas. Therefore I should recieve 672 - 500 = 172 Mampas in change.

17. Catherine is in charge of making smoothies for a large party. To make the smoothies, she uses four types of fruit. She adds 4 cups of strawberries, 1.5 times that amount of blueberries, and an equal amount of both mangoes and bananas. If Catherine added a total of 16 cups of fruit to her smoothie, what percent was just mangoes?

(A) 12.5 (B) 16 (C) 18.75 (D) 20.25 (E) 25

Solution: If Catherine adds 4 cups of strawberries, then she adds $4 \times 1.5 = 6$ cups of blueberries, giving her a total of 10 cups of fruit so far. Then to get a total of 16 cups of fruit, she must add 3 cups of mangoes and 3 cups of bananas. Therefore the entire smoothie is $100 \times \frac{3}{16} = 18.75$ % mango.

18. Grace is creating an art project for school. She cuts out two congruent circles *A* and *B* of radius 4 and a smaller circle *C*. Grace glues the three these circles on top of a larger circle so that they are all tangent (touching at one point) to each other. What is the radius of *C*?



(A) 2 (B) $\frac{9}{4}$ (C) $\frac{8}{3}$ (D) $\frac{5}{2}$ (E) $\frac{12}{5}$

Solution:



Let *O* be the center of the large circle and let *x* be the radius of circle *x*. Since the radius of the large circle is 4 + 4 = 8, the distance *OC* is 8 - x. Since circles *B* and *C* are tangent, the distance *BC* is 4 + x. Since the distance *OB* is 4, we can apply now apply the Pythagorean theorem on right triangle *OBC*.

This tells us that $4^2 + (8 - x)^2 = (4 + x)^2$ or $16 + 64 - 16x + x^2 = 16 + 8x + x^2$. Simplifying, we see that 24x = 64 so $x = \left\lfloor \frac{8}{3} \right\rfloor$.

19. If $x@y = \frac{1}{x} + \frac{1}{y}$ then what is the value of 2@(6@9)? (A) $\frac{27}{10}$ (B) $\frac{7}{2}$ (C) $\frac{18}{5}$ (D) $\frac{39}{10}$ (E) $\frac{41}{10}$

Solution: We have $6@9 = \frac{1}{6} + \frac{1}{9} = \frac{5}{18}$. Therefore the answer is

$$2@\left(\frac{5}{18}\right) = \frac{1}{2} + \frac{1}{\frac{5}{18}} = \frac{1}{2} + \frac{18}{5} = \left\lfloor\frac{41}{10}\right\rfloor.$$

20. A palindrome is a whole number that reads the same forwards and backwards. If one neglects the colon, certain times displayed on a digital watch in 12-hour format are palindromes. Three examples are: 1:01, 4:44, and 12:21. How many times during a 12-hour period will be palindromes?

(A) 40 **(B)** 45 **(C)** 48 **(D)** 54 **(E)** 57

Solution: If the time has three digits in it, the outer digits must be the same and the middle digit can be anything. The outer digit can be any number from 1 to 9 so there are 9 choices for the outer digit. The inner digit can be anything from 1 to 5 (because there are 60 minutes per hour) so there are 5 choices for the inner digit. Therefore there are $9 \times 5 = 45$ possibilities here.

If the time has three digits, the first two digits must be 10, 11, or 12. If the first two digits are 10, then it must be 10:01. If the first two digits are 11, it must be 11:11. And

if the first two digits are 12, it must be 12:21. Therefore there are 3 more possibilities here so the answer is 45 + 3 = 48.

21. There are 4 men and 5 women in a small office, but only a group of 2 men and 2 women can go to a special conference. How many different groups can be formed from the office?

(A) 52 **(B)** 60 **(C)** 72 **(D)** 80 **(E)** 96

Solution: First lets choose the group of 2 men out of the 4 men in the office. There are 4 ways to choose the first man and 3 ways to choose the second so $4 \times 3 = 12$ ways to choose the two men in order. However the order of the men does not matter so we need to divide by 2, giving us 6 ways to select two men for a group. Similarly, the number of ways to select 2 women out of the 5 women in the office is $5 \times 4 \times \frac{1}{2} = 10$. Therefore the number of ways to select a group of 2 men and 2 women is $6 \times 10 = 60$.

22. What is the value of $1 + 3 + 5 + \cdots + 2017 + 2019 - 2 - 4 - 6 - \cdots - 2016 - 2018$? (A) -2020 (B) -1010 (C) 0 (D) 1010 (E) 2020

Solution: Rearranging the numbers, we can rewrite the sum as

 $(1) + (3-2) + (5-4) + \dots + (2017 - 2016) + (2019 - 2018).$

The difference in each group is equal to 1 and since each number from 2 to 2019 is arranged in a pair, there are $1 + \frac{2018}{2} = 1010$ total groups. Therefore the value of the sum is $1 \times 1010 = 1010$.

23. A 20-gallon container is filled halfway with a mixture that is 90% vinegar and 10% water. How many gallons of water must be added for the mixture to become 60% vinegar and 40% water?

(A) 5 **(B)** 8 **(C)** 10 **(D)** 12 **(E)** 15

Solution: Half of 20 gallons is 10 gallons, so the container must start off with 9 gallons of vinegar and 1 gallon of water. Suppose we add *x* gallons of water to the container. Then there will be 9 gallons of vinegar and x + 1 gallons of water so the percent vinegar in the container is $100 \times \frac{9}{(x+1)+9}$.

If this value equals 60 then $100 \times \frac{9}{(x+1)+9} = 60$ so $\frac{9}{10+x} = \frac{3}{5}$, or 45 = 30 + 3x. Therefore x = 5 gallons. 24. A bag of gummy bears contains 8 blue, 5 red, and 3 orange. You pick two at random by first picking one, eating it, and then picking the second. What is the probability (chance) that both gummy bears picked will be blue?

(A) $\frac{1}{5}$ (B) $\frac{7}{30}$ (C) $\frac{1}{2}$ (D) $\frac{8}{15}$ (E) $\frac{5}{8}$

Solution: Since there are 8 blue bears and 8 + 5 + 3 = 16 total bears, the probability that the first bear is blue is $\frac{8}{16} = \frac{1}{2}$. After we pick the first blue bear, there will be 7 blue bears and 15 total bears, so the probability that the second bear is blue is $\frac{7}{15}$. Therefore the probability that both the first and second bears are blue is $\frac{1}{2} \times \frac{7}{15} = \left\lceil \frac{7}{30} \right\rceil$.

25. If I draw 3 lines and 1 circle on a sheet of paper, what is the maximum number of points on the paper that lie on at least two of the figures?

(A) 9 (B) 11 (C) 14 (D) 19 (E) 20

Solution: If I start off with the 3 lines, the maximum number of intersections I can get is 3 (when none of the lines are parallel and all three do not pass through the same point). Now if I draw a circle, each line can intersect the circle at two more points so since there are three lines, we can get 6 more intersections. Therefore the most intersections we can get is 3 + 6 = 9.

- 26. The sum of four numbers w, x, y, and z is 64, where w < x < y < z. The sum of z and w is twice the mean of all the numbers. The number x is half the largest number. What is the value of y if w = 4?
 - (A) 8 (B) 10 (C) 18 (D) 26 (E) 30

Solution: If the sum of the four numbers is 64, we know that w + x + y + z = 64. Because w = 4, we must have x + y + z = 60. Now the sum of z and w is twice the mean of the all the numbers. Because the sum of the numbers is 64, their mean must be $64 \div 4 = 16$ so $z + w = 2 \times 16 = 32$. Since w = 4, we know that z = 28. Because x is half of the largest number, $x = \frac{1}{2} \times z = \frac{1}{2} \times 28 = 14$. Finally, because x + y + z = 60, we have y = 60 - x - z = 60 - 28 - 14 = 18.

- 27. Jane rolls a fair six-sided dice three times. What is the probability (chance) that the second roll is greater than both the first roll and the third roll?
 - (A) $\frac{145}{216}$ (B) $\frac{1}{3}$ (C) $\frac{5}{16}$ (D) $\frac{25}{72}$ (E) $\frac{55}{216}$

Solution: First we find the number of ways the second roll can be greater than both the first roll and third roll.

If the second roll is a 1, the first and third rolls cannot be smaller.

If the second roll is a 2, the first and third rolls must both be 1 so there is 1 way here.

If the second roll is a 3, the first and third rolls must be a 1 or 2 so there are $2 \times 2 = 4$ ways here.

If the second roll is a 4, the first and third rolls must be a 1, 2, or 3 so there are $3 \times 3 = 9$ ways here.

If the second roll is a 5, the first and third rolls must be a 1, 2, 3, or 4 so there are $4 \times 4 = 16$ ways here.

Finally, if the second roll is a 6, the first and third rolls must be a 1, 2, 3, 4, or 5 so there are $5 \times 5 = 25$ ways here.

Therefore there are 1 + 4 + 9 + 16 + 25 = 55 ways the second roll can be greater than both the first and third. Because there are $6 \times 6 \times 6$ ways to choose 3 numbers for the first, second, and third rolls, the probability that out of those three numbers, the second is greater than the first and third is 55/216.

28. How many different patterns can be made by shading exactly two of the nine squares in a 3 by 3 grid? Patterns that can be matched by reflections and/or rotations are not considered different. For example, the patterns shown below only count once.





Solution: If one of the corners is selected, there are 5 distinct ways to select the second square:



If no corners are selected there are, there are three distinct possibilities:

Therefore the total number of ways to color two squares is $5 + 3 = \boxed{8}$.

29. In the cube *ABCDEFGH* shown below, *J* is the midpoint of *FB* and *I* is the midpoint of *HD*. If the cube has a side length of 4, what is the area of the shaded rhombus?



Solution: One formula for the area of a rhombus is $\frac{1}{2} \times a \times b$ where *a* and *b* are the lengths of the diagonals of the rhombus. In this rhombus, the diagonals are *EC* and *IJ*.

Since *EC* is the space diagonal of the cube, by the Pythagorean theorem, it has length $\sqrt{4^2 + 4^2 + 4^2} = 4\sqrt{3}$. Since *I* and *J* are the midpoints of *FB* and *HD* respectively, the length of *IJ* is the same as the length of *FH*. By the pythagorean theorem, *FH* has length $\sqrt{4^2 + 4^2} = 4\sqrt{2}$. Therefore the area of the rhombus is $\frac{1}{2} \times 4\sqrt{3} \times 4\sqrt{2} = \frac{8\sqrt{6}}{8\sqrt{6}}$.

30. A bike leaves City A travelling at a speed of 10 mph (miles per hour). 21 minutes later a car leaves City A travelling in the same direction at 40 mph. How much distance has each vehicle traveled when they meet?

(A)
$$\frac{15}{2}$$
 (B) 7 (C) $\frac{41}{6}$ (D) $\frac{14}{3}$ (E) $\frac{19}{5}$

Solution: Suppose that *x* hours have elapsed since the first bike leaves city *A*. Then the the first bike will be $10 \times x$ miles away from the city. Since the second bike leaves $\frac{21}{60}$ hours after the first bike leaves, the second bike will be $40 \times \left(x - \frac{21}{60}\right)$ miles away from the city. When the two bikes meet, they will be the same distance from the city, so $10 \times x = 40 \times \left(x - \frac{21}{60}\right)$ or $x = 4\left(x - \frac{7}{20}\right)$. Expanding, we have $x = 4x - \frac{7}{5}$ so $x = \frac{7}{15}$. After $\frac{7}{15}$ hours, the first bike would have traveled $10 \times \frac{7}{15} = \frac{14}{3}$ miles.