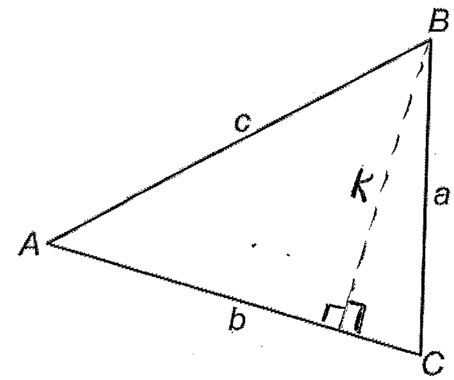
3.01 The Law of Sines

Name________________________

Right triangle trigonometry can be used to solve problems involving right triangles. However, many interesting problems involve non-right triangles. In this lesson, you will use right triangle trigonometry to develop the *Law of Sines*. The law of sines is important because it can be used to solve problems involving non-right triangles as well as right triangles.

→ non-right triangle

Consider oblique ΔABC shown to the right.



1. Sketch an altitude from vertex B.

2. Label the altitude k.

3. The altitude creates two right triangles inside ΔABC. Notice that $\angle A$ is contained in one of the right triangles, and $\angle C$ is contained in the other. Using right triangle trigonometry, write two equations, one involving sin A, and one involving sin C. (SOH-CAH-TOA)

$\sin A = \dfrac{k}{c}$ $\sin C = \dfrac{k}{a}$

4. Notice that each of the equations in Question 3 involves k. Why does this happen? Solve each equation for k.

$k = c(\sin A)$ $k = a(\sin C)$

5. Since both equations in Question 4 are equal to k, they can be set equal to each other. Why is this possible? Set the expressions equivalent to k equal to each other to form a new equation.

$a \sin C = c \sin A$

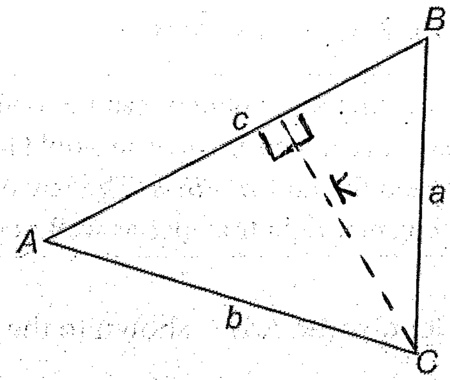
6. Notice that the equation in Question 5 no longer involves k. Rewrite the equation in Question 5, regrouping a with sin A and c with sin C.

$\dfrac{\sin C}{c} = \dfrac{\sin A}{a}$

Again, consider oblique ΔABC.

This time, sketch an altitude from vertex C.

Label the altitude k.



7. The altitude creates two right triangles inside ΔABC. Notice that $\angle A$ is contained in one of the right triangles and $\angle B$ is contained in the other. Using right triangle trigonometry, write two equations, one involving $\sin A$ and one involving $\sin B$.

$\sin A = \dfrac{k}{b}$ $\qquad\qquad$ $\sin B = \dfrac{k}{a}$

8. Again, each of the equations in Question 9 involves k. Solve each equation for k.

$k = b(\sin A)$ $\qquad\qquad$ $k = a(\sin B)$

9. Since both equations in Question 10 are equal to k, they can be set equal to each other. Set the equations equal to each other to form a new equation.

$b \sin A = a \sin B$

10. Now equation in Question 11 no longer involves k. Rewrite the equation in Question 11, regrouping a with $\sin A$ and b with $\sin B$.

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$

11. Use the equations in Question 6 and Question 12 to write a third equation involving b, c, $\sin B$, and $\sin C$.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Together, the equations in Questions 6, 12, and 13 form the *Law of Sines*. The law of sines is important, because it can be used to solve problems involving both right and non-right triangles, because it involves only the sides and angles of a triangle.

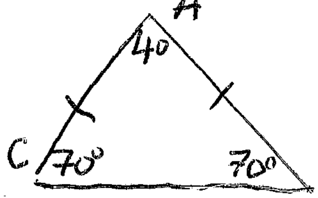
3.01 Law of Sines Practice Date: ____________

Draw and label each triangle. Then solve the triangle using the Law of Sines: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

1. A = 40° a = 20

Isosceles Δ B = 70° b = 29.238

C = 70° c = 29.238



$\frac{20}{\sin 40} = \frac{b}{\sin 70}$

$20\sin 70 = b\sin 40$

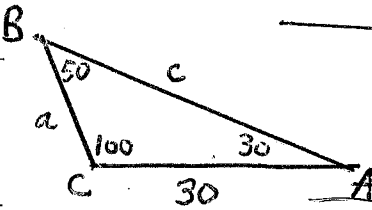
$\frac{20\sin 70}{\sin 40} = b$

$b = 29.238$

2. A = 30° a = 19.581

B = 50° b = 30

C = 100° c = 38.567



$\frac{a}{\sin 30} = \frac{30}{\sin 50}$

$a\sin 50 = 30\sin 30$

$a = \frac{30\sin 30}{\sin 50}$

$a = 19.581$

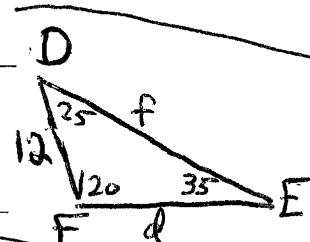
$\frac{c}{\sin 100} = \frac{30}{\sin 50} \rightarrow c = \frac{30\sin 100}{\sin 50}$

$c = 38.567$

3. D = 25° d = 8.842

E = 35° e = 12

F = 120° f = 18.118



$\frac{d}{\sin 25} = \frac{12}{\sin 35}$

$d = \frac{12\sin 25}{\sin 35} = 8.842$

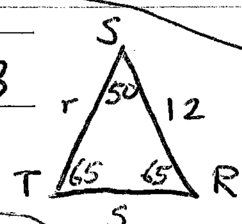
$\frac{f}{\sin 120} = \frac{12}{\sin 35}$

$f = \frac{12\sin 120}{\sin 35} = 18.118$

4. R = 65° r = 12

S = 50° s = 10.143

T = 65° t = 12



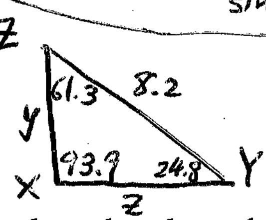
$\frac{r}{\sin 65} = \frac{12}{\sin 65}$ $r = 12$

$\frac{s}{\sin 50} = \frac{12}{\sin 65} \rightarrow s = \frac{12\sin 50}{\sin 65} = 10.143$

5. X = 93.9 x = 8.2

Y = 24.8° y = 3.447

Z = 61.3° z = 7.209



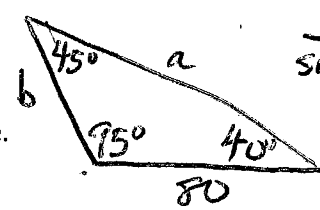
$\frac{y}{\sin 24.8} = \frac{8.2}{\sin 93.9} \rightarrow y = \frac{8.2\sin 24.8}{\sin 93.9} = 3.447$

$\frac{z}{\sin 61.3} = \frac{8.2}{\sin 93.9} \rightarrow z = \frac{8.2\sin 61.3}{\sin 93.9} = 7.209$

6. A landscaper wants to plant begonias along the edges of a triangular plot of land in Wills Park. Two of the angles of the triangle measure 95° and 40°. The side between these two angles is 80 feet long.

a) Find the measure of the third angle.

$180 - 95 - 40 = 45°$



b) Find the lengths of the other two sides of the triangle.

$\frac{a}{\sin 95} = \frac{80}{\sin 45}$

$a = \frac{80\sin 95}{\sin 45} =$

$\boxed{a = 112.707}$

$\frac{b}{\sin 40} = \frac{80}{\sin 45}$

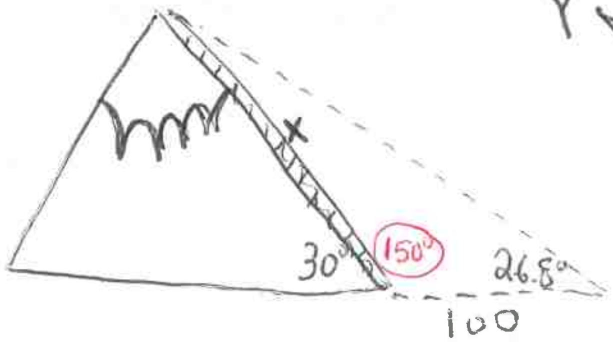
$b = \frac{80\sin 40}{\sin 45} = \boxed{72.723}$

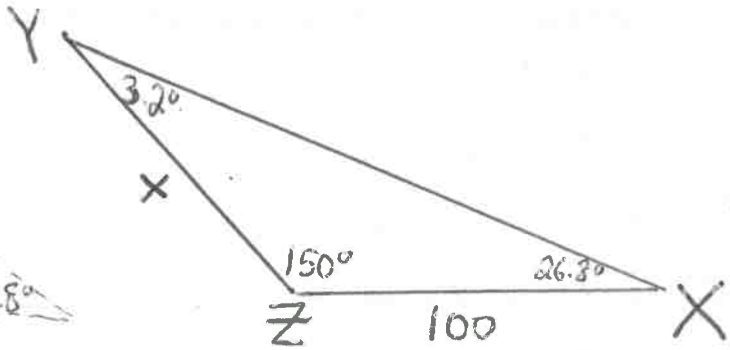
c) What is the perimeter of the triangular plot?

Perimeter = 80 + 112.707 + 72.723

$\boxed{\text{Perimeter} = 265.430\text{ft}}$

7. A cable car transports passengers up and down a mountain. The track used by the cable car has an angle of elevation of 30°. The angle of elevation from a point 100 feet from the base of the track (away from the mountain) to the top of the track is about 26.8°. Find the length of the track.





$$\frac{x}{\sin 26.8} = \frac{100}{\sin 3.2}$$

$$x = \frac{100 \cdot \sin 26.8}{\sin 3.2} = \boxed{807.713 \text{ ft.}}$$

Law of Sines:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

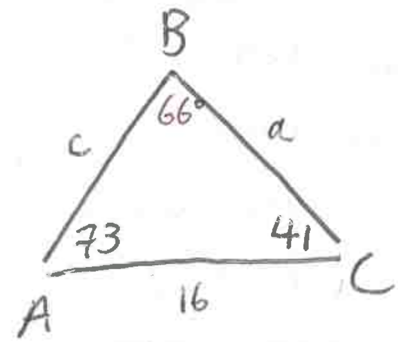
8) Check-In practice problem

Draw, label triangle, and solve

$A = 73°$ $a =$ ____

$B =$ ____ $b = 16$

$C = 41°$ $c =$ ____



$B = 66°$

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{a}{\sin 73} = \frac{16}{\sin 66}$$

$$a = \frac{16 \sin 73}{\sin 66}$$

$$\boxed{a = 16.749}$$

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$

$$\frac{c}{\sin 41} = \frac{16}{\sin 66}$$

$$c = \frac{16 \sin 41}{\sin 66}$$

$$\boxed{c = 11.49}$$