

3.02 Law of Sines - Ambiguous Case Notes

Name: _____

Remember: Law of Sines $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

In some cases, you'll be given a consecutive angle, side, and side (ASS). For these problems you have what's called The Ambiguous Case meaning: you don't know how many triangle solutions you'll have. There could be no triangle, one triangle, or even two triangles that work for your given measurements.

Here's what you do:

-Find your first missing angle (solution 1) using Law of Sines

-Check it to see if it makes sense

- given angle + solution 1 $\geq 180^\circ$ means **NO TRIANGLE**

- given angle + solution 1 $< 180^\circ$ means at least **1 TRIANGLE**

-If you have 1 triangle, you may have two. Find your second possible angle.

- 180° - solution 1 = second possible angle

- Check it to see if it makes sense

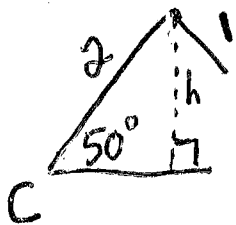
- given angle + solution 2 $\geq 180^\circ$ means this solution is no good; only **1 TRIANGLE**; solve

- given angle + solution 2 $< 180^\circ$ means **2 TRIANGLES**; make sure you solve both

Just remember: if you're given ASS, the problem is going to be a pain in the _____. ☹️

Examples:

1. $a = 2, c = 1, C = 50^\circ$



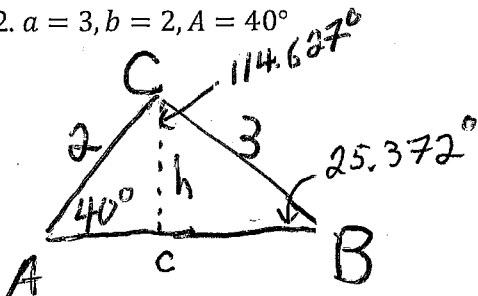
$$\sin 50 = \frac{h}{2}$$

$$h = 2 \sin 50$$

$$h = 1.532$$

No triangle

2. $a = 3, b = 2, A = 40^\circ$



$$\sin 40 = \frac{h}{2}$$

$$h = 1.286$$

$$\frac{2}{\sin B} = \frac{3}{\sin 40}$$

$$\frac{3 \sin B}{3} = \frac{2 \sin 40}{3}$$

$$\sin B = \frac{2 \sin 40}{3}$$

$$\sin B = 0.4285$$

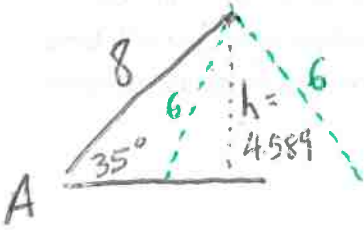
$$B = \sin^{-1}(0.4285) = 25.372^\circ$$

$$\frac{c}{\sin 114.627} = \frac{3}{\sin 40}$$

$$c = \frac{3 \sin 114.627}{\sin 40}$$

$$c = 4.242$$

$$3. a = 6, b = 8, A = 35^\circ$$



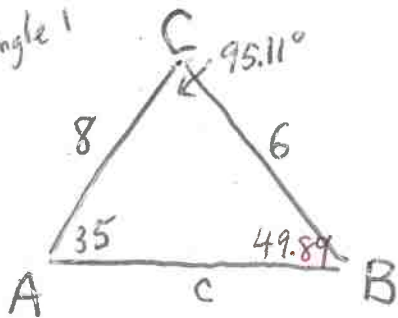
$$\sin 35 = \frac{h}{8}$$

$$h = 8 \sin 35$$

$$h = 4.589$$

2 triangles

Triangle 1



$$\frac{8}{\sin B} = \frac{6}{\sin 35}$$

$$6 \sin B = 8 \sin 35$$

$$\sin B = \frac{8 \sin 35}{6}$$

$$\sin B = 0.7647$$

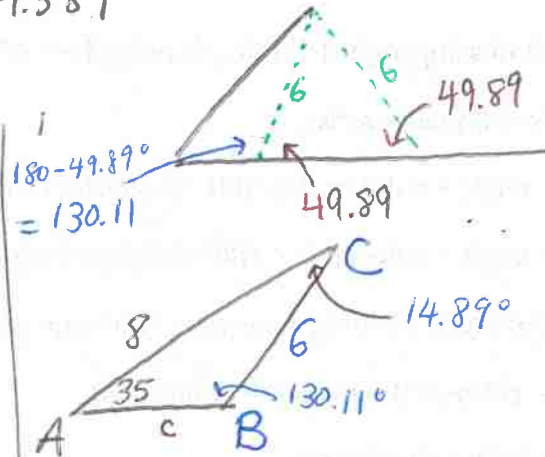
$$B = \sin^{-1}(0.7647)$$

$$B = 49.89^\circ$$

$$\frac{c}{\sin 95.11} = \frac{6}{\sin 35}$$

$$c = \frac{6 \sin 95.11}{\sin 35}$$

$$c = 10.419$$



$$\frac{c}{\sin 14.89} = \frac{6}{\sin 35}$$

$$c = \frac{6 \sin 14.89}{\sin 35}$$

$$c = 2.688$$

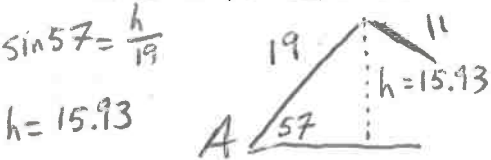
APC 3.02 Law of Sines - Ambiguous Case Practice

Date: _____

a) Determine the number of solutions for triangle ABC. b) If there is only 1 solution, solve the triangle.

1. $A = 57^\circ, a = 11, b = 19$

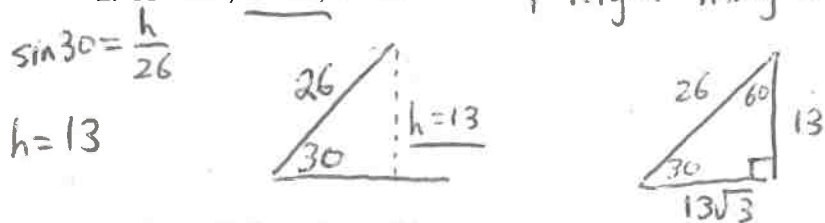
a) 0 Triangles



2. $A = 30^\circ, a = 13, b = 26$

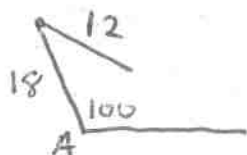
1 Right triangle.

a) 1 Triangles



3. $A = 100^\circ, b = 18, a = 12$

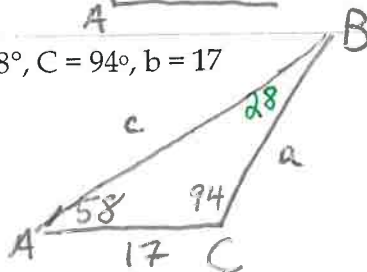
a) 0 Triangles



4. $A = 58^\circ, C = 94^\circ, b = 17$

$$\frac{a}{\sin 58} = \frac{17}{\sin 28} \rightarrow a = \frac{17 \sin 58}{\sin 28}$$

a) 1 Triangles
 $a = 30.709$

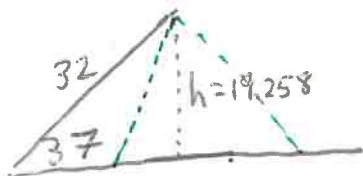


$$\frac{c}{\sin 94} = \frac{17}{\sin 28} \rightarrow c = \frac{17 \sin 94}{\sin 28}$$

$c = 36.123$

5. $A = 37^\circ, a = 27, b = 32$

a) 2 Triangles



$$\sin 37 = \frac{h}{32}$$

$$h = 32 \sin 37 = 19.258$$

6. $A = 65^\circ, a = 55, b = 57$

a) 2 Triangles



$$\sin 65 = \frac{h}{57} \quad h = 57 \sin 65 = 51.659$$