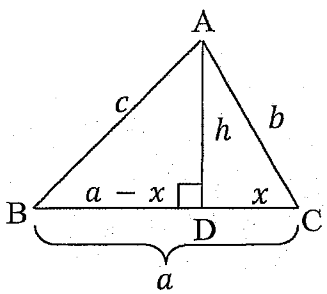
Accel Pre-Calculus
3.04 Notes: Law of Cosines

Let's look at that non-right triangle with the altitude again. The altitude breaks the side it intersects into two lengths, x and $a - x$. Apply the Pythagorean Theorem to ΔADB:



$$c^2 = (a - x)^2 + h^2$$

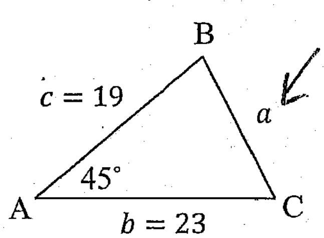
Law of Cosines:

$$c^2 = a^2 + b^2 - 2ab \cos C$$
$$b^2 = a^2 + c^2 - 2ac \cos B$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$

The law of cosines allows us to solve for oblique (non-right) triangles when we have 3 side lengths (SSS) or 2 side lengths and the included angle's measure (SAS).

Examples: Solve ΔABC. Round all answers to the nearest tenth.

1. $A = 45°, b = 23, and\ c = 19.$



$a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 = 23^2 + 19^2 - 2(23)(19)\cos 45$

$a^2 = 271.988$

$a = 16.492$

To find an angle, use Law of Sines:

$\dfrac{16.492}{\sin 45} = \dfrac{23}{\sin B}$

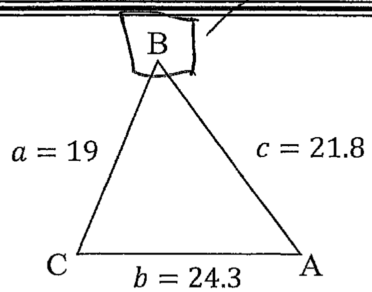
$\sin B = \dfrac{23 \sin 45}{16.492} \rightarrow B = 80.45°$

$\angle C = 180 - 80.45° - 45°$

$\angle C = 54.549°$

2. $a = 19, b = 24.3, \textit{and } c = 21.8$

start here



SSS information – ***find the angle opposite the longest side:***

$b^2 = a^2 + c^2 - 2ac\cos B$

$24.3^2 = 19^2 + 21.8^2 - 2(19)(21.8)\cos B$

$0.2966 = \cos B \rightarrow \underline{B = 72.743^\circ}$

To find another angle use the Law of Sines:

$\dfrac{19}{\sin A} = \dfrac{24.3}{\sin 72.743}$

$\sin A = \dfrac{19\sin 72.743}{24.3}$

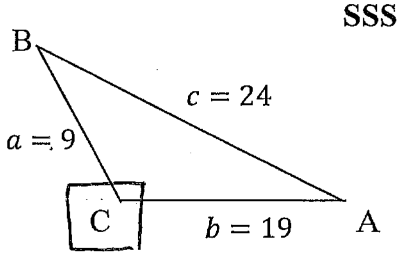
$\underline{A = 48.305^\circ}$

To find the last angle, subtract from 180°:

$\angle C = 180 - 72.743 - 48.305^\circ$

$\boxed{\angle C = 58.952^\circ}$

3. $a = 9, b = 19, c = 24$



SSS information – ***find the angle opposite the longest side:***

$c^2 = a^2 + b^2 - 2ab\cos C$

$24^2 = 9^2 + 19^2 - (2\cdot 9\cdot 19\cos C)$

$134 = -342\cos C$

$\underline{C = 113.067^\circ}$

To find another angle use the Law of Sines:

$\dfrac{9}{\sin A} = \dfrac{24}{\sin 113.067}$

$\sin A = \dfrac{9\sin 113.067}{24}$

To find the last angle, subtract from 180°:

$\underline{A = 20.183^\circ}$

$\angle B = 180 - 113.067^\circ - 20.183^\circ$

$\boxed{\angle B = 46.75^\circ}$

Accelerated Pre-Calculus

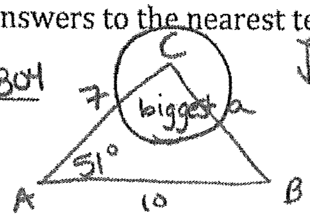
Law of Cosines Practice Key

Law of Cosines:

$$a^2 = b^2 + c^2 - 2bc\cos A \qquad \cos A = \frac{b^2+c^2-a^2}{2bc}$$

Solve each triangle. Round answers to the nearest tenth.

1. A = 51° ②B = 44.197° ③C = 84.803° | ① a = 7.804, b = 7, c = 10



$\sqrt{a^2} = \sqrt{7^2+10^2-2(7)(10)\cos 51^\circ} \approx 7.804$

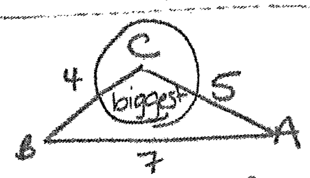
$\frac{\sin B}{7} = \frac{\sin 51^\circ}{a}$ $B = \sin^{-1}\left(\frac{7\sin 51^\circ}{\text{"ans"}}\right)$

$C = 180 - A - B$

SAS, Law of Cosines

2. A = 34.038° B = 44.415° ① C = 101.537° | a = 4, b = 5, c = 7

first or last!



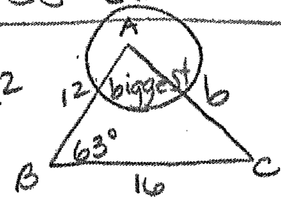
$7^2 = 4^2+5^2-2(4)(5)\cos C$ $C = \cos^{-1}\left(\frac{49-16-25}{-40}\right)$

$A = \sin^{-1}\left(\frac{a\sin C}{c}\right) = \sin^{-1}\left(\frac{4\sin \text{"ans"}}{7}\right)$

$B = 180 - A - C$

SSS Law of Cosines

3. ③A = 71.623° B = 63° ② C = 45.377° | a = 16, ① b = 15.022, c = 12



$b = \sqrt{12^2+16^2-2(12)(16)\cos 63^\circ}$

$C = \sin^{-1}\left(\frac{c\sin B}{b}\right) = \sin^{-1}\left(\frac{12\sin 63^\circ}{\text{"ans"}}\right)$

$A = 180 - B - C$

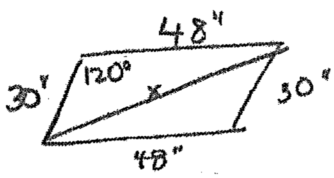
SAS, Law of Cosines

4. The sides of a triangle measure 14.9 cm, 23.8 cm and 36.9 cm. Find the angle with the least measure.

$A = \cos^{-1}\left(\frac{14.9^2-23.8^2-36.9^2}{-2(23.8)(36.9)}\right) \approx \boxed{13.759^\circ}$

across from shortest side

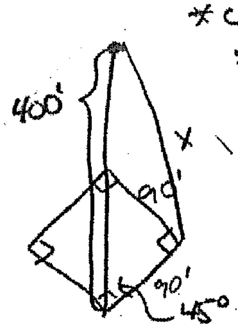
5. The lengths of two sides of a parallelogram are 48 inches and 30 inches. One angle measures 120°. Find the length of the longer diagonal.



$x = \sqrt{30^2+48^2-2(30)(48)\cos 120^\circ} \approx \boxed{68.147\text{ in}}$

6. In baseball, dead center field is the farthest point in the outfield on the straight line through home plate and second base. The distance between consecutive bases is 90 feet. In Wrigley Field in Chicago, dead center field is 400 feet from home plate. How far is dead center field from first base?

* corners of baseball diamond are 90° each, so bisecting homeplate makes a 45° angle



$x = \sqrt{400^2+90^2-2(400)(90)\cos 45^\circ}$

$\boxed{x \approx 342.328\text{ ft.}}$

Solve the following triangles using the law of sines and/or the law of cosines. 1) Draw and label each triangle. 2) Determine the # of solutions. 3) Then solve each triangle. Round answers to the nearest ~~tenth?~~ thousandth

7. P = 48°, Q = 96°, r = 12.1

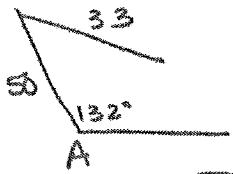
ASA → Law of Sines 1 Δ

$R = 180 - 48 - 96 = \boxed{36°}$

$p = \dfrac{12.1 \sin 48°}{\sin 36°}$ $q = \dfrac{12.1 \sin 96°}{\sin 36°}$

$\boxed{p \approx 15.298}$ $\boxed{q \approx 20.473}$

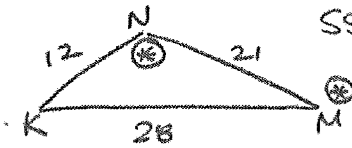
8. A = 132°, a = 33, b = 50



ASS w/ obtuse angle opposite side MUST be longest side but 33 < 50

$\boxed{\text{so, no } \Delta \text{ exists}}$

9. k = 21, m = 12, n = 28



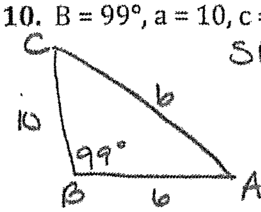
SSS 12+21 > 28 ✓
1 Δ
(*) find largest angle 1st

$N = \cos^{-1}\left(\dfrac{28^2 - 12^2 - 21^2}{-2(12)(21)}\right) \approx \boxed{113.256°}$

$K = \sin^{-1}\left(\dfrac{21 \sin N}{28}\right) \approx \boxed{43.556°}$

$M = 180° - N - K \approx \boxed{23.188°}$

10. B = 99°, a = 10, c = 6



SAS Law of Cosines 1 Δ

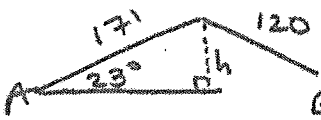
$b = \sqrt{10^2 + 6^2 - 2(10)(6)\cos 99°}$

$\boxed{b \approx 12.441}$

$A = \sin^{-1}\left(\dfrac{10 \sin 99}{b}\right) \approx \boxed{52.553°}$

$C = 180° - 99° - A \approx \boxed{28.447°}$

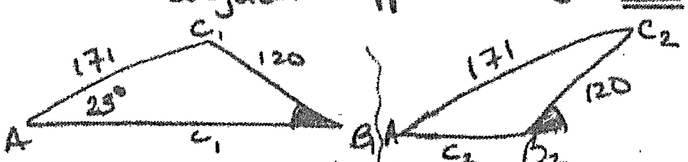
11. A = 23°, a = 120, b = 171



ASS with adj > opposite so check height

$h = 171 \sin 23° \approx 66.8$

adjacent > opposite > height: 2Δs



$B_1 = \sin^{-1}\left(\dfrac{171 \sin 23°}{120}\right)$

$\boxed{B_1 \approx 33.834°}$

$C_1 = 180 - 23 - B_1$

$\boxed{C_1 \approx 123.166°}$

$c_1 = \dfrac{120 \sin C_1}{\sin 23°}$

$\boxed{c_1 \approx 257.085}$

$B_2 = 180° - B_1$

$\boxed{B_2 \approx 146.166°}$

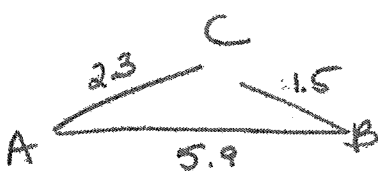
$C_2 = 180 - 23 - B_2$

$\boxed{C_2 \approx 10.834°}$

$c_1 = \dfrac{120 \sin C_2}{\sin 23°}$

$\boxed{c_1 \approx 57.728}$

12. a = 1.5, b = 2.3, c = 5.9



SSS but $\underbrace{1.5 + 2.3}_{3.8} \not> 5.9$

3.8 cannot reach across 5.9

$\boxed{\text{so, no } \Delta \text{ exists}}$