3.09 Applications of Law of Sines and Cosines, Area of Triangles Date _____

Law of Sines:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Law of Cosines:

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

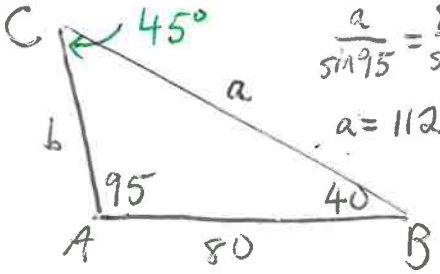
$$c^2 = a^2 + b^2 - 2ab\cos C$$

Area:

SAS: $\frac{1}{2}ab\sin C = \frac{1}{2}bc\sin A = \frac{1}{2}ac\sin B$

SSS: $\sqrt{s(s-a)(s-b)(s-c)}$, *where* $s = \frac{(a+b+c)}{2}$

1. The Alpharetta Community Garden Network wants to plant vegetables in a triangular plot of land in Wills Park. Two of the angles of the triangle measure 95° and 40°. The side between these two angles is 80 feet long.



$\frac{a}{\sin 95} = \frac{80}{\sin 45}$ $\quad a = 112.707$ $\quad \frac{b}{\sin 40} = \frac{80}{\sin 45}$ $\quad b = 72.723$

a) Find the measure of the third angle. 45°

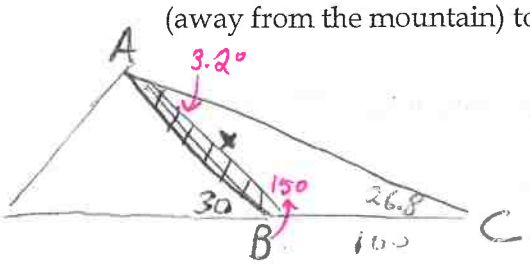
b) Find the lengths of the other two sides of the triangle.

$a = 112.707 \quad b = 72.723$

c) What is the perimeter of the triangular plot?

265.430 feet

2. A cable car transports passengers up and down a mountain. The track used by the cable car has an angle of elevation of 30°. The angle of elevation from a point 100 feet from the base of the track (away from the mountain) to the top of the track is about 26.8°. Find the length of the track.

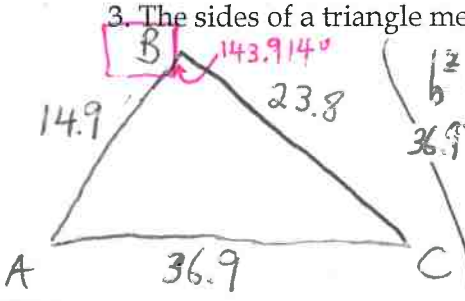


$\frac{x}{\sin 26.8} = \frac{100}{\sin 3.2}$

$x = \frac{100\sin 26.8}{\sin 3.2}$

$x = 807.7$ feet

3. The sides of a triangle measure 14.9 cm, 23.8 cm and 36.9 cm. Find the angle with the least measure.



$b^2 = a^2 + c^2 - 2ac\cos B$

$36.9^2 = 23.8^2 + 14.9^2 - 2(23.8)(14.9)\cos B$

$573.16 = -709.24\cos B$

$-0.808 = \cos B$

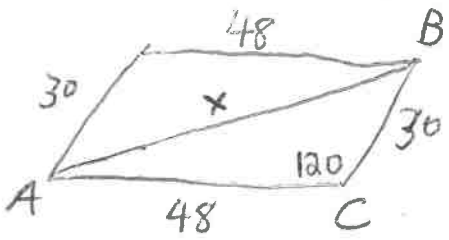
$B = \cos^{-1}(-0.808) = 143.914°$

$\frac{\sin C}{14.9} = \frac{\sin 143.914}{36.9}$

$\sin C = \frac{14.9\sin 143.914}{36.9}$

$C = \sin^{-1}(0.2378) = 13.759°$

4. The lengths of two sides of a parallelogram are 48 inches and 30 inches. One angle measures 120°. Find the length of the longer diagonal.

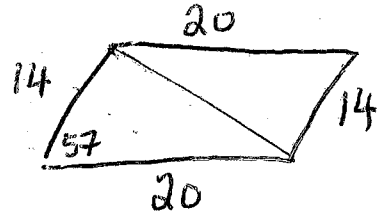


$x^2 = 30^2 + 48^2 - 2(30)(48)\cos 120$

$x^2 = 4644$

$x = 68.147$ inches

5. The adjacent sides of a parallelogram measure 14 cm and 20 cm and one angle measures 57°. Find the area of the parallelogram.

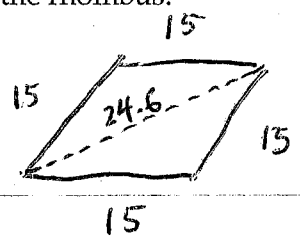


Area (of triangle) $= \frac{1}{2}(14)(20)\sin 57$

$= 117.414$

Area (of parallelogram) $= \boxed{234.828\ cm^2}$

6. The side of a rhombus is 15 cm long and the length of the longer diagonal is 24.6 cm. Find the area of the rhombus.



*555

$s = \frac{1}{2}(15+15+24.6)$

$s = 27.3$

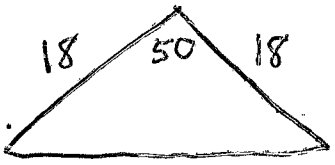
$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{27.3(27.3-15)(27.3-15)(27.3-24.6)}$

$= 105.601$

Area (rhombus) $= 2(105.601) = \boxed{211.2\ cm^2}$

7. The roof on a house has one side that is in the shape of an isosceles triangle. The sides of this part of the roof are 18 feet long and the angle at the peak is 50°. Find the area of this part of the roof.



$\text{Area} = \frac{1}{2}(18)(18)\sin 50$

$\boxed{\text{Area} = 124.099\ ft^2}$