Chapter 3 Curve Sketching 3.1 EVT Classwork Problems

Finding Extrema on a Closed Interval In Exercises 17–36, find the absolute extrema of the function on the closed interval.

19. $g(x) = 2x^2 - 8x, \quad [0, 6]$

21. $f(x) = x^3 - \frac{3}{2}x^2, \ [-1, 2]$

23. $y = 3x^{2/3} - 2x, \ [-1, 1]$

24. $g(x) = \sqrt[3]{x}, \ [-8, 8]$

26. $f(x) = \dfrac{2x}{x^2 + 1}, \ [-2, 2]$

28. $h(t) = \dfrac{t}{t + 3}, \ [-1, 6]$

Chapter 3 Curve Sketching 3.2 MVT and Rolle's Classwork Problems

If the Mean Value Theorem cannot be applied, explain why not.

39. $f(x) = x^3 + 2x, \quad [-1, 1]$

40. $f(x) = x^4 - 8x, \quad [0, 2]$

41. $f(x) = x^{2/3}, \quad [0, 1]$

42. $f(x) = \dfrac{x+1}{x}, \quad [-1, 2]$

35. Mean Value Theorem Consider the graph of the function $f(x) = -x^2 + 5$ (see figure on next page).

(a) Find the equation of the secant line joining the points $(-1, 4)$ and $(2, 1)$.

(b) Use the Mean Value Theorem to determine a point c in the interval $(-1, 2)$ such that the tangent line at c is parallel to the secant line.

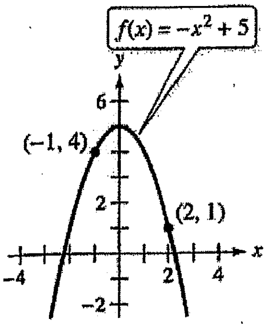

Figure for 35

Chapter 3 Curve Sketching 3.2 Rolle's Classwork Problems

Using Rolle's Theorem In Exercises 9–22, determine whether Rolle's Theorem can be applied to f on the closed interval $[a, b]$. If Rolle's Theorem can be applied, find all values of c in the open interval (a, b) such that $f'(c) = 0$. If Rolle's Theorem cannot be applied, explain why not.

9. $f(x) = -x^2 + 3x, \quad [0, 3]$

10. $f(x) = x^2 - 8x + 5, \quad [2, 6]$

13. $f(x) = x^{2/3} - 1, \quad [-8, 8]$

15. $f(x) = \dfrac{x^2 - 2x - 3}{x + 2}, \quad [-1, 3]$

16. $f(x) = \dfrac{x^2 - 1}{x}, \quad [-1, 1]$

Chapter 3 Curve Sketching 3.1 EVT Classwork Problems

Key

Finding Extrema on a Closed Interval In Exercises 17–36, find the absolute extrema of the function on the closed interval.

19. $g(x) = 2x^2 - 8x,\ [0, 6]$

$g(x)$ continuous $[0, 6]$

$g'(x) = 4x - 8$

$0 = 4x - 8$

$8 = 4x$

$x = 2$

$g(0) = 0$

$g(2) = -8$

$g(6) = 24$

Abs max is 24 at $x=6$

Abs min is -8 at $x=2$

21. $f(x) = x^3 - \frac{3}{2}x^2,\ [-1, 2]$

$f(x)$ continuous $[-1, 2]$

$f'(x) = 3x^2 - 3x$

$0 = 3x(x-1)$

$x = 0, 1$

$f(-1) = -5/2$

$f(0) = 0$

$f(1) = -1/2$

$f(2) = 2$

Abs max is 2 at $x=2$

Abs min is $-5/2$ at $x=-1$

23. $y = 3x^{2/3} - 2x,\ [-1, 1]$

$f(x)$ continuous on $[-1, 1]$

*set denom = 0

$x = 0$

$y'(x) = 3 \cdot \frac{2}{3}x^{-1/3} - 2$

$y'(x) = \frac{2}{x^{1/3}} - 2$

$2 = \frac{2}{x^{1/3}}$

$2x^{1/3} = 2$

$x^{1/3} = 1$

$x = 1$

$f(0) = 0$

$f(-1) = 5$

$f(1) = 1$

Abs max is 5 at $x=-1$

Abs min is 0 at $x=0$

24. $g(x) = \sqrt[3]{x},\ [-8, 8]$

$g(x)$ continuous $[-8, 8]$

$g(x) = x^{1/3}$

$g'(x) = \frac{1}{3}x^{-2/3}$

$g'(x) = \frac{1}{3x^{2/3}}$

$x = 0$

$g(-8) = \sqrt[3]{-8} = -2$

$g(0) = 0$

$g(8) = \sqrt[3]{8} = 2$

Abs max is 2 at $x=8$

Abs min is -2 at $x=-8$

26. $f(x) = \frac{2x}{x^2 + 1},\ [-2, 2]$

$f(x)$ continuous $[-2, 2]$

$$f'(x) = \frac{\overbrace{2}^{f'}\overbrace{(x^2+1)}^{g} - \overbrace{2x}^{f}\overbrace{(2x)}^{g'}}{\underbrace{(x^2+1)^2}_{g^2}} = \frac{2x^2+2-4x^2}{(x^2+1)^2}$$

$f'(x) = \frac{2-2x^2}{(x^2+1)^2}$

$2 - 2x^2 = 0$

$2 = 2x^2$

$x^2 = 1$

$x = \pm 1$

$f(-2) = -4/5$ $f(1) = 1$

$f(-1) = -1$ $f(2) = 4/5$

Abs max is 1 at $x=1$

Abs min is -1 at $x=-1$

28. $h(t) = \frac{t}{t + 3},\ [-1, 6]$

VA: $t = -3$

$h(t)$ continuous $[-1, 6]$

$h'(t) = \frac{(1)(t+3) - (t)(1)}{(t+3)^2}$

$h'(t) = \frac{t+3-t}{(t+3)^2} = \frac{3}{(t+3)^2}$

No critical point

$0 \neq \frac{+3}{(t+3)^2}$

$h(-1) = -1/2$

$h(6) = 2/3$

Abs max is $2/3$ at $t=6$

Abs min is $-1/2$ at $t=-1$

$\boxed{\text{Key}}$

Chapter 3 Curve Sketching 3.2 MVT and Rolle's Classwork Problems

If the Mean Value Theorem cannot be applied, explain why not. $f'(c) = \frac{f(b)-f(a)}{b-a}$

39. $f(x) = x^3 + 2x, \quad [-1, 1]$

f(x) continuous $[-1,1]$, differentiable $(-1,1)$

$f(-1) = -3$

$f(1) = 3$

slope: $\frac{3 - -3}{1 - -1} = \frac{6}{2}$

slope $= 3$

$f'(x) = 3x^2 + 2$

$3 = 3x^2 + 2$

$1 = 3x^2$

$\frac{1}{3} = x^2$

$x = \pm\sqrt{1/3}$

$\boxed{c = \sqrt{\tfrac{1}{3}}, -\sqrt{\tfrac{1}{3}}}$

40. $f(x) = x^4 - 8x, \quad [0, 2]$

f(x) continuous $[0,2]$, differentiable $(0,2)$

$f(0) = 0$

$f(2) = 0$

slope: $\frac{0-0}{2-0} = 0$

$f'(x) = 4x^3 - 8$

$0 = 4x^3 - 8$

$8 = 4x^3$

$2 = x^3$

$x = \sqrt[3]{2}$

$\boxed{c = \sqrt[3]{2}}$

41. $f(x) = x^{2/3}, \quad [0, 1]$

f(x) continuous $[0,1]$, differentiable $(0,1)$

$f(0) = 0$

$f(1) = 1$

slope:

$m = \frac{1-0}{1-0} = 1$

$f'(x) = \frac{2}{3}x^{-1/3}$

$1 = \frac{2}{3x^{1/3}}$

$3x^{1/3} = 2$

$x^{1/3} = \frac{2}{3}$

$(x^{1/3})^3 = \left(\frac{2}{3}\right)^3$

$x = \frac{8}{27}$

$\boxed{c = \frac{8}{27}}$

42. $f(x) = \frac{x+1}{x}, \quad [-1, 2]$

VA. at $x = 0$

f(x) not continuous $[-1,2]$

MVT does not apply.

35. Mean Value Theorem Consider the graph of the function $f(x) = -x^2 + 5$ (see figure on next page).

(a) Find the equation of the secant line joining the points $(-1, 4)$ and $(2, 1)$.

$\boxed{\text{(b)}}$ Use the Mean Value Theorem to determine a point c in the interval $(-1, 2)$ such that the tangent line at c is parallel to the secant line. f(x) continuous $[-1,2]$, differentiable $(-1,2)$

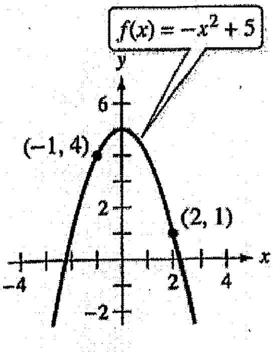

Figure for 35

$f(-1) = 4$

$f(2) = 1$

Slope: $m = \frac{1-4}{2-(-1)}$

$m = \frac{-3}{3} = -1$

$f(x) = -x^2 + 5$

$f'(x) = -2x$

$-1 = -2x$

$\frac{1}{2} = x$

$\boxed{c = 1/2}$

Chapter 3 Curve Sketching 3.2 Rolle's Classwork Problems

Key

Using Rolle's Theorem In Exercises 9–22, determine whether Rolle's Theorem can be applied to f on the closed interval $[a, b]$. If Rolle's Theorem can be applied, find all values of c in the open interval (a, b) such that $f'(c) = 0$. If Rolle's Theorem cannot be applied, explain why not.

9. $f(x) = -x^2 + 3x, \quad [0, 3]$

f(x) continuous [0,3], differentiable (0,3)

$f(0)=0$

$f(3)=0$

$f'(x)=-2x+3$

$0=-2x+3$

$2x=3$

$x=3/2$

$\boxed{c=\frac{3}{2}}$

10. $f(x) = x^2 - 8x + 5, \quad [2, 6]$

f(x) continuous [2,6], differentiable (2,6)

$f(2)=2^2-16+5=-7$

$f(6)=6^2-48=-7$

$m=\frac{-7-(-7)}{6-2}=\frac{0}{4}=\underline{0}$

$f'(x)=2x-8$

$0=2x-8$

$8=2x$

$\frac{8}{2}=x$

$\boxed{c=4}$

13. $f(x) = x^{2/3} - 1, \quad [-8, 8]$

f(x) continuous [-8,8] f(x) not differentiable (-8,8)

$f(-8)=3$

$f(8)=3$

$f'(x)=\frac{2}{3}x^{-1/3}$

$0=\frac{2}{3x^{1/3}}$

$2 \neq 0$

Rolle's theorem does not apply, f(x) not differentiable at $x=0$

15. $f(x) = \dfrac{x^2 - 2x - 3}{x + 2}, \quad [-1, 3]$

f(x) continuous [-1,3], differentiable (-1,3)

$f(-1)=0$

$f(3)=0$

$f'(x)=\dfrac{\overbrace{(2x-2)}^{f'}\overbrace{(x+2)}^{g}-\overbrace{(x^2-2x-3)}^{f}\overbrace{(1)}^{g'}}{\underbrace{(x+2)^2}_{g^2}}$

$f'(x)=\dfrac{2x^2+4x-2x-4-x^2+2x+3}{(x+2)^2}$

$f'(x)=\dfrac{x^2+4x-1}{(x+2)^2}$

$x^2+4x-1=0$

$\dfrac{-4\pm\sqrt{4^2-4(1)(-1)}}{2(1)}$

$x=-2\pm\sqrt{5}$

$\boxed{c=-2+\sqrt{5}}$

16. $f(x) = \dfrac{x^2 - 1}{x}, \quad [-1, 1]$

V.A. at $x=0$

f(x) not continuous [-1,1]

Rolle's Theorem does not apply.