Calculus Ch. 3.1-3.4 Concept Review Worksheet

I. **Extreme Value Theorem:** If a function is ___________ on a closed interval, then it has **both** an ________________ and an __________________ on the _____________

Steps for EVT:

1. Find critical values by ___________________________________
2. Confirm that the critical values are located _______________________
3. Find critical values by setting ____________ and _______________ equal to zero.
4. Find absolute extrema by plugging in _________________ and _______________ into f(x)
5. Max and Min values refer to the ________ values

II. **Mean Value Theorem**

If a function f(x) is ________________ on a, b and _________________ on a, b

then there must be at least one point, c in a, b where the slope of the _____________ is equal to the

slope of the _____________________

MVT Formula:

Steps to determine if function is continuous and differentiable in the interval (applicable for MVT And Rolle's)

a. To determine if whether function is not continuous on the closed interval, look for ______________ in the __________________ of the function
b. To determine whether function is not differentiable(sharp points, undefined slope) on the open interval, look for _________________ in the ________________ of the derivative function.
c.

III. **Rolle's Theorem**

If a function f(x) is ________________ on a, b and ______________ on a, b and __________

then there must be at least one point, c in a, b where the slope of the ___________ is equal to ___

IV. First Derivative Test

1. Find critical points by setting ____________________
2. Remember, critical points can come from the ______________ and the ______________ of the f '(x) derivative function
3. Put all critical points on a sign/number line
4. Choose values in each interval and plug into the ________________ to determine __________
5. Because Statements
   a. f(x) increasing in interval (a,b) b/c ______________________
   b. f(x) decreasing in interval (a,b) b/c ______________________
   c. Relative max at (**a, f(a)**) b/c ______________________
   d. Relative min at (**a, f(a)**) b/c ______________________

V. "Concavity Test"

1. Find critical points by setting ____________________
2. Remember, critical points can come from the ______________ and the ______________ of the ______________ function
3. Put all critical points on a sign/number line
4. Choose values in each interval and plug into the ________________ to determine __________
5. Because Statements
   a. Graph is Concave up in interval (a,b) b/c __________________
   b. Concave down in interval (a,b) b/c __________________
   c. Point of Inflection at (**a, f(a)**) b/c __________________

VI. 2nd Derivative Test

1. *The 2nd derivative test is a test for __________________and NOT for __________________
2. *The 2nd derivative test achieves the same as the 1st derivative test.

Steps:

a. Find critical points from ______________________
b. Take critical points and plug into the ______________________________
c. If $f'(a) = 0$ and $f''(a) > 0$, then there is a relative _______________ at $x = a$
d. If $f'(b) = 0$ and $f''(b) < 0$, then there is a relative _______________ at $x = b$
e. If $f'(c) = 0$ and $f''(c) = 0$, then ______________________

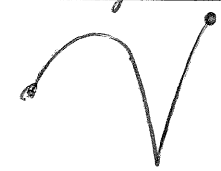
Calculus Ch. 3.1-3.4 **Concept Review Worksheet**

Key

I. **Extreme Value Theorem:** If a function is Continuous on a closed interval, then it has **both** an absolute max and an absolute min on the closed interval

Steps for EVT:

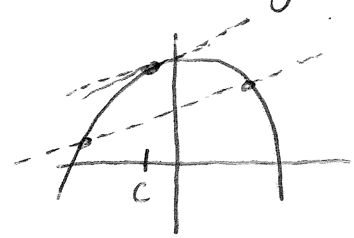
1. Find critical values by setting $f'(x)=0$ and denominator of $f'(x)=0$
2. Confirm that the critical values are located on the closed interval
3. Find critical values by setting $f'(x)$ numerator and $f'(x)$ denominator equal to zero.
4. Find absolute extrema by plugging in critical values and endpoints into f(x)
5. Max and Min values refer to the y- values



II. **Mean Value Theorem**

If a function f(x) is continuous on [a, b] and differentiable on (a, b) then there must be at least one point, c in (a, b) where the slope of the tangent is equal to the slope of the secant line

MVT Formula: $\boxed{f'(c) = \frac{f(b)-f(a)}{b-a}}$



Steps to determine if function is continuous and differentiable in the interval (applicable for MVT And Rolle's)

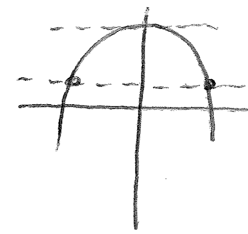
a. To determine if whether function is not continuous on the closed interval, look for V.A., holes in the denominator of the function f(x)

b. To determine whether function is not differentiable(sharp points, undefined slope) on the open interval, look for critical values in the denominator of the derivative function.

III. **Rolle's Theorem**

If a function f(x) is continuous on [a, b] and differentiable on (a, b) and $f(a)=f(b)$ then there must be at least one point, c in (a, b) where the slope of the tangent is equal to 0

$\boxed{f'(c)=0}$



IV. First Derivative Test

1. Find critical points by setting $f'(x)=0$
2. Remember, critical points can come from the numerator and the denominator of the f '(x) derivative function
3. Put all critical points on a sign/number line
4. Choose values in each interval and plug into the $f'(x)$ to determine slope
5. Because Statements
   a. f(x) increasing in interval (a,b) b/c $f'(x) > 0$
   b. f(x) decreasing in interval (a,b) b/c $f'(x) < 0$
   c. Relative max at (**a, f(a)**) b/c $f'(x)$ changes from + to −
   d. Relative min at (**a, f(a)**) b/c $f'(x)$ changes from − to +

V. "Concavity Test"

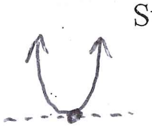
1. Find critical points by setting $f''(x)=0$
2. Remember, critical points can come from the numerator and the denominator of the $f''(x)$ function
3. Put all critical points on a sign/number line
4. Choose values in each interval and plug into the $f''(x)$ to determine concavity
5. Because Statements
   a. Graph is Concave up in interval (a,b) b/c $f''(x) > 0$
   b. Concave down in interval (a,b) b/c $f''(x) < 0$
   c. Point of Inflection at (**a, f(a)**) b/c $f''(x)$ changes sign

VI. 2nd Derivative Test

(relative max/min)

1. *The 2nd derivative test is a test for relative extrema and NOT for P.O.I.
2. *The 2nd derivative test achieves the same as the 1st derivative test.

Steps:



concave up (relative min)

a. Find critical points from $f'(x)$, set $f'(x)=0$ (set numerator of $f'(x)=0$)
b. Take critical points and plug into the $f''(x)$ function
c. If f '(a) = 0 and f ''(a) > 0 , then there is a relative minimum at x = a
d. If f '(b) = 0 and f ''(b) < 0 , then there is a relative maximum at x = b
e. If f '(c) = 0 and f ''(c) =0 , then 2nd derivative test is inconclusive

concave down (relative max)

AP Calculus AB — Chapter 3.1-3.4 Concept Review

I. Extreme Value Theorem

1. Check continuity
2. Check f(x) is a closed function
3. Find f'(x).
   a. Find Critical Points:
   b. Set numerator and denominator of f'(x) = 0
4. Plug critical points and endpoints into f(x) to find absolute max/min

* max and min values refer to y-values

II. Mean Value Theorem

1. Check continuity on closed interval [a,b]
   a. Does f(x) have variables in the denominator? (V.A. or holes)
   b. If so, then look to see if the x-value lies in the **closed** interval [a, b]
   c. If the x lies between the interval, then function is not continuous on the interval, MVT fails
2. Check Differentiability on open interval (a, b)
   a. Does f'(x) have variables in the denominator? (sharp points, slope undefined)
   b. If so, then look to see if the x-value lies in the **open** interval (a, b)
   c. If the x lies between the interval, then function is not differentiable on the interval, MVT fails
3. Find m_{avg}. (This is the slope between your endpoints)
4. Set $f'(x) = m_{avg}$ and solve for x
5. Make sure the x value(s) lies in the open interval (a, b)

III. Rolle's Theorem

1. Check continuity on closed interval [a,b]
2. Check Differentiability on open interval (a, b)
3. Check endpoints. Does f(a) = f(b) ? If not , then Rolle's fails
4. Set f'(x) = 0 and solve for x
5. Make sure the x value(s) lies in the open interval (a, b)

IV. 1st Derivative Test (Finds inc/dec and relative max/min)

1. Find f '(x), set equal to zero
   a. Find critical points from BOTH numerator and denominator
2. Put all critical points on sign line
3. Test intervals
   a. Plug values into f '(x) to determine slope
      i. Positive (+) means increasing slope
      ii. Negative(–) means decreasing slope
4. Write Because Statements
   a. f(x) increasing in interval (a,b) b/c f '(x) > 0
   b. f(x) decreasing in interval (a,b) b/c f '(x) < 0
   c. Relative max at (a, f(a)) b/c f '(x) changes from + to –
   d. Relative min at (a, f(a)) b/c f '(x) changes from – to +

V. Finding Intervals of Concave Up/Down and Points of Inflection (POI) / "Concavity Test"

1. Find f ''(x), set equal to zero
   a. Find critical points from BOTH numerator and denominator
2. Put all critical points on sign line
3. Test intervals
   a. Plug values into f ''(x) to determine concavity
      i. Positive (+) means concave up
      ii. Negative(–) means concave down
4. Write Because Statements
   a. Concave up in interval (a,b) b/c f ''(x) > 0
   b. Concave down in interval (a,b) b/c f ''(x) < 0
   c. Point of Inflection at (a, f(a)) b/c f ''(x) changes signs

VI. 2nd Derivative Test (Finding relative max/min)

1. Find f '(x) , set equal to zero
   a. Find critical points . (These are candidates for relative max/min)
2. Find f ''(x)
3. Plug the critical points (from step #1) into f ''(x).
   a. If result is positive value, then f''(x) > 0, concave up, and therefore relative minimum exists at x-value
      - **Relative Minimum at x = a because f '(a) = 0 and f ''(a) > 0**
   b. If result is negative value, then f''(x) < 0, concave down, and therefore relative maximum exists at x-value
      - **Relative Maximum at x = b because f '(b) = 0 and f ''(b) < 0**
   c. If result is zero, then since f''(x) = 0, then this test is inconclusive. We cannot determine whether relative extrema exists. (Use First Derivative Test)

VII. Absolute Extrema on an Open Interval

1. Find f '(x)
2. Find critical number (only 1)
3. Make sign line
4. Write because statements
5. An Absolute Min occurs at (a, f(a)) b/c f ' (x) < 0 for all x < a and f '(x) > 0 for all x > a
6. An Absolute Max occurs at (a, f(a)) b/c f ' (x) > 0 for all x < a and f '(x) < 0 for all x > a