AP Calculus AB — Chapter 3.1-3.4 Concept Review

I. Extreme Value Theorem

1. Check continuity
2. Check f(x) is a closed function
3. Find f'(x).
   a. Find Critical Points:
   b. Set numerator and denominator of f'(x) = 0
4. Plug critical points and endpoints into f(x) to find absolute max/min

II. Mean Value Theorem

1. Check continuity:
   a. Does f(x) have variables in the denominator? (V.A. or holes)
   b. If so, then look to see if the x-value lies in the **closed** interval [a, b]
   c. If the x lies between the interval, then function is not continuous on the interval, MVT fails
2. Check differentiability:
   a. Does f'(x) have variables in the denominator? (sharp points, slope undefined)
   b. If so, then look to see if the x-value lies in the **open** interval (a, b)
   c. If the x lies between the interval, then function is not differentiable on the interval, MVT fails
3. Find m_{avg}. (This is the slope between your endpoints)
4. Set f'(x) = m_{avg} and solve for x
5. Make sure the x value(s) lies between the intervals

III. Rolle's Theorem

1. Check continuity
2. Check Differentiability
3. Check endpoints. Does f(a) = f(b) ? If not , then Rolle's fails
4. Set f'(x) = 0 and solve for x
5. Make sure the x value(s) lies between the intervals

IV. 1st Derivative Test (Finds inc/dec and relative max/min)

1. Find f '(x), set equal to zero
   a. Find critical points from BOTH numerator and denominator
2. Put all critical points on sign line
3. Test intervals
   a. Plug values into f '(x) to determine slope
      i. Positive (+) means increasing slope
      ii. Negative(–) means decreasing slope
4. Write Because Statements
   a. f(x) increasing in interval (a,b) b/c f '(x) > 0
   b. f(x) decreasing in interval (a,b) b/c f '(x) < 0
   c. Relative max at (a, f(a)) b/c f '(x) changes from + to –
   d. Relative min at (a, f(a)) b/c f '(x) changes from – to +

V. Finding Intervals of Concave Up/Down and Points of Inflection (POI) / "Concavity Test"

1. Find f ''(x), set equal to zero
   a. Find critical points from BOTH numerator and denominator
2. Put all critical points on sign line
3. Test intervals
   a. Plug values into f ''(x) to determine concavity
      i. Positive (+) means concave up
      ii. Negative(–) means concave down
4. Write Because Statements
   a. Concave up in interval (a,b) b/c f ''(x) > 0
   b. Concave down in interval (a,b) b/c f ''(x) < 0
   c. Point of Inflection at (a, f(a)) b/c f ''(x) changes signs

VI. 2nd Derivative Test (Finding relative max/min)

1. Find f'(x) , set equal to zero
   a. Find critical points . (These are candidates for relative max/min)
2. Find f ''(x)
3. Plug the critical points (from step #1) into f ''(x).
   a. If result is positive value, then f''(x) > 0, concave up, and therefore relative minimum exists at x-value
   b. If result is negative value, then f''(x) < 0, concave down, and therefore relative maximum exists at x-value
   c. If result is zero, then since f''(x) = 0, then this test is inconclusive. We cannot determine whether relative extrema exists. (Use First Derivative Test)

VII. Absolute Extrema on an Open Interval

1. Find f '(x)
2. Find critical number (only 1)
3. Make sign line
4. Write because statements
5. An Absolute Min occurs at (a, f(a)) b/c f ' (x) < 0 for all x < a and f '(x) > 0 for all x > a
6. An Absolute Max occurs at (a, f(a)) b/c f ' (x) > 0 for all x < a and f '(x) < 0 for all x > a

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I. Sketching 1st Derivative and 2nd Derivative Graphs (Given the f (x) graph)

1. Given the f(x) graph
2. Make a sign line for f '(x) graph
   a. Label Critical points (relative max, relative min, or where slope = 0) on sign line
   b. Find intervals where graph is increasing (rising) and decreasing (falling)
   c. Use + and ↗ arrow on the sign line to indicate increasing slope
   d. Use – and ↘ arrow on the sign line to indicate decreasing slope
3. Sketch f ' (x) graph
   a. Plot critical points on the graph as x – intercepts (where slope = 0)
   b. Sketch portions of graph above the x-axis (positive slope) or below x-axis (negative slope) using the information on your sign line.

4. Make a sign line for f ''(x) graph
   a. Locate Points of Inflection on your f (x) graph.
      i. This is where graph transitions from concave up to down or from concave down to up.
   b. Label critical point on your sign line
      i. Where graph resembles parabola opening up , use + and ∪ to indicate concave up
      ii. Where graph resembles parabola opening up , use – and ∩ to indicate concave down

5. Sketch f ''(x) graph
   a. Plot critical points on the graph as x – intercepts (POI and where f '' (x) = 0)
   b. Sketch portions of graph above the x-axis (concave up) or below x-axis (concave down) using the information on your sign line.

II. Sketching f(x) graph and 2nd Derivative Graph (Given the f '(x) graph)

1. Given the f '(x) graph
2. Make a sign line for f '(x) graph
   a. Label Critical points **(x-intercepts)** on sign line
   b. Find intervals where graph is increasing(above x-axis) and decreasing(below x-axis)
   c. Use + and ↗ arrow on the sign line to indicate increasing slope
   d. Use – and ↘ arrow on the sign line to indicate decreasing slope

3. Sketch f (x) graph
   a. Follow the directional arrows on your sign line to draw the f(x) graph, along with the relative max (hills) and relative min (valleys) of your graph

4. Make a sign line for f ''(x) graph
   a. Locate critical points (Points of Inflection) on your f '(x) graph
      i. Points of Inflections are the relative max (hills) and relative mins (valleys) of your f '(x) graph

   b. Label critical point on your sign line

      i. Where f '(x) graph is increasing(rising), use + and ∪ to indicate concave up
      ii. Where f '(x) graph is decreasing(falling) , use – and ∩ to indicate concave down

5. Sketch f ''(x) graph

   a. Plot critical points on the graph as x – intercepts (POI and where f '' (x) = 0)
   b. Sketch portions of graph above the x-axis (concave up) or below x-axis (concave down) using the information on your sign line.