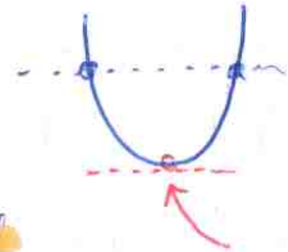
WS #4 10/18/2022

1) Rolle's theorem $[0,6]$

$$f(x)=\frac{x^2-6x}{x+2}$$

$f(x)$ continuous $[0,6]$
$f(x)$ differentiable $(0,6)$



$x+2=0$
$x=-2$
VA: $x=-2$

$f(0)=0$
$f(6)=0$ $\rightarrow \frac{0-0}{6-0} \rightarrow \boxed{m=0}$

$$f'(x)=\frac{(2x-6)(x+2)-(x^2-6x)(1)}{(x+2)^2} \rightarrow \frac{2x^2+4x-6x-12-x^2+6x}{(x+2)^2}$$

$$f'(x)=\frac{x^2+4x-12}{(x+2)^2}$$

$0=x^2+4x-12$
$0=(x+6)(x-2)$
$x=\cancel{-6}, \boxed{x=2}$

$\boxed{c=2}$

2) $f(x)=\frac{x+1}{x}$ $\left[\frac{1}{2},2\right]$ MVT

$x=0$
VA: $x=0$

$f(x)$ is continuous $[1/2, 2]$
$f(x)$ is differentiable $(1/2, 2)$

$f(1/2)=\frac{\frac{1}{2}+1}{1/2}=\frac{3/2}{1/2}=3$

$f(2)=\frac{2+1}{2}=3/2$

$m=\frac{\frac{3}{2}-3}{2-1/2} \rightarrow \frac{-\frac{3}{2}}{\frac{3}{2}}\boxed{=-1}$

$f'(x)=\frac{(1)(x)-(x+1)(1)}{x^2} \rightarrow \frac{x-x-1}{x^2} \rightarrow f'(x)=\frac{-1}{x^2}$

$\frac{-1}{1}=\frac{-1}{x^2}$
$-x^2=-1$
$\sqrt{x^2}=\pm\sqrt{1}$
$x=1$
$x=-1$

$\boxed{c=1}$

3) $f(x) = \frac{1}{4}x^4 - 2x^3 + 6$ (2nd derivative test)

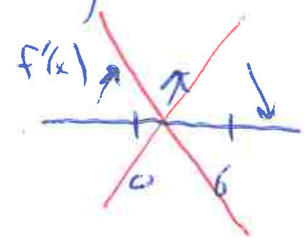
$f'(x) = \frac{1}{4} \cdot 4x^3 - 2 \cdot 3x^2 + 0$

$f'(x) = x^3 - 6x^2$

$f'(x) = x^2(x-6)$

$0 = x^2(x-6)$

$\boxed{x = 0,\ x = 6}$



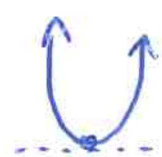
$f''(x) = 3x^2 - 12x$

$f''(0) = 3(0)^2 - 12(0) = 0$ (inconclusive)

$f''(6) = 3(6)^2 - 12(6)$

$f''(6) = 36 > 0$

Concave up



Relative minimum at $x = 6$

$\cancel{(6, 36)}$

$(6, \quad)$

3b) $f(2) = 9$

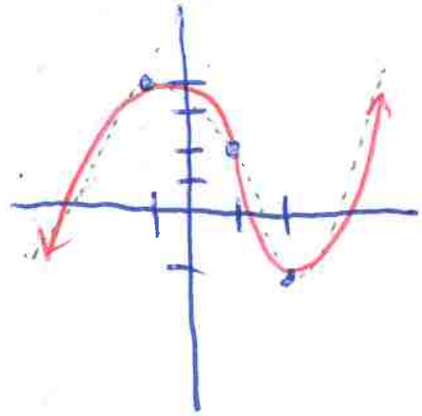
$f'(2) = 0$ (slope = 0)

$f''(2) = -16 < 0$ concave down

(2,9)

$\boxed{\text{Rel. maximum at } (2,9)}$

4) Sketch graph with characteristics



a) $f(-1)=4$ $\quad f(2)=-1$

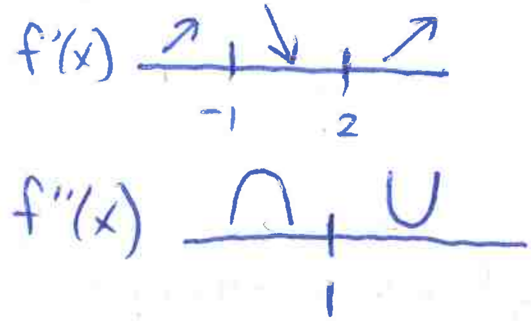
b) $f'(x)>0$ $\quad x<-1$ and $x>2$

c) $f'(x)<0$ $\quad -1<x<2$

d) POI at $(1,2)$

e) $f''(x)<0$ $\quad x<1$

f) $f''(x)>0$ $\quad x>1$



5) Find Abs max/min $\quad f(x)=\dfrac{x^2-4}{x^2+4}$ $\quad [-4,4]$

$x^2+4=0$

$\sqrt{x^2}=\sqrt{4}$

EVT, $f(x)$ continuous $[-4,4]$

$$f'(x)=\frac{(2x)(x^2+4)-(x^2-4)(2x)}{(x^2+4)^2} \rightarrow \frac{\cancel{2x^3}+8x-\cancel{2x^3}+8x}{(x^2+4)^2}$$

$$\boxed{f'(x)=\frac{16x}{(x^2+4)^2}}$$

$16x=0$ $\quad$ | $\quad (x^2+4)^2=0$

$x=0$

$$f(-4)=\frac{(-4)^2-4}{(-4)^2+4}=\frac{12}{20}=3/5$$

$$f(0)=\frac{0-4}{0+4}=-1$$

$$f(4)=\frac{4^2-4}{4^2+4}=\frac{12}{20}=3/5$$

Abs max is $3/5$

Abs min is -1