3.1-3.4 Recovery/Corrections Help Session

1. Find all critical numbers for $h(x) = 2x^{5/3} - x^{2/3} + 3$

$h'(x) = \frac{10}{3}x^{2/3} - \frac{2}{3}x^{-1/3}$

$h'(x) = \frac{10x^{2/3}}{3} - \frac{2}{3x^{1/3}}$

$\frac{10x^{2/3} \cdot x^{1/3}}{3x^{1/3}} - \frac{2}{3x^{1/3}}$

$h'(x) = \frac{10x - 2}{3x^{1/3}}$

$10x - 2 = 0$ | $3x^{1/3} = 0$

$x = \frac{2}{10} = \boxed{\frac{1}{5}}$ | $\boxed{x = 0}$

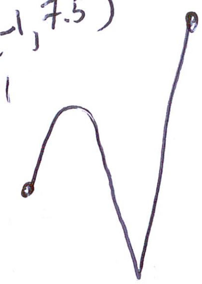
2. Find the **value(s)** of the absolute extrema of the function $f(x) = x^3 - \frac{9}{2}x^2 - 12x + 1$ on the interval [-2, 3].

max = 7.5 at $x = -1$

min = -48.5

$f(-1) = 7.5$

$(-1, 7.5)$



Include theorem and condition. EVT: $f(x)$ continuous $[-2, 3]$

$f'(x) = 3x^2 - 2 \cdot \frac{9}{2}x - 12$

$f'(x) = 3x^2 - 9x - 12$

$0 = 3(x^2 - 3x - 4)$

$0 = 3(x-4)(x+1)$

$\cancel{x = 4}$, $x = -1$

$f(-2) = -1$

$f(-1) = 7.5$ (max)

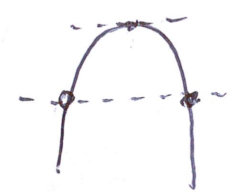
$f(3) = -48.5$ (min)

3. If $f(x) = \frac{x^2}{8x-15}$ on [3, 5], determine if Rolle's Theorem can be applied. If yes, find the value(s) of c defined on Rolle's Theorem.

VA: $8x - 15 = 0$

$x = 15/8 \approx 1.8$

$f(x)$ continuous $[3, 5]$ $f(x)$ differentiable $(3, 5)$



$f(3) = \frac{3^2}{24-15} = \frac{9}{9} = 1$

$f(5) = \frac{25}{40-15} = \frac{25}{25} = 1$

slope $= 0$

$f'(x) = \frac{2x(8x-15) - x^2(8)}{(8x-15)^2}$

$f'(x) = \frac{16x^2 - 30x - 8x^2}{(8x-15)^2}$

$f'(x) = \frac{8x^2 - 30x}{(8x-15)^2} = 0$

$8x^2 - 30x = 0$

$2x(4x - 15) = 0$

$\cancel{x = 0}$, $x = 15/4$

$\boxed{c = 15/4}$

in $(3, 5)$

open interval

4. If $g(x) = x^3 - x^2 - 2x$ on [-1, 1]. Determine if the Mean Value Theorem can be applied. If yes, find the value(s) of c defined in the Mean Value Theorem.

$g(x)$ continuous $[-1, 1]$, $g(x)$ differentiable on $(-1, 1)$

$g(-1) = -1 - 1 + 2 = 0$

$g(1) = 1 - 1 - 2 = -2$

$m = \frac{-2 - 0}{1 - (-1)} = \frac{-2}{2} = \boxed{-1}$

$g'(x) = 3x^2 - 2x - 2$

$3x^2 - 2x - 2 = -1$

$3x^2 - 2x - 1 = 0$

$(3x + 1)(x - 1) = 0$

$x = -1/3$

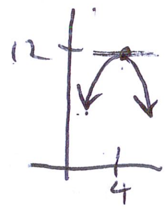
$\cancel{x = 1}$

$\boxed{c = -1/3}$

2nd derivative → finds relative max/mins

7) Suppose $f(4) = 12, f'(4) = 0$, and $f''(4) = -23$. If $f'(x)$ is never equal to 0 or undefined for any other values of x, use the Second Derivative Test to state all relative extrema for $f(x)$.

at point (4, 12), slope at point is 0, and graph is concave down



$\boxed{\text{Rel. max at } x = 4}$

7b) Given that $f(x) = \frac{1}{3}x^3 + \frac{5}{2}x^2 + 6x - 1$ If $f'(x)$ is never equal to 0 or undefined for any other values of x, use the Second Derivative Test to state all relative extrema for $f(x)$.

$f'(x) = x^2 + 5x + 6$ → $f''(x) = 2x + 5$

$0 = (x+3)(x+2)$

critical pts. $\boxed{x = -3, -2}$

$f''(-2) = -4 + 5 = 1 > 0$, concave up — Rel. min at $x = -2$

$f''(-3) = 2(-3) + 5 = -1 < 0$, concave down — Rel. max at $x = -3$

8. Sketch a labeled graph of a function, f, with the following characteristics:

$f(-6) = -6, \ f(-2) = -3, \ f(2) = 5, \quad f(5) = -1$

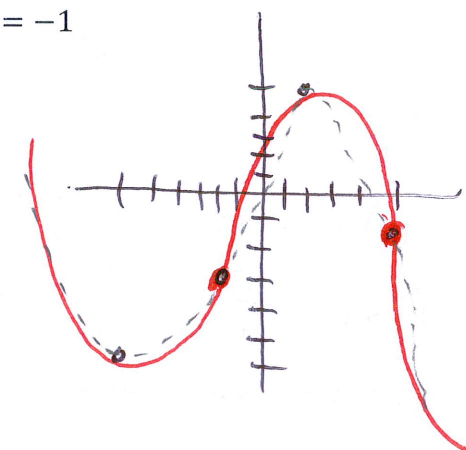
$f'(x) < 0$ if $x < -6$, $x > 2$

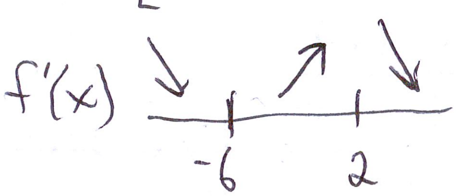
$f'(x)$ is continuous for all x

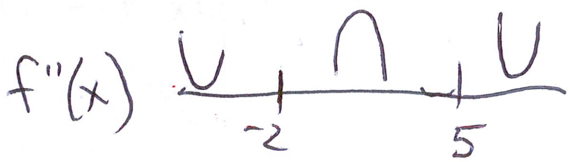
$f'(x) > 0$ if $-6 < x < 2$

$f''(x) < 0$ if $-2 < x < 5$

$f''(x) > 0$ if $x < -2$ and $x > 5$







9. Clearly and fully explain what it means in terms of the graph of a function for intervals where $\frac{dy}{dx}$ is negative and $\frac{d^2y}{dx^2}$ is negative at the same time. (sketch a portion of graph demonstrating these properties)

decreasing slope becoming more negative

$f'(x) > 0$, $f''(x) > 0$ | $f'(x) < 0$, $f''(x) > 0$ | $f'(x) > 0$, $f''(x) < 0$ | $f'(x) < 0$, $f''(x) < 0$