

## AP Calculus AB - Quiz Review #3.1 - 3.4

(WS #1)

①

1. Find the critical numbers of the function  $f(x) = 2x^3 - 8x^2 + 10x$
2. Find the absolute extrema of  $f(x) = 2x^3 - 3x^2 - 12x + 1$  on the interval  $[-2, 3]$ .
3. If  $f(x) = \frac{-x^2 + 4}{4x}$  on  $[-2, 2]$ , determine if Rolle's Theorem applies. If yes, find the value(s) of  $c$  defined by Rolle's Theorem. If no, explain why not.
4. If  $f(x) = -(25 - 5x)^{\frac{1}{2}}$  on  $[3, 5]$ , determine if the Mean Value Theorem applies. If yes, find the value(s) of  $c$  defined by the Mean Value Theorem. If no, explain why not.
5. If  $f(x) = x^3 - 4x^2 + 5x$ , find the interval(s) where  $f(x)$  is Increasing and Decreasing

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6. If  $f(x) = x^3 - 12x^2 - 27x$ , use the First Derivative Test to find all relative extrema (x-values).

7. If  $f(x) = x^3 + x^2 - 21x$ , find the intervals where  $f(x)$  is concave up and concave down, and find all points of inflection.

8. If  $f(x) = 2x^3 - x^2 + 20x - 10$ , use the Second Derivative Test to find all relative extrema.

9. Sketch a labeled graph of a function,  $f$ , with the following characteristics:

$$f(0) = f(2) = 0$$

$$f'(x) > 0 \text{ if } x < 1$$

$f'(1)$  does not exist

$$f'(x) < 0 \text{ if } x > 1$$

$$f''(x) > 0, x \neq 1$$

AP Calculus AB - Quiz #3.1 - 3.4

Review

Review KEY

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1. Find the critical numbers of the function  $f(x) = 2x^3 - 8x^2 + 10x$

$$f'(x) = 6x^2 - 16x + 10$$

$$= 2(3x^2 - 8x + 5)$$

$$0 = 2(3x - 5)(x - 1)$$

$x = \frac{5}{3}, 1$

$f(x)$  is continuous on  $[-2, 3]$

2. Find the absolute extrema of  $f(x) = 2x^3 - 3x^2 - 12x + 1$  on the interval  $[-2, 3]$

$$f'(x) = 6x^2 - 6x - 12$$

$$0 = 6(x^2 - x - 2)$$

$$0 = 6(x - 2)(x + 1)$$

$x = 2, -1$

$$f(-2) = -3$$

$$f(2) = -19$$

$$f(3) = -8$$

$$f(-1) = 8$$

Absolute minimum is  $-19$  at  $x = 2$   
 Absolute maximum is  $8$  at  $x = -1$

3. If  $f(x) = \frac{-x^2 + 4}{4x}$  on  $[-2, 2]$ , determine if Rolle's Theorem applies. If yes, find the value(s) of  $c$

defined by Rolle's Theorem. If no, explain why not.

Check conditions? not continuous on  $[-2, 2]$  since  $f(x)$  is not continuous at  $x = 0$ . Rolle's theorem does not apply.

4. If  $f(x) = -(25 - 5x)^{\frac{1}{2}}$  on  $[3, 5]$ , determine if the Mean Value Theorem applies. If yes, find the value(s) of  $c$  defined by the Mean Value Theorem. If no, explain why not.

$$f'(x) = -\frac{1}{2}(25 - 5x)^{-\frac{1}{2}}(-5)$$

$$= \frac{+5}{2\sqrt{25 - 5x}}$$

$f(x)$  is continuous on  $[3, 5]$  and differentiable on  $(3, 5)$  so MVT applies

$$25 - 5x = 0$$

$$5x = 25 \quad x = 5$$

$f(3) = -\sqrt{10}$     \* set  $f'(x) = M_{Avg}$

$f(5) = 0$

$$M_{Avg} = \frac{-\sqrt{10} - 0}{3 - 5} = \frac{-\sqrt{10}}{-2} = \frac{\sqrt{10}}{2}$$

$$\frac{+5}{2\sqrt{25 - 5x}} = \frac{\sqrt{10}}{2}$$

$$+10 = 2\sqrt{10(25 - 5x)}$$

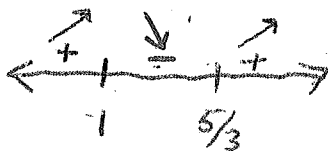
$$+5 = \sqrt{10(25 - 5x)} \rightarrow (-5)^2 = (\sqrt{10(25 - 5x)})^2$$

5. If  $f(x) = x^3 - 4x^2 + 5x$ , find the interval(s) where  $f(x)$  is Increasing and Decreasing

$$f'(x) = 3x^2 - 8x + 5$$

$$0 = (3x - 5)(x - 1)$$

$x = \frac{5}{3}, 1$



$f(x)$  increasing  $(-\infty, 1) \cup (\frac{5}{3}, \infty)$  b/c  $f'(x) > 0$   
 $f(x)$  decreasing  $(1, \frac{5}{3})$  b/c  $f'(x) < 0$

$$25 = 10(25 - 5x)$$

$$5 = 2(25 - 5x)$$

$$5 = 50 - 10x$$

$$-45 = -10x$$

$$\frac{9}{2} = x$$

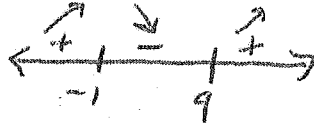
$c = 4.5$

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$$x^3 - 12x^2 - 27x$$

6. If  $f(x) = x^3 - 12x^2 + 16x$ , use the First Derivative Test to find all relative extrema. (x-values)

$$f'(x) = 3x^2 - 24x + 16$$



$$0 = 3(x^2 - 8x + 9)$$

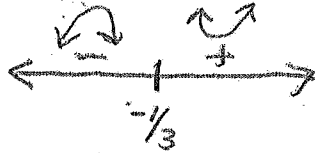
$$= 3(x-9)(x+1)$$

$$x = 9, -1$$

Relative max at  $x=1$  b/c  $f'(x)$  change signs from + to -  
Relative min at  $x=9$  b/c  $f'(x)$  change signs from - to +

7. If  $f(x) = x^3 + x^2 - 21x$ , find the intervals where  $f(x)$  is concave up and concave down, and find all points of inflection.

$$f'(x) = 3x^2 + 2x - 21$$



$$f''(x) = 6x + 2$$

$$0 = 6x + 2$$

$$x = -1/3$$

$f(x)$  concave up  $(-1/3, \infty)$  b/c  $f''(x) > 0$

$f(x)$  concave down  $(-\infty, -1/3)$  b/c  $f''(x) < 0$

POI at  $(-1/3, 7.074)$  b/c  $f''(x)$  change signs

8. If  $f(x) = 2x^3 - x^2 - 20x - 10$ , use the Second Derivative Test to find all relative extrema.

Find Rel max/min

$$f'(x) = 6x^2 - 2x - 20$$

$$= 2(3x^2 - x - 10)$$

$$2(3x + 5)(x - 2)$$

$$x = -5/3, 2$$

$$f''(x) = 12x - 2$$

$$f''(-5/3) = 12(-5/3) - 2 = -22 < 0, \text{ concave down}$$

Relative max at  $x = -5/3$

$$f''(2) = 12(2) - 2 = 22 > 0, \text{ concave up}$$

Relative min at  $x = 2$

9. Sketch a labeled graph of a function,  $f$ , with the following characteristics:

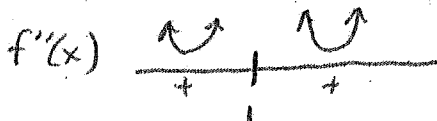
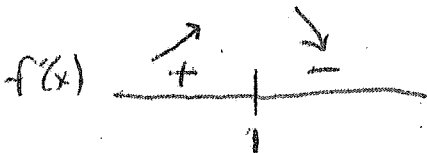
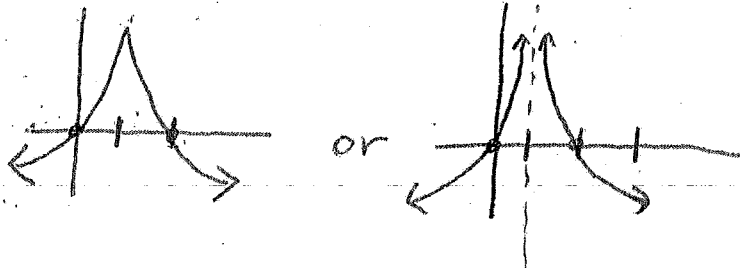
$$f(0) = f(2) = 0$$

$$f'(x) > 0 \text{ if } x < 1$$

$f'(1)$  does not exist

$$f'(x) < 0 \text{ if } x > 1$$

$$f''(x) > 0, x \neq 1$$





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5) If  $f(x) = \frac{x^2}{x^2-9}$  find the following (where appropriate): Intervals where  $f(x)$  is increasing, decreasing, relative maximum points, and relative minimum points. Justify your answer(s)

6) If  $f(x) = \frac{1}{2}x^4 + 2x^3$  Find the intervals where  $f(x)$  is concave up and concave down, and find all points of inflection. (Justify your answers)

7) If  $f(x) = -x^3 + 7x^2 - 15x$  use the Second Derivative Test to find all relative extrema. Justify your answers.

8) Sketch a labeled graph of a function,  $f$ , with the following characteristics:

$$f(0) = 4, f(6) = 0, f(2) = 0, f(4) = 2$$

$$f'(x) < 0 \text{ for } x < 2 \text{ or } x > 4$$

$f'(2)$  does not exist

$$f'(4) = 0$$

$$f'(x) > 0 \text{ for } 2 < x < 4$$

$$f''(x) < 0 \text{ for } x \neq 2$$

1) Find all critical numbers for  $g(x) = 16\sqrt{x} - x^2$

$$g(x) = 16x^{1/2} - x^2$$

$$g'(x) = 16 \cdot \frac{1}{2} x^{-1/2} - 2x$$

$$g'(x) = \frac{8}{x^{1/2}} - 2x$$

$$g'(x) = \frac{8 - 2x^{3/2}}{x^{1/2}}$$

$$\begin{array}{l|l} 8 - 2x^{3/2} = 0 & x^{1/2} = 0 \\ -2x^{3/2} = -8 & x = 0 \\ x^{3/2} = 4 & x = (4)^{2/3} \end{array}$$

critical points:  
 $x = 4^{2/3}, x = 0$

2) Find the value(s) of the absolute extrema of the function  $f(x) = x^3 + 6x^2$  on the interval  $[-3, 1]$ . State theorem and conditions

\*Apply EVT:  $f(x)$  continuous  $[-3, 1]$

$$f'(x) = 3x^2 + 12x$$

$$0 = 3x(x+4)$$

$$x = 0, x = -4 \text{ (outside interval)}$$

$$f(-3) = (-3)^3 + 6(-3)^2 = 27$$

$$f(0) = 0$$

$$f(1) = 1 + 6 = 7$$

Abs maximum is 27 at  $x = -3$   
 Abs minimum is 0 at  $x = 0$

3) If  $f(x) = x^2 - 8x + 5$  on  $[2, 6]$ , determine if Rolle's Theorem can be applied. If yes, find the value(s) of  $c$  defined on Rolle's Theorem. State conditions and show steps.  $f(x)$  continuous on  $[2, 6]$ ,  $f(x)$  differentiable  $(2, 6)$

$$f(2) = -7 \quad f(6) = -7 \quad \checkmark$$

$$f'(x) = 2x - 8$$

$$\text{set } f'(x) = 0$$

$$2x - 8 = 0$$

$$x = 4$$

By Rolle's Theorem  
 $c = 4$

4) If  $g(x) = x^3 - x$  on  $[1, 2]$ , determine if the Mean Value Theorem can be applied. If yes, find the value(s) of  $c$  defined in the Mean Value Theorem. State conditions and show steps.  $g(x)$  continuous  $[1, 2]$ ,  $g(x)$  differentiable  $(1, 2)$

\*MVT:  $g'(c) = \frac{g(b) - g(a)}{b - a}$

$$g(1) = 1 - 1 = 0$$

$$g(2) = 2^3 - 2 = 6$$

$$\frac{g(2) - g(1)}{2 - 1} = \frac{6 - 0}{1} = 6$$

$$g'(x) = 3x^2 - 1$$

$$6 = 3x^2 - 1$$

$$7 = 3x^2$$

$$\frac{7}{3} = x^2$$

$x^2 = 7/3$   
 $x = \pm\sqrt{7/3}$   
 $c = \sqrt{7/3}, c = -\sqrt{7/3}$   
 By MVT,  $c = \sqrt{7/3}$

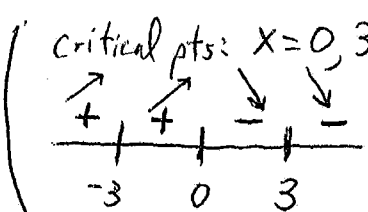
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5) If  $f(x) = \frac{x^2}{x^2-9}$  find the following (where appropriate): intervals where  $f(x)$  is increasing, decreasing, relative maximum points, and relative minimum points. Justify your answer(s)

$$f'(x) = \frac{2x(x^2-9) - x^2(2x)}{(x^2-9)^2}$$

$$f'(x) = \frac{2x^3 - 18x - 2x^3}{(x^2-9)^2}$$

$$f'(x) = \frac{-18x}{(x^2-9)^2} \quad \begin{array}{l} -18x=0 \\ x=0 \end{array} \quad \begin{array}{l} x^2-9=0 \\ x=3, -3 \end{array}$$



$f(x)$  increasing  $(-\infty, -3), (-3, 0)$   
 $b/c f'(x) > 0$   
 $f(x)$  decreasing  $(0, 3), (3, \infty)$   
 $b/c f'(x) < 0$   
 Rel. max at  $(0, 0)$   $b/c$   
 $f'(x)$  changes from  $+$  to  $-$

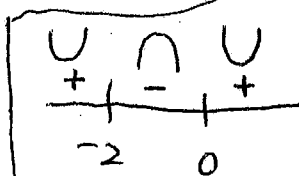
6) If  $f(x) = \frac{1}{2}x^4 + 2x^3$  Find the intervals where  $f(x)$  is concave up and concave down, and find all points of inflection. (Justify your answers)

$$f'(x) = \frac{1}{2} \cdot 4x^3 + 6x^2 = 2x^3 + 6x^2$$

$$f''(x) = 6x^2 + 12x$$

$$0 = 6x(x+2)$$

$$x = 0, -2$$



$f(x)$  concave up  $(-\infty, -2), (0, \infty)$   
 $b/c f''(x) > 0$

$f(x)$  concave down  $(-2, 0)$   $b/c$   
 $f''(x) < 0$

POI at  $(-2, f(-2))$  and  $(0, f(0))$   
 $b/c f''(x)$  change signs.

7) If  $f(x) = -x^3 + 7x^2 - 15x$  use the Second Derivative Test to find all relative extrema. Justify your answers.

$$f'(x) = -3x^2 + 14x - 15$$

$$0 = -(3x^2 - 14x + 15)$$

$$0 = -(3x-5)(x-3)$$

$$x = 5/3, x = 3$$

$$f''(x) = -6x + 14$$

$$f''(5/3) = -6(5/3) + 14 > 0, \text{ concave up}$$

Rel. minimum at  $(5/3, f(5/3))$

$$f''(3) = -6(3) + 14 = -4 < 0 \text{ concave down}$$

Rel. maximum at  $(3, f(3))$

8) Sketch a labeled graph of a function,  $f$ , with the following characteristics:

$$f(0) = 4, f(6) = 0, f(2) = 0, f(4) = 2$$

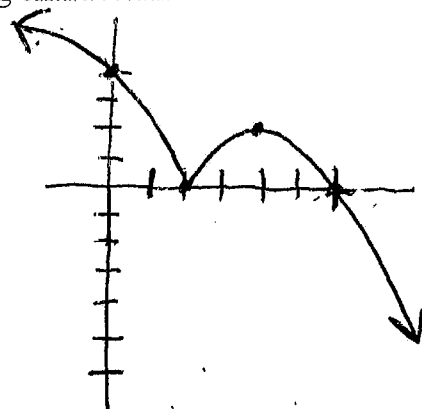
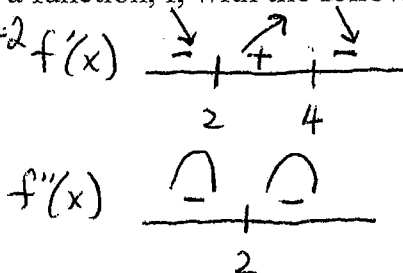
$$f'(x) < 0 \text{ for } x < 2 \text{ or } x > 4$$

$$f'(2) \text{ does not exist}$$

$$f'(4) = 0$$

$$f''(x) > 0 \text{ for } 2 < x < 4$$

$$f''(x) < 0 \text{ for } x \neq 2$$







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5) If  $f(x) = 2x^3 + 3x^2 - 12x$  find the following (where appropriate): Intervals where  $f(x)$  is increasing, decreasing, relative maximum points, and relative minimum points. Justify your answer(s)

6) If  $f(x) = x^5 - 5x^4 + 3x + 7$  Find the intervals where  $f(x)$  is concave up and concave down, and find all points of inflection. (Justify your answers)

7) Sketch a labeled graph of a function,  $f$ , with the following characteristics:

$$f(-4) = 5, f(-1) = -2, f(0) = 0, f(2) = 4$$

$$f'(x) < 0 \text{ for } x < -1 \text{ and } x > 2$$

$$f'(-1) = 0, f'(2) = 0$$

$$f'(x) > 0 \text{ for } -1 < x < 2$$

$$f''(x) < 0 \text{ for } x < -4, x > 0$$

$$f''(x) > 0 \text{ for } -4 < x < 0$$

1) Find the critical points of  $f(x) = x^{8/5} + x^{3/5}$

$$f'(x) = \frac{8}{5}x^{3/5} + \frac{3}{5}x^{-2/5}$$

$$f'(x) = \frac{8x^{3/5}}{5} + \frac{3}{5x^{2/5}}$$

$$f'(x) = \frac{8x^1 + 3}{5x^{2/5}}$$

$$8x + 3 = 0$$

$$5x^{2/5} = 0$$

$$x = \frac{-3}{8}$$

$$x = 0$$

2) Find the value(s) of the absolute extrema of the function  $f(x) = 2x^3 - 3x^2 - 12x + 1$  on the interval  $[-2, 3]$ . State theorem and conditions

$f(x)$  continuous  $[-2, 3]$ , EVT

$$f'(x) = 6x^2 - 6x - 12 \quad \left| \quad f(-2) = -3 \right.$$

$$f'(x) = 6(x^2 - x - 2) \quad \left| \quad f(-1) = 8 \right.$$

$$0 = 6(x-2)(x+1) \quad \left| \quad f(2) = -19 \right.$$

$$x = 2, x = -1 \quad \left| \quad f(3) = -8 \right.$$

Abs max is 8 (at  $x = -1$ )  
Abs min is -19 (at  $x = 2$ )

3) If  $f(x) = \frac{x^2 - 2x - 3}{x + 2}$  on  $[-1, 3]$ , determine if Rolle's Theorem can be applied. If yes, find the value(s) of  $c$  defined on Rolle's Theorem. State conditions and show steps.

V.A. at  $x = -2$  (outside interval)

i)  $f(x)$  continuous  $[-1, 3]$

ii)  $f(x)$  differentiable  $(-1, 3)$

iii)  $f(-1) = 0$   
 $f(3) = 0 \Rightarrow f(-1) = f(3)$

Rolle's theorem applies:

Set  $f'(x) = 0$

$$f'(x) = \frac{f' \cdot g - f \cdot g'}{g^2} = \frac{(2x-2)(x+2) - (x^2-2x-3)(1)}{(x+2)^2} = \frac{2x^2+2x-4-x^2+2x+3}{(x+2)^2}$$

$$f'(x) = \frac{x^2+4x-1}{(x+2)^2}$$

$$x^2 + 4x - 1 = 0$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-4 \pm \sqrt{16 - 4(1)(-1)}}{2(1)}$$

$$\frac{-4 \pm \sqrt{20}}{2} = x$$

$$x = \frac{-4 + \sqrt{20}}{2}$$

$$x = \frac{-4 - \sqrt{20}}{2}$$

$$c = \frac{-2 + \sqrt{20}}{2}$$

4) If  $g(x) = x^3 - x - 1$  on  $[-1, 2]$ , determine if the Mean Value Theorem can be applied. If yes, find the value(s) of  $c$  defined in the Mean Value Theorem. State conditions and show steps.

$g(x)$  continuous  $[-1, 2]$ ,  $g(x)$  differentiable  $(-1, 2)$

MVT:  $f'(c) = \frac{f(b) - f(a)}{b - a}$

$$g(-1) = -1 + 1 - 1 = -1$$

$$g(2) = 2^3 - 2 - 1 = 5$$

$$\text{slope: } \frac{5 - (-1)}{2 - (-1)} = \frac{6}{3} = 2$$

$$g'(x) = 3x^2 - 1$$

$$3x^2 - 1 = 2$$

$$3x^2 = 3$$

$$x^2 = 1$$

$$x = \pm 1$$

$$c = 1$$

$$c \neq -1$$

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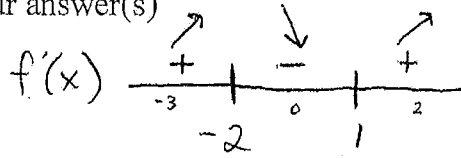
4) If  $f(x) = 2x^3 + 3x^2 - 12x$  find the following (where appropriate): Intervals where  $f(x)$  is increasing, decreasing, relative maximum points, and relative minimum points. Justify your answer(s)

$$f'(x) = 6x^2 + 6x - 12$$

$$0 = 6(x^2 + x - 2)$$

$$0 = 6(x+2)(x-1)$$

$$x = -2, x = 1$$



Rel. max  $(-2, 20)$  b/c  $f'(x)$  changes from + to -

Rel. min  $(1, -7)$  b/c  $f'(x)$  changes from - to +

$f(x)$  increasing  $(-\infty, -2) \cup (1, \infty)$  b/c  $f'(x) > 0$

$f(x)$  decreasing  $(-2, 1)$  b/c  $f'(x) < 0$

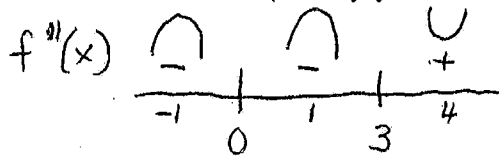
5) If  $f(x) = x^5 - 5x^4 + 3x + 7$  Find the intervals where  $f(x)$  is concave up and concave down, and find all points of inflection. (Justify your answers)

$$f'(x) = 5x^4 - 20x^3 + 3$$

$$f''(x) = 20x^3 - 60x^2$$

$$0 = 20x^2(x-3)$$

$$x = 0, x = 3$$



Point of Inflection at  $(3, -146)$  b/c  $f''(x)$  change signs.

$f(x)$  is concave up  $(3, \infty)$  b/c  $f''(x) > 0$

$f(x)$  is concave down  $(-\infty, 0) \cup (0, 3)$  b/c  $f''(x) < 0$ .

6) Sketch a labeled graph of a function,  $f$ , with the following characteristics:

$$f(-4) = 5, f(-1) = -2, f(0) = 0, f(2) = 4$$

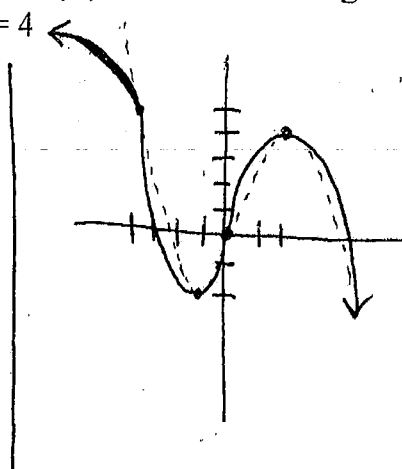
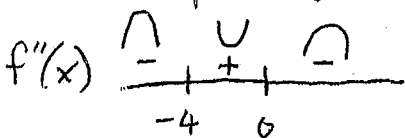
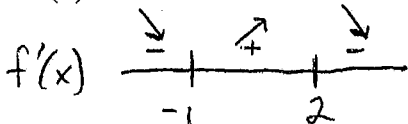
$$f'(x) < 0 \text{ for } x < -1 \text{ or } x > 2$$

$$f'(-1) = 0, f'(2) = 0$$

$$f'(x) > 0 \text{ for } -1 < x < 2$$

$$f''(x) < 0 \text{ for } x < -4, x > 0$$

$$f''(x) > 0 \text{ for } -4 < x < 0$$



(WS #4)

1. If  $f(x) = \frac{x^2 - 6x}{x^2 + 2}$ , does Rolle's theorem apply to the function on the interval  $[0, 6]$ . If yes, find the value of  $c$  defined by Rolle's theorem.

2. Determine whether the Mean Value Theorem can be applied to  $f(x) = \frac{x+1}{x}$  on  $\left[\frac{1}{2}, 2\right]$ .

- if so, then find all values of  $c$  on  $(a, b)$  defined by MVT.

3. Use the Second Derivative Test to classify the critical points of  $y = \frac{1}{4}x^4 - 2x^3 + 6$ .

4. Sketch a labeled graph with the following characteristics:

- $f(-1) = 4$  and  $f(2) = -1$
- $f'(x) > 0$  if  $x < -1$  and if  $x > 2$
- $f'(x) < 0$  if  $-1 < x < 2$
- POI at point  $(1, 2)$
- $f''(x) < 0$  if  $x < 1$
- $f''(x) > 0$  if  $x > 1$

5. Find the absolute maximum and absolute minimum of the function  $f$  on the given interval:

$$f(x) = \frac{x^2 - 4}{x^2 + 4} \quad \text{on } [-4, 4]$$

KEY

Ch. 3.1-3.4 Quiz Review

AP Calculus AB

1. If  $f(x) = \frac{x^2-6x}{x+2}$ , does Rolle's theorem apply to the function on the interval  $[0,6]$ . If yes, find the value of  $c$  defined by Rolle's theorem.  $f(x)$  continuous, differentiable on interval  $[0,6]$

$f(x) = \frac{(2x-6)(x+2)}{(x+2)^2}$   
 $f(0) = 0$     $f(6) = f(6)$ , Rolle's applies  
 $f'(x) = 0$   
 $2x^2 - 2x - 12 - x^2 + 6x$   
 $\frac{(x+2)^2}{(x+2)^2} = 0$   
 $\frac{x^2 + 4x - 12}{(x+2)^2} = 0$   
 $\frac{(x+6)(x-2)}{(x+2)^2} = 0$   
 $x = 2, -6$     **$c = 2$**

2. Determine whether the Mean Value Theorem can be applied to  $f(x) = \frac{x^2+1}{x}$  on  $[\frac{1}{2}, 2]$ . If so, then find all values of  $c$  on  $(a, b)$  defined by MVT.  $x \neq 0$ ,  $f(x)$  is continuous, differentiable on interval  $[\frac{1}{2}, 2]$ .

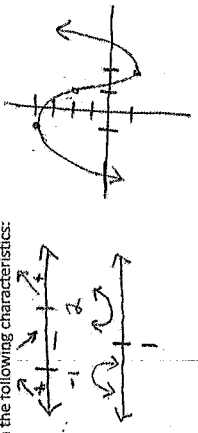
$f(x) = \frac{(1)(x) - (x+1)(1)}{x^2}$   
 $f'(x) = \frac{-1}{x^2}$   
 $\frac{-1}{x^2} = -1$     $x = \pm 1$   
**Final endpoints**  
 $f'(1) = 3$     $M_{Avg} = \frac{3 - \frac{3}{2}}{\frac{1}{2} - 2} = \frac{\frac{3}{2}}{-\frac{3}{2}} = -1$     **$c = 1$**   
 $f(2) = \frac{3}{2}$

3. Use the Second Derivative Test to classify the critical points of  $y = \frac{1}{3}x^3 - 2x^2 + 0$ .

$y'(x) = \frac{1}{3} \cdot 4x^2 - 6x^2 = x^3 - 6x^2 = x^2(x-6)$     **$x = 0, 6$**   
 $y''(x) = 3x^2 - 12x$   
 $y''(0) = 0 - 0$  inconclusive test at  $x = 0$   
 $y''(6) = 3(6)^2 - 12(6) = 36 - 72 = -36 < 0$ , concave up   **Relative min at  $x = 6$**

4. Sketch a labeled graph with the following characteristics:

- a)  $f(-1) = 4$  and  $f(2) = -1$
- b)  $f'(x) > 0$  if  $x < -1$  and if  $x > 2$
- c)  $f'(x) < 0$  if  $-1 < x < 2$
- d) POI at point  $(1, 2)$
- e)  $f''(x) < 0$  if  $x < 1$
- f)  $f''(x) > 0$  if  $x > 1$



5) Find the absolute max and absolute min of function  $f$  on the given interval:  $f(x) = \frac{x^2-4}{x^2+4}$ ;  $[-4, 4]$

$f'(x) = \frac{(2x)(x^2+4) - (x^2-4)(2x)}{(x^2+4)^2} = \frac{2x^3+8x-2x^3+8x}{(x^2+4)^2} = \frac{16x}{(x^2+4)^2}$

\*  $f(x)$  is continuous on  $[-4, 4]$

find critical pts:  $\frac{16x}{(x^2+4)^2} = 0 \rightarrow x = 0$   
 $16x = 0 \rightarrow x = 0$   
 $(x^2+4)^2 = 0 \rightarrow$  no critical pts

$f(-4) = \frac{(-4)^2-4}{4^2+4} = \frac{12}{20} = \frac{3}{5}$

$f(4) = \frac{(4)^2-4}{4^2+4} = \frac{12}{20} = \frac{3}{5}$

$f(0) = \frac{0-4}{0+4} = -\frac{4}{4} = -1$

Abs. max is  $\frac{3}{5}$  at  $x = 4, x = -4$   
 Abs. min is  $-1$  at  $x = 0$