

Key

1) Find the critical points of  $f(x) = x^{8/5} + x^{3/5}$

$$f'(x) = \frac{8}{5}x^{3/5} + \frac{3}{5}x^{-2/5}$$

$$f'(x) = \frac{8x^{3/5}}{5} + \frac{3}{5x^{2/5}}$$

$$f'(x) = \frac{8x' + 3}{5x^{2/5}}$$

$$8x + 3 = 0 \quad \left| \quad 5x^{2/5} = 0 \right.$$

$$x = \frac{-3}{8} \quad \left| \quad x = 0 \right.$$

2) Find the value(s) of the absolute extrema of the function  $f(x) = 2x^3 - 3x^2 - 12x + 1$  on the interval  $[-2, 3]$ . State theorem and conditions  $f(x)$  continuous  $[-2, 3]$ , EVT

$$f'(x) = 6x^2 - 6x - 12$$

$$f(-2) = -3$$

$$f'(x) = 6(x^2 - x - 2)$$

$$f(-1) = 8$$

$$0 = 6(x-2)(x+1)$$

$$f(2) = -19$$

$$x = 2, x = -1$$

$$f(3) = -8$$

Abs max is 8 (at  $x = -1$ )

Abs min is -19 (at  $x = 2$ )

3) If  $f(x) = \frac{x^2 - 2x - 3}{x + 2}$  on  $[-1, 3]$ , determine if Rolle's Theorem can be applied. If yes, find the value(s) of  $c$  defined on Rolle's Theorem. State conditions and show steps.

V.A. at  $x = -2$  (outside interval)

i)  $f(x)$  continuous  $[-1, 3]$

ii)  $f(x)$  differentiable  $(-1, 3)$

iii)  $f(-1) = 0$   
 $f(3) = 0 \rightarrow f(-1) = f(3)$

Rolle's theorem applies:  
 Set  $f'(x) = 0$

$$f'(x) = \frac{f' \cdot g - f \cdot g'}{g^2} = \frac{(2x-2)(x+2) - (x^2-2x-3)(1)}{(x+2)^2} = \frac{2x^2+2x-4-x^2+2x+3}{(x+2)^2}$$

$$f'(x) = \frac{x^2+4x-1}{(x+2)^2}$$

$$x^2 + 4x - 1 = 0$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-4 \pm \sqrt{16 - 4(1)(-1)}}{2(1)}$$

$$\frac{-4 \pm \sqrt{20}}{2} = x$$

$$x = \frac{-4 \pm \sqrt{20}}{2}$$

$$x = \frac{-4 - \sqrt{20}}{2}$$

$c = -2 + \frac{\sqrt{20}}{2}$

*\*quotient rule*

4) If  $g(x) = x^3 - x - 1$  on  $[-1, 2]$ , determine if the Mean Value Theorem can be applied. If yes, find the value(s) of  $c$  defined in the Mean Value Theorem. State conditions and show steps.

$g(x)$  continuous  $[-1, 2]$ ,  $g(x)$  differentiable  $(-1, 2)$

MVT:  $f'(c) = \frac{f(b) - f(a)}{b - a}$

$$g(-1) = -1 + 1 - 1 = -1$$

$$g(2) = 2^3 - 2 - 1 = 5$$

slope:  $\frac{5 - (-1)}{2 - (-1)} = \frac{6}{3} = 2$

$$g'(x) = 3x^2 - 1$$

$$3x^2 - 1 = 2$$

$$3x^2 = 3$$

$$x^2 = 1$$

$$x = \pm 1$$

$c = 1$

$c \neq -1$

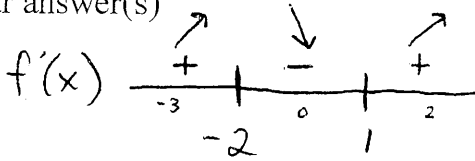
4) If  $f(x) = 2x^3 + 3x^2 - 12x$  find the following (where appropriate): Intervals where  $f(x)$  is increasing, decreasing, relative maximum points, and relative minimum points. Justify your answer(s)

$$f'(x) = 6x^2 + 6x - 12$$

$$0 = 6(x^2 + x - 2)$$

$$0 = 6(x+2)(x-1)$$

$$x = -2, x = 1$$



Rel. max  $(-2, 20)$  b/c  $f'(x)$  changes from + to -  
 Rel. min  $(1, -7)$  b/c  $f'(x)$  changes from - to +  
 $f(x)$  increasing  $(-\infty, -2) \cup (1, \infty)$  b/c  $f'(x) > 0$   
 $f(x)$  decreasing  $(-2, 1)$  b/c  $f'(x) < 0$

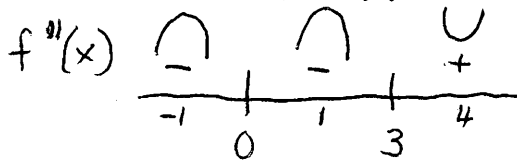
5) If  $f(x) = x^5 - 5x^4 + 3x + 7$  Find the intervals where  $f(x)$  is concave up and concave down, and find all points of inflection. (Justify your answers)

$$f'(x) = 5x^4 - 20x^3 + 3$$

$$f''(x) = 20x^3 - 60x^2$$

$$0 = 20x^2(x-3)$$

$$x = 0, x = 3$$



Point of Inflection at  $(3, -146)$  b/c  $f''(x)$  change signs.  
 $f(x)$  is concave up  $(3, \infty)$  b/c  $f''(x) > 0$   
 $f(x)$  is concave down  $(-\infty, 0) \cup (0, 3)$  b/c  $f''(x) < 0$ .

6) Sketch a labeled graph of a function,  $f$ , with the following characteristics:

$$f(-4) = 5, f(-1) = -2, f(0) = 0, f(2) = 4$$

$$f'(x) < 0 \text{ for } x < -1 \text{ or } x > 2$$

$$f'(-1) = 0, f'(2) = 0$$

$$f'(x) > 0 \text{ for } -1 < x < 2$$

$$f''(x) < 0 \text{ for } x < -4, x > 0$$

$$f''(x) > 0 \text{ for } -4 < x < 0$$

