Key

AP Calculus AB — Quiz Review 3.1-3.4 Worksheet 3

1) Find the critical points of $f(x) = x^{8/5} + x^{3/5}$

$f'(x) = \frac{8}{5}x^{3/5} + \frac{3}{5}x^{-2/5}$

$f'(x) = \frac{8x^{3/5}}{5} + \frac{3}{5x^{2/5}}$

$f'(x) = \frac{8x^1 + 3}{5x^{2/5}}$

$8x+3=0$

$\boxed{x = \frac{-3}{8}}$

$5x^{2/5} = 0$

$\boxed{x=0}$

2) Find the value(s) of the absolute extrema of the function $f(x) = 2x^3 - 3x^2 - 12x + 1$ on the interval [-2, 3]. State theorem and conditions $f(x)$ continuous $[-2,3]$, EVT

$f'(x) = 6x^2 - 6x - 12$

$f'(x) = 6(x^2 - x - 2)$

$0 = 6(x-2)(x+1)$

$x = 2,\ x = -1$

$f(-2) = -3$

$f(-1) = 8$

$f(2) = -19$

$f(3) = -8$

$\boxed{\text{Abs max is } 8 \text{ (at } x=-1) \quad \text{Abs min is } -19 \text{ (at } x=2)}$

3) If $f(x) = \frac{x^2-2x-3}{x+2}$ on [-1,3], determine if Rolle's Theorem can be applied. If yes, find the value(s) of c defined on Rolle's Theorem. State conditions and show steps.

V.A. at $x=-2$ (outside interval)

i) $f(x)$ continuous $[-1,3]$

ii) $f(x)$ differentiable $(-1,3)$

iii) $\left.\begin{array}{l} f(-1)=0 \\ f(3)=0 \end{array}\right] \rightarrow f(-1)=f(3)$

Rolle's theorem applies:

$\boxed{\text{Set } f'(x)=0}$

*quotient rule

$f'(x) = \frac{\overbrace{(2x-2)}^{f'}\overbrace{(x+2)}^{g} - \overbrace{(x^2-2x-3)}^{f}\overbrace{(1)}^{g'}}{\underbrace{(x+2)^2}_{g^2}} = \frac{2x^2+2x-4-x^2+2x+3}{(x+2)^2}$

$f'(x) = \frac{x^2+4x-1}{(x+2)^2}$

$\downarrow$

$x^2+4x-1=0$

$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$\frac{-4 \pm \sqrt{16-4(1)(-1)}}{2(1)}$

$\frac{-4 \pm \sqrt{20}}{2} = x$

$x = \frac{-4+\sqrt{20}}{2}$

$\cancel{x = \frac{-4-\sqrt{20}}{2}}$

$\boxed{c = -2 + \frac{\sqrt{20}}{2}}$

4) If $g(x) = x^3 - x - 1$ on [-1, 2], determine if the Mean Value Theorem can be applied. If yes, find the value(s) of c defined in the Mean Value Theorem. State conditions and show steps.

$g(x)$ continuous $[-1,2]$, $g(x)$ differentiable $(-1,2)$

$\boxed{\text{MVT: } f'(c) = \frac{f(b)-f(a)}{b-a}}$

$g(-1) = -1+1-1 = -1$

$g(2) = 2^3 - 2 - 1 = 5$

slope: $\frac{5--1}{2--1} = \frac{6}{3} = \boxed{2}$

$g'(x) = \boxed{3x^2-1}$

$3x^2 - 1 = 2$

$3x^2 = 3$

$x^2 = 1$

$x = \pm 1$

$\boxed{c=1}$

$\cancel{c=-1}$

4) If $f(x) = 2x^3 + 3x^2 - 12x$ find the following (where appropriate): Intervals where f(x) is increasing, decreasing, relative maximum points, and relative minimums points. Justify your answer(s)

$f'(x) = 6x^2 + 6x - 12$

$0 = 6(x^2 + x - 2)$

$0 = 6(x+2)(x-1)$

$x = -2,\ x = 1$

$f'(x)$: $+$ (test -3) | -2 | $-$ (test 0) | 1 | $+$ (test 2)

Rel. max $(-2, 20)$ b/c $f'(x)$ changes from $+$ to $-$

Rel. min $(1, -7)$ b/c $f'(x)$ changes from $-$ to $+$

$f(x)$ increasing $(-\infty, -2) \cup (1, \infty)$ b/c $f'(x) > 0$

$f(x)$ decreasing $(-2, 1)$ b/c $f'(x) < 0$

5) If $f(x) = x^5 - 5x^4 + 3x + 7$ Find the intervals where f(x) is concave up and concave down, and find all points of inflection. (Justify your answers)

$f'(x) = 5x^4 - 20x^3 + 3$

$f''(x) = 20x^3 - 60x^2$

$0 = 20x^2(x-3)$

$x = 0,\ x = 3$

$f''(x)$: $\cap$ $-$ (test -1) | 0 | $\cap$ $-$ (test 1) | 3 | $\cup$ $+$ (test 4)

Point of Inflection at $(3, -146)$ b/c $f''(x)$ change signs.

$f(x)$ is concave up $(3, \infty)$ b/c $f''(x) > 0$

$f(x)$ is concave down $(-\infty, 0) \cup (0, 3)$ b/c $f''(x) < 0$.

6) Sketch a labeled graph of a function, f, with the following characteristics:

f(-4) = 5, f(-1) = -2, f(0) = 0, f(2) = 4

f '(x) < 0 for x < -1 or x > 2

f '(-1) = 0 , f '(2) = 0

f '(x) > 0 for -1 < x < 2

f ''(x) < 0 for x < −4 , x > 0

f ''(x) > 0 for - 4 < x < 0

$f'(x)$: $-$ | -1 | $+$ | 2 | $-$

$f''(x)$: $\cap$ $-$ | -4 | $\cup$ $+$ | 0 | $\cap$ $-$