AP Calculus AB Test Review 3.1-3.4 Worksheet #4

1) Find the value(s) of the absolute extrema of the function $f(x) = 2x^3 - 3x^2 - 12x + 1$ on the interval $[-2, 3]$. State theorem and conditions

2) If $f(x) = \frac{x^2-2x-3}{x+2}$ on [-1,3], determine if Rolle's Theorem can be applied. If yes, find the value(s) of c defined on Rolle's Theorem. State conditions and show steps.

3) If $g(x) = x^3 - x - 1$ on [-1, 2], determine if the Mean Value Theorem can be applied. If yes, find the value(s) of c defined in the Mean Value Theorem. State conditions and show steps.

4) If $f(x) = 2x^3 + 3x^2 - 12x$ find the following (where appropriate): Intervals where f(x) is increasing, decreasing, relative maximum points, and relative minimums points. Justify your answer(s)

5) If $f(x) = x^5 - 5x^4 + 3x + 7$ Find the intervals where f(x) is concave up and concave down, and find all points of inflection. (Justify your answers)

6) Sketch a labeled graph of a function, f, with the following characteristics:
$f(-4) = 5$, $f(-1) = -2$, $f(0) = 0$, $f(2) = 4$
$f'(x) < 0$ for $x < -1$ and $x > 2$
$f'(-1) = 0$, $f'(2) = 0$
$f'(x) > 0$ for $-1 < x < 2$
$f''(x) < 0$ for $x < -4$, $x > 0$
$f''(x) > 0$ for $-4 < x < 0$

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Key

1) Find the value(s) of the absolute extrema of the function $f(x) = 2x^3 - 3x^2 - 12x + 1$ on the interval [-2, 3]. State theorem and conditions

f(x) continuous [-2, 3] *Apply EVT

$f'(x) = 6x^2 - 6x - 12$

$0 = 6(x^2 - x - 2)$

$0 = 6(x-2)(x+1)$

$x = 2, \ x = -1$

*both critical pts inside interval [-2, 3]

$f(-2) = -3$

$f(-1) = 8$ (max)

$f(2) = -19$ (min)

$f(3) = -8$

$\boxed{\text{Abs max is } 8 \text{ at } x=-1 \quad \text{Abs min is } -19 \text{ at } x=2}$

2) If $f(x) = \frac{x^2-2x-3}{x+2}$ on [-1,3], determine if Rolle's Theorem can be applied. If yes, find the value(s) of *c* defined on Rolle's Theorem. State conditions and show steps.

V.A. at $x=-2$ (outside interval) f(x) continuous [-1, 3], differentiable (-1, 3)

$f(-1) = 0$

$f(3) = 0$

$f(-1) = f(3)$

Rolle's theorem applies.

$f'(x) = \frac{(2x-2)(x+2)-(x^2-2x-3)(1)}{(x+2)^2} = \frac{2x^2+2x-4-x^2+2x+3}{(x+2)^2}$

$f'(x) = \frac{x^2+4x-1}{(x+2)^2}$

$0 = x^2+4x-1$

$\frac{-4 \pm \sqrt{16-4(1)(-1)}}{2(1)} = \frac{-4 \pm \sqrt{20}}{2}$

$x = \frac{-4 \pm 2\sqrt{5}}{2}$ $x = -2+\sqrt{5}$, $x = -2-\sqrt{5}$

$\boxed{c = -2+\sqrt{5}}$

3) If $g(x) = x^3 - x - 1$ on [-1, 2], determine if the Mean Value Theorem can be applied. If yes, find the value(s) of *c* defined in the Mean Value Theorem. State conditions and show steps.

g(x) continuous [-1, 2], differentiable (-1, 2)

* MVT: $f'(c) = \frac{f(b)-f(a)}{b-a}$

$g(-1) = -1$

$g(2) = 5$

slope: $\frac{5-(-1)}{2-(-1)} = \frac{6}{3} = 2$

$g'(x) = 3x^2 - 1$

$2 = 3x^2 - 1$

$3 = 3x^2$

$1 = x^2$

$c = 1, \ c = -1$

$\boxed{c = 1 \text{ in the open interval } (-1, 2)}$

4) If $f(x) = 2x^3 + 3x^2 - 12x$ find the following (where appropriate): Intervals where f(x) is increasing, decreasing, relative maximum points, and relative minimums points. Justify your answer(s)

$f'(x) = 6x^2 + 6x - 12$

$0 = 6(x^2 + x - 2)$

$0 = 6(x+2)(x-1)$

$x = -2, x = 1$

$f'(x)$: $\nearrow$ $+$ (test -3) | -2 | $\searrow$ $-$ (test 0) | 1 | $\nearrow$ $+$ (test 2)

Rel. max $(-2, \underline{20})$ b/c $f'(x)$ changes from $+$ to $-$

Rel. min $(1, \underline{-7})$ b/c $f'(x)$ changes from $-$ to $+$

$f(x)$ increasing $(-\infty, -2) \cup (1, \infty)$ b/c $f'(x) > 0$

$f(x)$ decreasing $(-2, 1)$ b/c $f'(x) < 0$

5) If $f(x) = x^5 - 5x^4 + 3x + 7$ Find the intervals where f(x) is concave up and concave down, and find all points of inflection. (Justify your answers)

$f'(x) = 5x^4 - 20x^3 + 3$

$f''(x) = 20x^3 - 60x^2$

$0 = 20x^2(x-3)$

$x = 0, x = 3$

$f''(x)$: $\cap$ $-$ (test -1) | 0 | $\cap$ $-$ (test 1) | 3 | $\cup$ $+$ (test 4)

Point of Inflection at $(3, \underline{-146})$ b/c $f''(x)$ change signs.

$f(x)$ is concave up $(3, \infty)$ b/c $f''(x) > 0$

$f(x)$ is concave down $(-\infty, 0) \cup (0, 3)$ b/c $f''(x) < 0$.

6) Sketch a labeled graph of a function, f, with the following characteristics:

f(-4) = 5, f(-1) = -2, f(0) = 0, f(2) = 4

f '(x) < 0 for x < -1 or x > 2

f '(-1) = 0 , f '(2) = 0

f '(x) > 0 for -1 < x < 2

f ''(x) < 0 for x < -4 , x > 0

f ''(x) > 0 for - 4 < x < 0

$f'(x)$: $\searrow$ $-$ | -1 | $\nearrow$ $+$ | 2 | $\searrow$ $-$

$f''(x)$: $\cap$ $-$ | -4 | $\cup$ $+$ | 0 | $\cap$ $-$