

AP Calculus AB **Test Review 3.1-3.4 Worksheet #4**

1) Find the value(s) of the absolute extrema of the function

$f(x) = 2x^3 - 3x^2 - 12x + 1$ on the interval $[-2, 3]$. State theorem and conditions

2) If $f(x) = \frac{x^2 - 2x - 3}{x + 2}$ on $[-1, 3]$, determine if Rolle's Theorem can be applied. If yes, find the value(s) of c defined on Rolle's Theorem. State conditions and show steps.

3) If $g(x) = x^3 - x - 1$ on $[-1, 2]$, determine if the Mean Value Theorem can be applied. If yes, find the value(s) of c defined in the Mean Value Theorem. State conditions and show steps.

4) If $f(x) = 2x^3 + 3x^2 - 12x$ find the following (where appropriate): Intervals where $f(x)$ is increasing, decreasing, relative maximum points, and relative minimum points. Justify your answer(s)

5) If $f(x) = x^5 - 5x^4 + 3x + 7$ Find the intervals where $f(x)$ is concave up and concave down, and find all points of inflection. (Justify your answers)

6) Sketch a labeled graph of a function, f , with the following characteristics:

$$f(-4) = 5, f(-1) = -2, f(0) = 0, f(2) = 4$$

$$f'(x) < 0 \text{ for } x < -1 \text{ and } x > 2$$

$$f'(-1) = 0, f'(2) = 0$$

$$f'(x) > 0 \text{ for } -1 < x < 2$$

$$f''(x) < 0 \text{ for } x < -4, x > 0$$

$$f''(x) > 0 \text{ for } -4 < x < 0$$

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Key

1) Find the value(s) of the absolute extrema of the function $f(x) = 2x^3 - 3x^2 - 12x + 1$ on the interval $[-2, 3]$. State theorem and conditions

$f(x)$ continuous $[-2, 3]$ *Apply EVT

$f'(x) = 6x^2 - 6x - 12$

$0 = 6(x^2 - x - 2)$

$0 = 6(x-2)(x+1)$

$x = 2, x = -1$

*both critical pts inside interval $[-2, 3]$

$f(-2) = -3$

$f(-1) = 8$ (max)

$f(2) = -19$ (min)

$f(3) = -8$

Abs max is 8 at $x = -1$
Abs min is -19 at $x = 2$

2) If $f(x) = \frac{x^2 - 2x - 3}{x + 2}$ on $[-1, 3]$, determine if Rolle's Theorem can be applied. If yes, find the value(s) of c defined on Rolle's Theorem. State conditions and show steps.

V.A. at $x = -2$ (outside interval) $f(x)$ continuous $[-1, 3]$, differentiable $(-1, 3)$

$f(-1) = 0$
 $f(3) = 0$ } $f(-1) = f(3)$

$f'(x) = \frac{(2x-2)(x+2) - (x^2-2x-3)(1)}{(x+2)^2} = \frac{2x^2+2x-4-x^2+2x+3}{(x+2)^2}$

$f'(x) = \frac{x^2+4x-1}{(x+2)^2}$

Rolle's theorem applies.

$0 = x^2 + 4x - 1$

$\frac{-4 \pm \sqrt{16 - 4(1)(-1)}}{2(1)} = \frac{-4 \pm \sqrt{20}}{2}$

$x = \frac{-4 \pm 2\sqrt{5}}{2}$ $x = -2 + \sqrt{5}$ $x = -2 - \sqrt{5}$

3) If $g(x) = x^3 - x - 1$ on $[-1, 2]$, determine if the Mean Value Theorem can be applied. If yes, find the value(s) of c defined in the Mean Value Theorem. State conditions and show steps.

$c = -2 + \sqrt{5}$

$g(x)$ continuous $[-1, 2]$, differentiable $(-1, 2)$

*MVT:

$f'(c) = \frac{f(b) - f(a)}{b - a}$

$g(-1) = -1$

$g(2) = 5$

slope: $\frac{5 - (-1)}{2 - (-1)} = \frac{6}{3} = 2$

$g'(x) = 3x^2 - 1$

$2 = 3x^2 - 1$

$3 = 3x^2$

$1 = x^2$

$c = 1, c = -1$

$c = 1$ in the open interval $(-1, 2)$

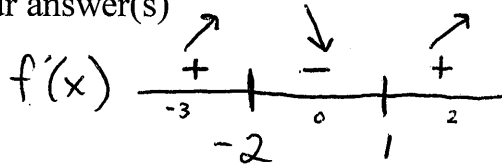
4) If $f(x) = 2x^3 + 3x^2 - 12x$ find the following (where appropriate): Intervals where $f(x)$ is increasing, decreasing, relative maximum points, and relative minimum points. Justify your answer(s)

$$f'(x) = 6x^2 + 6x - 12$$

$$0 = 6(x^2 + x - 2)$$

$$0 = 6(x+2)(x-1)$$

$$x = -2, x = 1$$



Rel. max $(-2, 20)$ b/c $f'(x)$ changes from + to -

Rel. min $(1, -7)$ b/c $f'(x)$ changes from - to +

$f(x)$ increasing $(-\infty, -2) \cup (1, \infty)$ b/c $f'(x) > 0$

$f(x)$ decreasing $(-2, 1)$ b/c $f'(x) < 0$

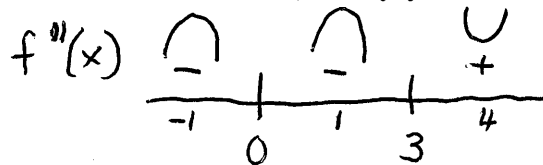
5) If $f(x) = x^5 - 5x^4 + 3x + 7$ Find the intervals where $f(x)$ is concave up and concave down, and find all points of inflection. (Justify your answers)

$$f'(x) = 5x^4 - 20x^3 + 3$$

$$f''(x) = 20x^3 - 60x^2$$

$$0 = 20x^2(x-3)$$

$$x = 0, x = 3$$



Point of Inflection at $(3, -146)$ b/c $f''(x)$ change signs.

$f(x)$ is concave up $(3, \infty)$ b/c $f''(x) > 0$

$f(x)$ is concave down $(-\infty, 0) \cup (0, 3)$ b/c $f''(x) < 0$.

6) Sketch a labeled graph of a function, f , with the following characteristics:

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