AP Calculus AB   Quiz Review 3.1-3.4 Worksheet #2

1) Find all critical numbers for $g(x) = 16\sqrt{x} - x^2$

2) Find the value(s) of the absolute extrema of the function $f(x) = x^3 + 6x^2$ on the interval [-3, 1] . State theorem and conditions

3) If $f(x) = x^2 - 8x + 5$ on [2,6], determine if Rolle's Theorem can be applied. If yes, find the value(s) of c defined on Rolle's Theorem. State conditions and show steps.

4) If $g(x) = x^3 - x$ on [1, 2], determine if the Mean Value Theorem can be applied. If yes, find the value(s) of c defined in the Mean Value Theorem. State conditions and show steps.

5) If $f(x) = \frac{x^2}{x^2-9}$ find the following (where appropriate): Intervals where f(x) is increasing, decreasing, relative maximum points, and relative minimums points. Justify your answer(s)

6) If $f(x) = \frac{1}{2}x^4 + 2x^3$ Find the intervals where f(x) is concave up and concave down, and find all points of inflection. (Justify your answers)

7) If $f(x) = -x^3 + 7x^2 - 15x$ use the Second Derivative Test to find all relative extrema. Justify your answers.

8) Sketch a labeled graph of a function, f, with the following characteristics:

$f(0) = 4$, $f(6) = 0$, $f(2) = 0$, $f(4) = 2$

$f'(x) < 0$ for $x < 2$ or $x > 4$

$f'(2)$ does not exist

$f'(4) = 0$

$f'(x) > 0$ for $2 < x < 4$

$f''(x) < 0$ for $x \neq 2$

Solution Key

AP Calculus AB **Quiz Review 3.1-3.4 Worksheet #2**

1) Find all critical numbers for $g(x) = 16\sqrt{x} - x^2$

$g(x) = 16x^{1/2} - x^2$

$g'(x) = 16 \cdot \frac{1}{2} x^{-1/2} - 2x$

$g'(x) = \frac{8}{x^{1/2}} - 2x$

$g'(x) = \frac{8 - 2x^{3/2}}{x^{1/2}}$

$8 - 2x^{3/2} = 0$

$-2x^{3/2} = -8$

$x^{3/2} = 4 \quad x = (4)^{2/3}$

$x^{1/2} = 0$

$x = 0$

$\boxed{\text{critical points: } x = 4^{2/3}, x = 0}$

2) Find the value(s) of the absolute extrema of the function $f(x) = x^3 + 6x^2$ on the interval [-3, 1]. State theorem and conditions

*Apply EVT: f(x) continuous [-3, 1]

$f'(x) = 3x^2 + 12x$

$0 = 3x(x+4)$

$x = 0, \cancel{x = -4}$ outside interval

$f(-3) = (-3)^3 + 6(-3)^2 = 27$

$f(0) = 0$

$f(1) = 1 + 6 = 7$

$\boxed{\text{Abs maximum is } 27 \text{ at } x = -3 \quad \text{Abs minimum is } 0 \text{ at } x = 0}$

3) If $f(x) = x^2 - 8x + 5$ on [2,6], determine if Rolle's Theorem can be applied. If yes, find the value(s) of c defined on Rolle's Theorem. State conditions and show steps. f(x) continuous on [2,6], f(x) differentiable (2,6)

$f(2) = -7$

$f(6) = -7$

$f(2) = f(6)$ ✓

$f'(x) = 2x - 8$

set $f'(x) = 0$

$2x - 8 = 0$

$x = 4$

$\boxed{\text{By Rolle's theorem } c = 4}$

4) If $g(x) = x^3 - x$ on [1, 2], determine if the Mean Value Theorem can be applied. If yes, find the value(s) of c defined in the Mean Value Theorem. State conditions and show steps. g(x) continuous [1, 2] g(x) differentiable (1, 2)

*MVT: $g'(c) = \frac{g(b) - g(a)}{b - a}$

$g(1) = 1 - 1 = 0$

$g(2) = 2^3 - 2 = 6$

$\frac{g(2) - g(1)}{2 - 1} = \frac{6 - 0}{1} = 6$

$g'(x) = 3x^2 - 1$

$6 = 3x^2 - 1$

$7 = 3x^2$

$\frac{7}{3} = x^2$

$x^2 = 7/3$

$x = \pm\sqrt{7/3}$

$c = \sqrt{7/3}, \cancel{c = -\sqrt{7/3}}$

$\boxed{\text{By MVT, } c = \sqrt{7/3}}$

5) If $f(x) = \frac{x^2}{x^2-9}$ find the following (where appropriate): Intervals where f(x) is increasing, decreasing, relative maximum points, and relative minimums points. Justify your answer(s)

$VA: x=3, x=-3$

$f(x) = \frac{x^2}{(x+3)(x-3)}$

$f'(x) = \frac{2x(x^2-9)-x^2(2x)}{(x^2-9)^2}$

$f'(x) = \frac{2x^3-18x-2x^3}{(x^2-9)^2}$

$f'(x) = \frac{-18x}{(x^2-9)^2}$ $\quad -18x=0$, $x=0$ $\quad x^2-9=0$, $x=3,-3$

Critical pts: $x=0, 3, -3$

f': $+$ | -3 | $+$ | 0 | $-$ | 3 | $-$

$f(x)$ increasing $(-\infty,-3), (-3,0)$ b/c $f'(x)>0$

$f(x)$ decreasing $(0,3), (3,\infty)$ b/c $f'(x)<0$

Rel. max at $(0,0)$ b/c $f'(x)$ changes from $+$ to $-$

6) If $f(x) = \frac{1}{2}x^4 + 2x^3$ Find the intervals where f(x) is concave up and concave down, and find all points of inflection. (Justify your answers)

$f'(x) = \frac{1}{2}\cdot 4x^3 + 6x^2 = 2x^3+6x^2$

$f''(x) = 6x^2+12x$

$0 = 6x(x+2)$

$x = 0, -2$

f'': $\cup$ $+$ | -2 | $\cap$ $-$ | 0 | $\cup$ $+$

$f(x)$ concave up $(-\infty,-2), (0,\infty)$ b/c $f''(x)>0$

$f(x)$ concave down $(-2,0)$ b/c $f''(x)<0$

POI at $(-2, f(-2))$ and $(0, f(0))$ b/c $f''(x)$ change signs.

7) If $f(x) = -x^3 + 7x^2 - 15x$ use the Second Derivative Test to find all relative extrema. Justify your answers.

$f'(x) = -3x^2+14x-15$

$0 = -(3x^2-14x+15)$

$0 = -(3x-5)(x-3)$

$\boxed{x = 5/3,\ x=3}$

$f''(x) = -6x+14$

$f''(5/3) = -6(5/3)+14 > 0$, concave up

$\boxed{\text{Rel. minimum at } (5/3, f(5/3))}$

$f''(3) = -6(3)+14 = -4 < 0$ concave down

$\boxed{\text{Rel. maximum at } (3, f(3))}$

8) Sketch a labeled graph of a function, f, with the following characteristics:

f(0) = 4, f(6) = 0, f(2) = 0, f(4) = 2

f '(x) < 0 for x < 2 or x > 4

f '(2) does not exist

f '(4) = 0

f '(x) > 0 for 2 < x < 4

f ''(x) < 0 for x $\neq$ 2

$f'(x)$: $-$ | 2 | $+$ | 4 | $-$

$f''(x)$: $\cap$ $-$ | 2 | $\cap$ $-$