

5) If $f(x) = \frac{x^2}{x^2-9}$ find the following (where appropriate): Intervals where $f(x)$ is increasing, decreasing, relative maximum points, and relative minimum points. Justify your answer(s)

6) If $f(x) = \frac{1}{2}x^4 + 2x^3$ Find the intervals where $f(x)$ is concave up and concave down, and find all points of inflection. (Justify your answers)

7) If $f(x) = -x^3 + 7x^2 - 15x$ use the Second Derivative Test to find all relative extrema. Justify your answers.

8) Sketch a labeled graph of a function, f , with the following characteristics:

$$f(0) = 4, f(6) = 0, f(2) = 0, f(4) = 2$$

$$f'(x) < 0 \text{ for } x < 2 \text{ or } x > 4$$

$f'(2)$ does not exist

$$f'(4) = 0$$

$$f'(x) > 0 \text{ for } 2 < x < 4$$

$$f''(x) < 0 \text{ for } x \neq 2$$

AP Calculus AB

Quiz Review 3.1-3.4 Worksheet #2

Solution Key

1) Find all critical numbers for $g(x) = 16\sqrt{x} - x^2$

$$g(x) = 16x^{1/2} - x^2$$

$$g'(x) = 16 \cdot \frac{1}{2} x^{-1/2} - 2x$$

$$g'(x) = \frac{8}{x^{1/2}} - 2x$$

$$g'(x) = \frac{8 - 2x^{3/2}}{x^{1/2}}$$

$$\begin{array}{l|l} 8 - 2x^{3/2} = 0 & x^{1/2} = 0 \\ -2x^{3/2} = -8 & x = 0 \\ x^{3/2} = 4 & x = (4)^{2/3} \end{array}$$

critical points:
 $x = 4^{2/3}, x = 0$

2) Find the value(s) of the absolute extrema of the function $f(x) = x^3 + 6x^2$ on the interval $[-3, 1]$. State theorem and conditions

*Apply EVT: $f(x)$ continuous $[-3, 1]$

$$f'(x) = 3x^2 + 12x$$

$$0 = 3x(x + 4)$$

$$x = 0, x = -4 \text{ (outside interval)}$$

$$f(-3) = (-3)^3 + 6(-3)^2 = 27$$

$$f(0) = 0$$

$$f(1) = 1 + 6 = 7$$

Abs maximum is 27 at $x = -3$
 Abs minimum is 0 at $x = 0$

3) If $f(x) = x^2 - 8x + 5$ on $[2, 6]$, determine if Rolle's Theorem can be applied. If yes, find the value(s) of c defined on Rolle's Theorem. State conditions and show steps.

$f(x)$ continuous on $[2, 6]$, $f(x)$ differentiable $(2, 6)$

$$f(2) = -7 \quad f(6) = -7 \quad \checkmark$$

$$f'(x) = 2x - 8$$

$$\text{set } f'(x) = 0$$

$$2x - 8 = 0$$

$$x = 4$$

By Rolle's Theorem
 $c = 4$

4) If $g(x) = x^3 - x$ on $[1, 2]$, determine if the Mean Value Theorem can be applied. If yes, find the value(s) of c defined in the Mean Value Theorem. State conditions and show steps.

$g(x)$ continuous $[1, 2]$ $g(x)$ differentiable $(1, 2)$

*MVT: $g'(c) = \frac{g(b) - g(a)}{b - a}$

$$g(1) = 1 - 1 = 0$$

$$g(2) = 2^3 - 2 = 6$$

$$\frac{g(2) - g(1)}{2 - 1} = \frac{6 - 0}{1} = 6$$

$$g'(x) = 3x^2 - 1$$

$$6 = 3x^2 - 1$$

$$7 = 3x^2$$

$$\frac{7}{3} = x^2$$

$$x^2 = \frac{7}{3}$$

$$x = \pm \sqrt{\frac{7}{3}}$$

$$c = \sqrt{\frac{7}{3}}, c = -\sqrt{\frac{7}{3}}$$

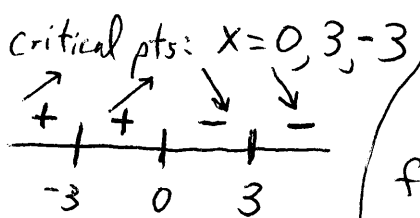
By MVT, $c = \sqrt{\frac{7}{3}}$

5) If $f(x) = \frac{x^2}{x^2-9}$ find the following (where appropriate): Intervals where $f(x)$ is increasing, decreasing, relative maximum points, and relative minimum points. Justify your answer(s)

$$f'(x) = \frac{2x(x^2-9) - x^2(2x)}{(x^2-9)^2}$$

$$f'(x) = \frac{2x^3 - 18x - 2x^3}{(x^2-9)^2}$$

$$f'(x) = \frac{-18x}{(x^2-9)^2} \quad \begin{array}{l} -18x=0 \\ x=0 \end{array} \quad \begin{array}{l} x^2-9=0 \\ x=3, -3 \end{array}$$



$f(x)$ increasing $(-\infty, -3), (-3, 0)$
 b/c $f'(x) > 0$
 $f(x)$ decreasing $(0, 3), (3, \infty)$
 b/c $f'(x) < 0$
 Rel. max at $(0, 0)$ b/c $f'(x)$ changes from + to -

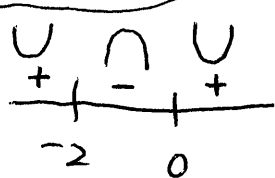
6) If $f(x) = \frac{1}{2}x^4 + 2x^3$ Find the intervals where $f(x)$ is concave up and concave down, and find all points of inflection. (Justify your answers)

$$f'(x) = \frac{1}{2} \cdot 4x^3 + 6x^2 = 2x^3 + 6x^2$$

$$f''(x) = 6x^2 + 12x$$

$$0 = 6x(x+2)$$

$$x = 0, -2$$



$f(x)$ concave up $(-\infty, -2), (0, \infty)$
 b/c $f''(x) > 0$

$f(x)$ concave down $(-2, 0)$ b/c $f''(x) < 0$

POI at $(-2, f(-2))$ and $(0, f(0))$
 b/c $f''(x)$ change signs.

7) If $f(x) = -x^3 + 7x^2 - 15x$ use the Second Derivative Test to find all relative extrema. Justify your answers.

$$f'(x) = -3x^2 + 14x - 15$$

$$0 = -(3x^2 - 14x + 15)$$

$$0 = -(3x - 5)(x - 3)$$

$$x = \frac{5}{3}, x = 3$$

$$f''(x) = -6x + 14$$

$$f''(\frac{5}{3}) = -6(\frac{5}{3}) + 14 > 0 \text{ , concave up } \curvearrowright$$

Rel. minimum at $(\frac{5}{3}, f(\frac{5}{3}))$

$$f''(3) = -6(3) + 14 = -4 < 0 \text{ concave down } \curvearrowleft$$

Rel. maximum at $(3, f(3))$

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