Key

3.1 AP Practice Problems (p. 235) – The Chain Rule

1. If $f(x) = \underbrace{(x+3)}_{f}\underbrace{(x^2-2)^4}_{g}$, then $f'(x) =$

(A) $8x(x^2-2)^3$

(B) $8x(x+3)(x^2-2)^3$

(C) $8x(x^2+x+1)(x^2-2)^3$

$\boxed{\text{(D)}}$ $(9x^2+24x-2)(x^2-2)^3$

*product Rule

chain Rule

out: $(\ )^4$
in: x^2-2

$f'(x) = \underbrace{(1)}_{f'}\underbrace{(x^2-2)^4}_{g} + \underbrace{(x+3)}_{f}\cdot\underbrace{4(x^2-2)^3\cdot(2x)}_{g'}$

$f'(x) = (x^2-2)^4 + 8x(x+3)(x^2-2)^3$

$f'(x) = (x^2-2)^3\left[x^2-2+8x(x+3)\right]$

$\boxed{f'(x) = (x^2-2)^3(9x^2+24x-2)}$

2. If $f(x) = \sec(4x)$, then $f'\left(\frac{\pi}{6}\right) =$

(A) $\frac{\sqrt{3}}{2}$ (B) $2\sqrt{3}$ $\boxed{\text{(C)}}$ $8\sqrt{3}$ (D) $-8\sqrt{3}$

$f'(x) = \sec(4x)\tan(4x)\cdot 4 = 4\sec(4x)\tan(4x)$

$f'\left(\frac{\pi}{6}\right) = 4\sec\left(4\cdot\frac{\pi}{6}\right)\tan\left(4\cdot\pi/6\right)$

$f'(\pi/6) = 4\sec\left(\frac{2\pi}{3}\right)\tan(2\pi/3)$

$f'(\pi/6) = 4\cdot(-2)\cdot(-\sqrt{3}) = \boxed{8\sqrt{3}}$

3. $\frac{d}{dx}e^{-3/x} =$

$\boxed{\text{(A)}}$ $\frac{3e^{-3/x}}{x^2}$ (B) $-3e^{-3/x}$

(C) $-\frac{3e^{-3/x}}{x^2}$ (D) $-3x^2e^{-3/x}$

$y = e^{-3/x}$

$y = e^{-3x^{-1}}$

$y' = e^{-3x^{-1}}\cdot 3x^{-2}$

$y' = \frac{3}{x^2}\cdot e^{-3/x}$

$\boxed{\frac{dy}{dx} = \frac{3e^{-3/x}}{x^2}}$

4. $\frac{d}{dx}\tan e^{-x} =$

(A) $\sec^2 e^{-x}$ (B) $-x\sec^2 e^{-x}$

(C) $e^{-x}\sec^2 e^{-x}$ $\boxed{\text{(D)}}$ $-e^{-x}\sec^2 e^{-x}$

$y = \tan(e^{-x})$

$y' = \sec^2(e^{-x})\cdot e^{-x}\cdot(-1)$

$\boxed{\frac{dy}{dx} = -e^{-x}\sec^2(e^{-x})}$

5. An equation of the normal line to the graph of $f(x) = 2(10-x)^2$ at the point (9, 2) is

perpendicular to tangent line

$\boxed{\text{(A)}}$ $y-2 = \frac{1}{4}(x-9)$ (B) $y-2 = -\frac{1}{4}(x-9)$

(C) $y-2 = -4(x-9)$ (D) $y-2 = 4(x-9)$

$f(x) = 2(10-x)^2$

out: $2(\ )^2$
in: $10-x$

$f'(x) = 4(10-x)^1\cdot(-1)$

$f'(x) = -4(10-x)$

$f'(9) = -4(10-9)$

$f'(9) = -4$

point: $(9,2)$

slope: $m = -4$

slope (normal line)

$\boxed{m_2 = 1/4}$

$\boxed{y-2 = \frac{1}{4}(x-9)}$

6. If $y = \left(\frac{e^{4x}}{2x}\right)^2$, the instantaneous rate of change of y with respect to x is:

$\boxed{\text{(A)}}$ $\frac{(4x-1)e^{8x}}{2x^3}$ (B) $\frac{(2x-1)e^{8x}}{x^3}$

(C) $\frac{(4x-1)e^{8x}}{x^3}$ (D) $\frac{(x-1)e^{8x}}{x^3}$

*chain Rule out: $(\ )^2$ in: $\frac{e^{4x}}{2x}$

$$y' = 2\left(\frac{e^{4x}}{2x}\right)^1 \cdot \frac{\overbrace{e^{4x}\cdot 4}^{f'} \cdot \overbrace{2x}^{g} - \overbrace{e^{4x}}^{f} \cdot \overbrace{2}^{g'}}{\underbrace{(2x)^2}_{g^2}}$$

$$y' = \frac{2e^{4x}(8xe^{4x} - 2e^{4x})}{2x \cdot 4x^2} \rightarrow \frac{2e^{4x}\cdot 2e^{4x}(4x-1)}{8x^3}$$

$$\boxed{y' = \frac{e^{8x}(4x-1)}{2x^3}}$$

7. An equation of the tangent line to the graph of $g(x) = \sin(2x)$ at $x = \frac{\pi}{3}$ is

(A) $x + y = \frac{2\pi - 3\sqrt{3}}{6}$ $\boxed{\text{(B)}}$ $x + y = \frac{3\sqrt{3} + 2\pi}{6}$

(C) $x + y = \frac{2\sqrt{3} + 3\pi}{6}$ (D) $y - x = \frac{2\pi - 3\sqrt{3}}{6}$

$g'(x) = \cos(2x)\cdot 2$

$g'(\pi/3) = 2\cos(2\cdot\pi/3)$

$g'(\pi/3) = 2(-1/2) = -1$

$g(\pi/3) = \sin(2\pi/3)$

$g(\pi/3) = \frac{\sqrt{3}}{2}$

point: $(\pi/3, \frac{\sqrt{3}}{2})$

slope: $m = -1$

$y - \frac{\sqrt{3}}{2} = -1(x - \pi/3)$

$6[y - \frac{\sqrt{3}}{2} = -1x + \pi/3]$

$6y - 3\sqrt{3} = -6x + 2\pi$

$6x + 6y = 3\sqrt{3} + 2\pi$

$6(x+y) = 3\sqrt{3} + 2\pi$

$\boxed{x + y = \frac{3\sqrt{3} + 2\pi}{6}}$

8) $y = [\cos(x^2)]^3$ $y' = 3[\cos(x^2)]^2 \cdot -\sin(x^2)\cdot 2x$

8. If $y = \cos^3(x^2)$, then $\frac{dy}{dx} =$

(A) $-6x\cos^2(x^2)$ $\boxed{\text{(B)}}$ $-6x\cos^2(x^2)\sin x^2$

(C) $6x\cos^2(x^2)\sin x^2$ (D) $-3\cos^2(x^2)\sin x$

$\boxed{y' = -6x\cos^2(x^2)\sin(x^2)}$

9. If $f(x) = e^{\sin^2 x}$, then $f'(x) =$

$\boxed{\text{(A)}}$ $2e^{\sin^2 x}\sin x\cos x$ (B) $e^{\sin x(\sin x + 2\cos x)}$

(C) $2e^x \sin x\cos x$ (D) $e^x \sin^2 x$

$f(x) = e^{(\sin x)^2}$ *chain Rule out: $e^{(\ )}$ in: $(\sin x)^2$

$f'(x) = e^{(\sin x)^2}\cdot 2(\sin x)\cdot\cos x$

$\boxed{f'(x) = 2e^{(\sin^2 x)}\cdot\sin x\cos x}$

10. An object is moving along the x-axis. Its position (in kilometers) at time $t \geq 0$ (in hours) is given by $s(t) = \sin(3t) - \cos(4t)$. What is the acceleration of the object at time $t = \frac{\pi}{2}$?

(A) 7 km/h^2 (B) 9 km/h^2

(C) -7 km/h^2 $\boxed{\text{(D)}}$ 25 km/h^2

$s'(t) = \cos(3t)\cdot 3 - -\sin(4t)\cdot 4 = 3\cos(3t) + 4\sin(4t)$

$s''(t) = -3\sin(3t)\cdot 3 + 4\cos(4t)\cdot 4$

$s''(t) = -9\sin(3t) + 16\cos(4t)$

$s''(\pi/2) = -9\sin(\frac{3\pi}{2}) + 16\cos(4\cdot\frac{\pi}{2})$

$s''(\pi/2) = -9(-1) + 16(1)$

$\boxed{s''(\pi/2) = 25}$

$*\ \frac{d}{dx}a^u = \ln a \cdot a^u \cdot u'$

11. $\frac{d}{dx}5^x =$

(A) $5^x \ln 5$ (B) $(5^{x-1})x$ (C) 5^{x-1} (D) $\frac{5^x}{\ln 5}$

$y = 5^x$

$\frac{dy}{dx} = \ln 5 \cdot 5^x \cdot (1)$

$\boxed{y' = 5^x \cdot \ln 5}$

12. If $y = e^{kx}$, then $\frac{d^k}{dx^k}(e^{kx}) =$

(A) k^2e^{kx} (B) k^ke^x (C) k^ke^{kx} (D) $k!\,e^{kx}$

$y' = e^{kx} \cdot k = ke^{kx}$

$y'' = ke^{kx} \cdot k \to k^2e^{kx}$

$y''' = k^2e^{kx} \cdot k = k^3e^{kx}$

$y^4(x) = k^3e^{kx} \cdot k = k^4e^{kx}$

$\boxed{\frac{d^k}{dx^k}(e^{kx}) = k^ke^{kx}}$

13. If the functions f and g are both twice differentiable and if $h(x) = (f \circ g)(x) = f(g(x))$, then $h''(x) =$

(A) $[f'(g(x)) \cdot g'(x)]^2$

(B) $f'(g(x)) \cdot g''(x) + g'(x)f''(g(x))$

(C) $f'(g(x)) \cdot g''(x) + [g'(x)]^2 f''(g(x))$

(D) $f'(g(x)) \cdot g'(x)$

chain Rule

out: $f'[g(x)]$

in: $g(x)$

$h(x) = f[g(x)]$

$h'(x) = \underbrace{f'[g(x)]}_{f} \cdot \underbrace{g'(x)}_{g}$

$h''(x) = \overbrace{f''[g(x)] \cdot g'(x) \cdot g'(x)}^{f'} + \overbrace{f'[g(x)]}^{f} \cdot \overbrace{g''(x)}^{g'}$

$\boxed{h''(x) = f''[g(x)](g'(x))^2 + f'(g(x)) \cdot g''(x)}$

14. The velocity v (in meters/second) of an object moving on a line is given by $v(t) = 3 - 1.5^{-t^2}$, $t \geq 0$. What is the acceleration of the object at $t = 4$ seconds?

(A) 0.005 m/s² (B) −0.005 m/s²

(C) 0.012 m/s² (D) 2.998 m/s²

$*\ \frac{d}{dx}a^u = (\ln a) \cdot a^u \cdot u'$

$v(t) = 3 - 1.5^{-t^2}$

$v'(t) = a(t) = 0 - \ln(1.5) \cdot (1.5)^{-t^2} \cdot (-2t)$

$a(4) = v'(4) = -\ln(1.5) \cdot 1.5^{(-4)^2} \cdot -2(4) = \boxed{0.005\ m/s^2}$