Ch. 3.1 Exercise Problems - Chain Rule

p. 231-235 #19-45 odd, 55, 57, 75

19) $g(x) = (x^2+5)^4$

out: $(\ )^4$

in: x^2+5

$g'(x) = 4(\ \ )^3 \cdot (2x)$

$g'(x) = 4(x^2+5)^3 \cdot 2x$

$\boxed{g'(x) = 8x(x^2+5)^3}$

21) $f(u) = \left(u - \frac{1}{u}\right)^3$

out: $(\ )^3$

in: $u - u^{-1}$

$f'(u) = 3(\ \ \ )^2 \cdot (1 - -1u^{-2})$

$\boxed{f'(u) = 3\left(u - \frac{1}{u}\right)^2\left(1 + \frac{1}{u^2}\right)}$

23) $g(x) = (4x + e^x)^3$

out: $(\ )^3$

in: $4x + e^x$

$\boxed{g'(x) = 3(4x+e^x)^2 \cdot (4+e^x)}$

25) $f(x) = \tan^2 x$

$f(x) = (\tan x)^2$

out: $(\ )^2$

in: $\tan x$

$f'(x) = 2(\ \ \ ) \cdot \sec^2 x$

$\boxed{f'(x) = 2\tan x \sec^2 x}$

27) $f(z) = (\tan z + \cos z)^2$

out: $(\ )^2$

in: $\tan z + \cos z$

$f'(z) = 2(\ \ \ \ ) \cdot (\sec^2 z - \sin z)$

$\boxed{f'(z) = 2(\tan z + \cos z)(\sec^2 z - \sin z)}$

$\boxed{3.1}$

29) $y = \underbrace{(x^2+4)^2}_{f}\underbrace{(2x^3-1)^3}_{g}$

* product Rule
* chain Rule

$y' = \underbrace{2(x^2+4)(2x)}_{f'} \cdot \underbrace{(2x^3-1)^3}_{g} + \underbrace{(x^2+4)^2}_{f} \cdot \underbrace{3(2x^3-1)^2(6x^2)}_{g'}$

pull out common factors

$y' = 4x(x^2+4)(2x^3-1)^3 + 18x^2(x^2+4)^2(2x^3-1)^2$

$y' = 2x(x^2+4)(2x^3-1)^2\left[2(2x^3-1) + 9x(x^2+4)\right]$

Reduces to $4x^3-2+9x^3+36x$

$\boxed{y' = 2x(x^2+4)(2x^3-1)^2(13x^3+36x-2)}$

31) $y = \left(\frac{\sin x}{x}\right)^2$

out: $(\ )^2$

in: $\frac{\sin x}{x}$

* chain Rule
* quotient Rule

$y' = 2(\quad) \cdot \left[\frac{\overbrace{\cos x}^{f'} \cdot \overbrace{x}^{g} - \overbrace{\sin x}^{f}\,\overbrace{(1)}^{g'}}{\underbrace{x^2}_{g^2}}\right]$

$y' = 2\left[\frac{\sin x}{x}\right]\left[\frac{x\cos x - \sin x}{x^2}\right]$

$\boxed{y' = \frac{2\sin x(x\cos x - \sin x)}{x^3}}$

33) $y = \sin(4x)$

out: $\sin(\ )$

in: $4x$

$y' = \cos(\ ) \cdot 4$

$y' = \cos(4x) \cdot 4$

$\boxed{y' = 4\cos(4x)}$

35) $y = 2\sin(x^2+2x-1)$

out: $2\sin(\ )$

in: x^2+2x-1

$y' = 2\cos(\ ) \cdot (2x+2)$

$y' = 2\cos(x^2+2x-1) \cdot (2x+2)$

$\boxed{y' = (4x+4)\cos(x^2+2x-1)}$

$\boxed{3.1}$

37) $y = \sin\left(\frac{1}{x}\right)$

out: $\sin(\ )$
in: $\frac{1}{x} \to x^{-1}$

$y' = \cos(\ ) \cdot -1x^{-2}$

$y' = \cos\left(\frac{1}{x}\right) \cdot -\frac{1}{x^2}$

$$\boxed{y' = -\frac{1}{x^2}\cos\left(\frac{1}{x}\right)}$$

39) $y = \sec(4x)$

out: $\sec(\ )$
in: $4x$

$y' = \sec(\ )\tan(\ ) \cdot 4$

$$\boxed{y' = 4\sec(4x)\tan(4x)}$$

41) $y = e^{1/x}$

$\frac{d}{dx}e^u = e^u \cdot u'$

out: $e^{(\ )}$
in: $\frac{1}{x} \to x^{-1}$

$y' = e^{(\ )} \cdot -1x^{-2}$

$y' = e^{1/x} \cdot -1x^{-2}$

$$\boxed{y' = \frac{-e^{1/x}}{x^2}}$$

43) $y = \frac{1}{x^4 - 2x + 1}$

* can use chain or quotient rule

$y = (x^4 - 2x + 1)^{-1}$

out: $(\ )^{-1}$
in: $x^4 - 2x + 1$

$y' = -1(\quad)^{-2} \cdot (4x^3 - 2)$

$y' = -1(x^4 - 2x + 1)^{-2}(4x^3 - 2)$

$$\boxed{y' = \frac{-(4x^3 - 2)}{(x^4 - 2x + 1)^2}}$$

45) $y = \frac{100}{1 + 99e^{-x}}$

$y = 100(1 + 99e^{-x})^{-1}$

out: $100(\ )^{-1}$
in: $1 + 99e^{-x}$

$y' = -100(\quad)^{-2}(99e^{-x})(-1)$

$y' = -100(1 + 99e^{-x})^{-2}(99e^{-x})(-1)$

$$\boxed{y' = \frac{9900}{(1 + 99e^{-x})^2 e^{-x}}}$$

$\boxed{3.1}$

Find y'

55) $y = \underbrace{e^{-ax}}_{f}\underbrace{\sin(bx)}_{g}$ $\quad$ *product Rule

$*\ \frac{d}{dx}e^{u} = e^{u}\cdot u'$

$\frac{d}{dx}\sin(u) = \cos(u)\cdot u'$

$$y' = \overbrace{e^{-ax}(-a)}^{f'}\cdot\overbrace{\sin(bx)}^{g} + \overbrace{e^{-ax}}^{f}\cdot\overbrace{\cos(bx)\cdot b}^{g'}$$

$$\boxed{y' = -ae^{-ax}\sin(bx) + be^{-ax}\cos(bx)}$$

57) $y = \dfrac{e^{ax}-1}{e^{ax}+1}$

$$y' = \frac{\overbrace{e^{ax}(a)}^{f'}\cdot\overbrace{(e^{ax}+1)}^{g} - \overbrace{(e^{ax}-1)}^{f}\overbrace{(a\cdot e^{ax})}^{g'}}{\underbrace{(e^{ax}+1)^2}_{g^2}}$$

$$y' = \frac{ae^{2ax} + ae^{ax} - ae^{2ax} + ae^{ax}}{(e^{ax}+1)^2} \rightarrow \boxed{y' = \frac{2ae^{ax}}{(e^{ax}+1)^2}}$$

75) $f(x) = \dfrac{x}{(x^2-1)^3}$ at $\left(2, \frac{2}{27}\right)$

a) Find equation of tangent line at given point.

$$f'(x) = \frac{\overbrace{(1)}^{f'}\overbrace{(x^2-1)^3}^{g} - \overbrace{x}^{f}\cdot\overbrace{3(x^2-1)^2(2x)}^{g'}}{\underbrace{(x^2-1)^6}_{g^2}} \rightarrow \frac{(x^2-1)^2\left[x^2-1-6x^2\right]}{(x^2-1)^6} \rightarrow \frac{(-1-5x^2)}{(x^2-1)^4}$$

$$f'(2) = \frac{-1-5(2)^2}{(4-1)^4} \rightarrow \frac{-21}{81} \rightarrow \frac{-7}{27}$$

point: $(2, \frac{2}{27})$

slope: $m = -\frac{7}{27}$

$$\boxed{y - \frac{2}{27} = \frac{-7}{27}(x-2)}$$

b) Find equation of normal line (perpendicular to tangent line)

slope: $m_2 = \frac{27}{7}$ $\quad$ point: $(2, \frac{2}{27})$

$$\boxed{y - \frac{2}{27} = \frac{27}{7}(x-2)}$$

c) graph tangent and normal line