Key

AP Calculus – 3.1 Notes - Chain Rule

Composite Functions:

$f(g(x))$ out: $f(\ )$ in: $g(x)$

$\sin(x^2)$	$\sqrt{\ln x} = (\ln x)^{1/2}$	$\cos(\sin(5x))$
out: $\sin(\ )$	out: $(\ )^{1/2}$	out: $\cos(\ )$
in: x^2	in: $\ln x$	in: $\sin(\ )$

The Chain Rule (derivative of a composite function)

$$\frac{d}{dx}f(g(x)) = f'[g(x)] \cdot g'(x)$$

Find the derivative

1. $f(x) = (x^2-5)^4$

out: $(\ )^4$

in: x^2-5

$f'(x) = 4(x^2-5)^3 \cdot (2x)$

$\boxed{f'(x) = 8x(x^2-5)^3}$

2. $g(x) = \sqrt{4x-3}$

$g(x) = (4x-3)^{1/2}$

out: $(\ )^{1/2}$

in: $4x-3$

$g'(x) = \frac{1}{2}(4x-3)^{-1/2}(4) \rightarrow \frac{4}{2\sqrt{4x-3}} \rightarrow \boxed{\frac{2}{\sqrt{4x-3}}}$

3. $h(x) = \sin^2 5x$

$h(x) = [\sin(5x)]^2$

out: $[\ ]^2$

in: $\sin(5x)$

$h'(x) = 2[\sin(5x)] \cdot \cos(5x) \cdot 5$

$\boxed{h'(x) = 10\sin(5x)\cos(5x)}$

4. $y = \ln(x^3)$

$*\frac{d}{dx}\ln u = \frac{u'}{u}$

$y' = \frac{3x^2}{x^3}$

$\boxed{y' = \frac{3}{x}}$

5. $y = \ln(x^3)$

*OR expand first $\ln a^n = n \cdot \ln a$

$y = 3\ln x \quad y' = 3\left(\frac{1}{x}\right)$ $\boxed{y' = \frac{3}{x}}$

6. $f(x) = \left(\frac{t^2+1}{2t-5}\right)^3$

out: $(\ )^3$

in: $\frac{t^2+1}{2t-5}$

*chain Rule

*quotient Rule

$f'(x) = 3\left(\frac{t^2+1}{2t-5}\right)^2 \cdot \left[\frac{(2t)(2t-5)-(t^2+1)(2)}{(2t-5)^2}\right]$ $4t^2-10t-2t^2-2$

$f'(x) = \frac{3(t^2+1)^2(2t^2-10t-2)}{(2t-5)^2(2t-5)^2}$

$f'(x) = \frac{3(t^2+1)^2 \cdot 2(t^2-5t-1)}{(2t-5)^4}$

$\boxed{f'(x) = \frac{6(t^2+1)^2(t^2-5t-1)}{(2t-5)^4}}$

7. If $g(x) = 2x\sqrt{1-x}$ find $g'(-3)$.

$g(x) = \underbrace{2x}_{f}\underbrace{(1-x)^{1/2}}_{g}$

* product Rule
* chain Rule

$g'(x) = \underbrace{2}_{f'}\underbrace{(1-x)^{1/2}}_{g} + \underbrace{2x}_{f}\underbrace{\tfrac{1}{2}(1-x)^{-1/2}(-1)}_{g'}$ ← chain Rule: out: $(\ )^{1/2}$, in: $1-x$

$g'(x) = 2\sqrt{1-x} - \dfrac{x}{\sqrt{1-x}}$

$g'(-3) = 2\sqrt{1-(-3)} - \dfrac{-3}{\sqrt{1-(-3)}}$

$g'(-3) = 2\sqrt{4} + \dfrac{3}{\sqrt{4}}$

$= 2(2) + \dfrac{3}{2} = \dfrac{8}{2} + \dfrac{3}{2}$

$g'(-3) = \dfrac{11}{2}$

8. Given the following table of values, find $f'(4)$ for each function.

x	$g(x)$	$g'(x)$	$h(x)$	$h'(x)$
3	−1	7	−2	−3
4	3	−2	9	5

$f(x) = (g(x))^2$ out: $(\ )^2$, in: $g(x)$

$f'(x) = 2(g(x)) \cdot g'(x)$

$f'(4) = 2[g(4)] \cdot g'(4)$

$f'(4) = 2(3)(-2) = \boxed{-12}$

$f(x) = \sqrt{h(x)}$

$f(x) = [h(x)]^{1/2}$ out: $[\ ]^{1/2}$, in: $h(x)$

$f'(x) = \frac{1}{2}[h(x)]^{-1/2} \cdot h'(x)$

$f'(4) = \frac{1}{2}[h(4)]^{-1/2} \cdot h'(4) \rightarrow \frac{1}{2}(9)^{-1/2}(5) \rightarrow \frac{1}{2}\left(\frac{1}{3}\right)(5) \rightarrow \boxed{5/6}$

$f(x) = h(g(x))$ out: $h(\ )$, in: $g(x)$

$f'(x) = h'(g(x)) \cdot g'(x)$

$f'(4) = h'[g(4)] \cdot g'(4)$

$f'(4) = h'[3] \cdot g'(4)$

$f'(4) = (-3)\cdot(-2) = \boxed{6}$

Practice Problems:

Find the derivative of each function.

1. $g(x) = (3x^2 - 1)^5$

out: $(\ )^5$, in: $3x^2-1$

$g'(x) = 5(3x^2-1)^4(6x)$

$\boxed{g'(x) = 30x(3x^2-1)^4}$

2. $y = \sin(2x)$

* $\frac{d}{dx}\sin u = \cos u \cdot u'$

out: $\sin(\ )$, in: $2x$

$y' = \cos(2x) \cdot 2$

$\boxed{y' = 2\cos(2x)}$

3. $h(r) = \sqrt[3]{5r^2 - 2r + 1}$

out: $(\ )^{1/3}$, in: $5r^2-2r+1$

$h'(r) = \frac{1}{3}(5r^2-2r+1)^{-2/3}(10r-2)$

$\boxed{h'(r) = \dfrac{10r-2}{3(5r^2-2r+1)^{2/3}}}$

4. $y = \sqrt{4 - \cos(x^2)}$

$y = [4-\cos(x^2)]^{1/2}$ out: $[\ ]^{1/2}$, in: $4-\cos(x^2)$

$y' = \frac{1}{2}[4-\cos(x^2)]^{-1/2}(+\sin(x^2)\cdot 2x)$

$\boxed{y' = \dfrac{x\sin(x^2)}{\sqrt{4-\cos(x^2)}}}$

5. $h(x) = \ln(5^x)$

$h(x) = x \cdot \ln 5$

$h(x) = (\ln 5)x$

$\boxed{h'(x) = \ln 5}$

6. $g(x) = \ln(2x^3)$

$\frac{d}{dx}\ln u = \frac{u'}{u}$

$g'(x) = \dfrac{6x^2}{2x^3}$

$\boxed{g'(x) = \dfrac{3}{x}}$

7. $f(x) = \sqrt{\tan(2x)}$

$f(x) = [\tan(2x)]^{1/2}$ out: $[\ ]^{1/2}$, in: $\tan(2x)$

$f'(x) = \frac{1}{2}[\tan(2x)]^{-1/2} \cdot \sec^2(2x) \cdot 2$

$\boxed{f'(x) = \dfrac{\sec^2(2x)}{\sqrt{\tan(2x)}}}$

8. $y = \cos^2 x$ out: $[\ ]^2$, in: $\cos x$

$y = [\cos x]^2$

$y' = 2[\cos x] \cdot -\sin x$

$\boxed{y' = -2\sin x \cos x}$

9. $y = \frac{1}{(7x^2-1)^2} = (7x^2-1)^{-2}$

out: $(\ )^{-2}$, in: $7x^2-1$

$y' = -2(7x^2-1)^{-3}(14x)$

$\boxed{y' = \dfrac{-28x}{(7x^2-1)^3}}$