CHAPTER 3
Applications of Differentiation

Section 3.1 Extrema on an Interval

1. $f(x) = \dfrac{x^2}{x^2 + 4}$

$$f'(x) = \frac{(x^2+4)(2x) - (x^2)(2x)}{(x^2+4)^2} = \frac{8x}{(x^2+4)^2}$$

$f'(0) = 0$

2. $f(x) = \cos \dfrac{\pi x}{2}$

$f'(x) = -\dfrac{\pi}{2} \sin \dfrac{\pi x}{2}$

$f'(0) = 0$

$f'(2) = 0$

3. $f(x) = x + \dfrac{4}{x^2} = x + 4x^{-2}$

$f'(x) = 1 - 8x^{-3} = 1 - \dfrac{8}{x^3}$

$f'(2) = 0$

4. $f(x) = -3x\sqrt{x+1}$

$$\begin{aligned} f'(x) &= -3x\left[\tfrac{1}{2}(x+1)^{-1/2}\right] + \sqrt{x+1}(-3) \\ &= -\tfrac{3}{2}(x+1)^{-1/2}[x + 2(x+1)] \\ &= -\tfrac{3}{2}(x+1)^{-1/2}(3x+2) \end{aligned}$$

$f'\left(-\tfrac{2}{3}\right) = 0$

5. $f(x) = (x+2)^{2/3}$

$f'(x) = \tfrac{2}{3}(x+2)^{-1/3}$

$f'(-2)$ is undefined.

6. Using the limit definition of the derivative,

$$\lim_{x\to 0^-} \frac{f(x) - f(0)}{x - 0} = \lim_{x\to 0^-} \frac{(4 - |x|) - 4}{x} = 1$$

$$\lim_{x\to 0^+} \frac{f(x) - f(0)}{x - 0} = \lim_{x\to 0^+} \frac{(4 - |x|) - 4}{x - 0} = -1$$

$f'(0)$ does not exist, because the one-sided derivatives are not equal.

7. Critical number: $x = 2$

$x = 2$: absolute maximum (and relative maximum)

8. Critical number: $x = 0$

$x = 0$: neither

9. Critical numbers: $x = 1, 2, 3$

$x = 1, 3$: absolute maxima (and relative maxima)

$x = 2$: absolute minimum (and relative minimum)

10. Critical numbers: $x = 2, 5$

$x = 2$: neither

$x = 5$: absolute maximum (and relative maximum)

11. $f(x) = x^3 - 3x^2$

$f'(x) = 3x^2 - 6x = 3x(x-2)$

Critical numbers: $x = 0, 2$

12. $g(x) = x^4 - 8x^2$

$g'(x) = 4x^3 - 16x = 4x(x^2 - 4)$

Critical numbers: $x = 0, -2, 2$

13. $g(t) = t\sqrt{4-t},\ t < 3$

$$\begin{aligned} g'(t) &= t\left[\frac{1}{2}(4-t)^{-1/2}(-1)\right] + (4-t)^{1/2} \\ &= \frac{1}{2}(4-t)^{-1/2}[-t + 2(4-t)] \\ &= \frac{8-3t}{2\sqrt{4-t}} \end{aligned}$$

Critical number: $t = \dfrac{8}{3}$

14. $f(x) = \dfrac{4x}{x^2+1}$

$$f'(x) = \frac{(x^2+1)(4) - (4x)(2x)}{(x^2+1)^2} = \frac{4(1-x^2)}{(x^2+1)^2}$$

Critical numbers: $x = \pm 1$

15. $h(x) = \sin^2 x + \cos x,\quad 0 < x < 2\pi$

$h'(x) = 2\sin x \cos x - \sin x = \sin x(2\cos x - 1)$

Critical numbers in $(0, 2\pi)$: $x = \dfrac{\pi}{3}, \pi, \dfrac{5\pi}{3}$

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16. $f(\theta) = 2\sec\theta + \tan\theta,\ 0 < \theta < 2\pi$

$$f'(\theta) = 2\sec\theta\tan\theta + \sec^2\theta$$
$$= \sec\theta(2\tan\theta + \sec\theta)$$
$$= \sec\theta\left[2\left(\frac{\sin\theta}{\cos\theta}\right) + \frac{1}{\cos\theta}\right]$$
$$= \sec^2\theta(2\sin\theta + 1)$$

Critical numbers in $(0, 2\pi)$: $\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$

17. $f(x) = 3 - x, \quad [-1, 2]$

$f'(x) = -1 \Rightarrow$ no critical numbers

Left endpoint: $(-1, 4)$ Maximum

Right endpoint: $(2, 1)$ Minimum

18. $f(x) = \frac{3}{4}x + 2, [0,4]$

$f'(x) = \frac{3}{4} \Rightarrow$ no critical numbers

Left endpoint: $(0, 2)$ Minimum

Right endpoint: $(4, 5)$ Maximum

19. $g(x) = 2x^2 - 8x, \ [0, 6]$

$g'(x) = 4x - 8 = 4(x - 2)$

Critical number: $x = 2$

Left endpoint: $(0, 0)$

Critical number: $(2, -8)$ Minimum

Right endpoint: $(6, 24)$ Maximum

20. $h(x) = 5 - x^2, [-3, 1]$

$h'(x) = -2x$

Critical number: $x = 0$

Left endpoint: $(-3, -4)$ Minimum

Critical number: $(0, 5)$ Maximum

Right endpoint: $(1, 4)$

21. $f(x) = x^3 - \frac{3}{2}x^2, [-1, 2]$

$f'(x) = 3x^2 - 3x = 3x(x - 1)$

Left endpoint: $\left(-1, -\frac{5}{2}\right)$ Minimum

Right endpoint: $(2, 2)$ Maximum

Critical number: $(0, 0)$

Critical number: $\left(1, -\frac{1}{2}\right)$

22. $f(x) = 2x^3 - 6x, [0, 3]$

$f'(x) = 6x^2 - 6 = 6(x^2 - 1)$

Critical number: $x = 1$ ($x = -1$ not in interval.)

Left endpoint: $(0, 0)$

Critical number: $(1, -4)$ Minimum

Right endpoint: $(3, 36)$ Maximum

23. $f(x) = 3x^{2/3} - 2x, \quad [-1, 1]$

$$f'(x) = 2x^{-1/3} - 2 = \frac{2(1 - \sqrt[3]{x})}{\sqrt[3]{x}}$$

Left endpoint: $(-1, 5)$ Maximum

Critical number: $(0, 0)$ Minimum

Right endpoint: $(1, 1)$

24. $g(x) = \sqrt[3]{x} = x^{1/3}, [-8, 8]$

$$g'(x) = \frac{1}{3x^{2/3}}$$

Critical number: $x = 0$

Left endpoint: $(-8, -2)$ Minimum

Critical number: $(0, 0)$

Right endpoint: $(8, 2)$ Maximum

25. $g(t) = \frac{t^2}{t^2 + 3}, \quad [-1, 1]$

$$g'(t) = \frac{6t}{(t^2 + 3)^2}$$

Left endpoint: $\left(-1, \frac{1}{4}\right)$ Maximum

Critical number: $(0, 0)$ Minimum

Right endpoint: $\left(1, \frac{1}{4}\right)$ Maximum

26. $f(x) = \frac{2x}{x^2 + 1}, \quad [-2, 2]$

$$f'(x) = \frac{(x^2 + 1)2 - 2x(2x)}{(x^2 + 1)^2} = \frac{2 - 2x^2}{(x^2 + 1)^2}$$

$$f'(x) = \frac{2(1 - x^2)}{(x^2 + 1)^2}$$

Left endpoint: $\left(-2, -\frac{4}{5}\right)$

Critical number: $(-1, -1)$ Minimum

Critical number: $(1, 1)$ Maximum

Right endpoint: $\left(2, \frac{4}{5}\right)$

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27. $h(s) = \frac{1}{s-2} = (s-2)^{-1}, [0, 1]$

$h'(s) = \frac{-1}{(s-2)^2}$

Left endpoint: $\left(0, -\frac{1}{2}\right)$ Maximum

Right endpoint: $(1, -1)$ Minimum

28. $h(t) = \frac{t}{t+3}, [-1, 6]$

$h'(t) = \frac{(t+3)(1) - t(1)}{(t+3)^2} = \frac{3}{(t+3)^2}$

No critical numbers

Left endpoint: $\left(-1, -\frac{1}{2}\right)$ Minimum

Right endpoint: $\left(6, \frac{2}{3}\right)$ Maximum

29. $y = 3 - |t - 3|, \quad [-1, 5]$

For $x < 3, y = 3 + (t - 3) = t$

and $y' = 1 \neq 0$ on $[-1, 3)$

For $x > 3, y = 3 - (t - 3) = 6 - t$

and $y' = -1 \neq 0$ on $(3, 5]$

So, $x = 3$ is the only critical number.

Left endpoint: $(-1, -1)$ Minimum

Right endpoint: $(5, 1)$

Critical number: $(3, 3)$ Maximum

30. $g(x) = |x + 4|, [-7, 1]$

g is the absolute value function shifted 4 units to the left. So, the critical number is $x = -4$.

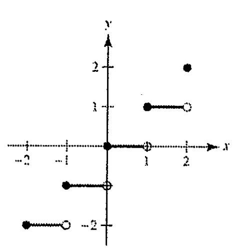
Left endpoint: $(-7, 3)$

Critical number: $(-4, 0)$ Minimum

Right endpoint: $(1, 5)$ Maximum

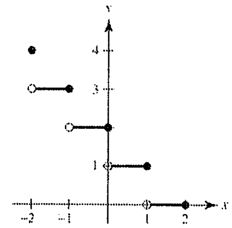
31. $f(x) = [\![x]\!], \quad [-2, 2]$

From the graph of f, you see that the maximum value of f is 2 for $x = 2$, and the minimum value is -2 for $-2 \leq x < -1$.



32. $h(x) = [\![2 - x]\!], \quad [-2, 2]$

From the graph you see that the maximum value of h is 4 at $x = -2$, and the minimum value is 0 for $1 < x \leq 2$.



33. $f(x) = \sin x, \left[\frac{5\pi}{6}, \frac{11\pi}{6}\right]$

$f'(x) = \cos x$

Critical number: $x = \frac{3\pi}{2}$

Left endpoint: $\left(\frac{5\pi}{6}, \frac{1}{2}\right)$ Maximum

Critical number: $\left(\frac{3\pi}{2}, -1\right)$ Minimum

Right endpoint: $\left(\frac{11\pi}{6}, -\frac{1}{2}\right)$

34. $g(x) = \sec x, \quad \left[-\frac{\pi}{6}, \frac{\pi}{3}\right]$

$g'(x) = \sec x \tan x$

Left endpoint: $\left(-\frac{\pi}{6}, \frac{2}{\sqrt{3}}\right) \approx \left(-\frac{\pi}{6}, 1.1547\right)$

Right endpoint: $\left(\frac{\pi}{3}, 2\right)$ Maximum

Critical number: $(0, 1)$ Minimum

35. $y = 3\cos x, \quad [0, 2\pi]$

$y' = -3\sin x$

Critical number in $(0, 2\pi)$: $x = \pi$

Left endpoint: $(0, 3)$ Maximum

Critical number: $(\pi, -3)$ Minimum

Right endpoint: $(2\pi, 3)$ Maximum

36. $y = \tan\left(\frac{\pi x}{8}\right), \quad [0, 2]$

$y' = \frac{\pi}{8}\sec^2\left(\frac{\pi x}{8}\right) \neq 0$

Left endpoint: $(0, 0)$ Minimum

Right endpoint: $(2, 1)$ Maximum

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37. $f(x) = 2x - 3$

(a) Minimum: $(0, -3)$

Maximum: $(2, 1)$

(b) Minimum: $(0, -3)$

(c) Maximum: $(2, 1)$

(d) No extrema

38. $f(x) = 5 - x$

(a) Minimum: $(4, 1)$

Maximum: $(1, 4)$

(b) Maximum: $(1, 4)$

(c) Minimum: $(4, 1)$

(d) No extrema

39. $f(x) = x^2 - 2x$

(a) Minimum: $(1, -1)$

Maximum: $(-1, 3)$

(b) Maximum: $(3, 3)$

(c) Minimum: $(1, -1)$

(d) Minimum: $(1, -1)$

40. $f(x) = \sqrt{4 - x^2}$

(a) Minima: $(-2, 0)$ and $(2, 0)$

Maximum: $(0, 2)$

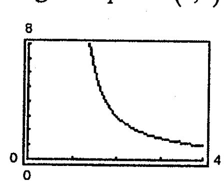
(b) Minimum: $(-2, 0)$

(c) Maximum: $(0, 2)$

(d) Maximum: $(1, \sqrt{3})$

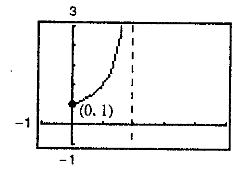
41. $f(x) = \dfrac{3}{x - 1}, \quad (1, 4]$

Right endpoint: $(4, 1)$ Minimum

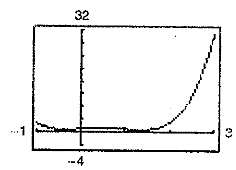


42. $f(x) = \dfrac{2}{2 - x}, \quad [0, 2)$

Left endpoint: $(0, 1)$ Minimum



43. $f(x) = x^4 - 2x^3 + x + 1, \quad [-1, 3]$



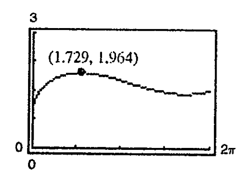
$f'(x) = 4x^3 - 6x^2 + 1 = (2x - 1)(2x^2 - 2x - 1) = 0$

$x = \dfrac{1}{2}, \dfrac{1 \pm \sqrt{3}}{2} \approx 0.5, -0.366, 1.366$

Right endpoint: $(3, 31)$ Maximum

Critical points: $\left(\dfrac{1 \pm \sqrt{3}}{2}, \dfrac{3}{4}\right)$ Minima

44. $f(x) = \sqrt{x} + \cos\dfrac{x}{2}, \quad [0, 2\pi]$



$f'(x) = \dfrac{1}{2\sqrt{x}} - \dfrac{1}{2}\sin\dfrac{x}{2}$

Left endpoint: $(0, 1)$ Minimum

Graphing utility: $(1.729, 1.964)$ Maximum

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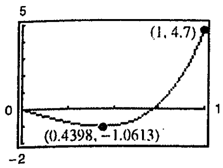
45. (a)



Minimum: $(0.4398, -1.0613)$

(b)

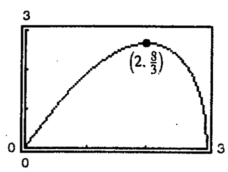
$$f(x) = 3.2x^5 + 5x^3 - 3.5x, \quad [0, 1]$$
$$f'(x) = 16x^4 + 15x^2 - 3.5$$
$$16x^4 + 15x^2 - 3.5 = 0$$
$$x^2 = \frac{-15 \pm \sqrt{(15)^2 - 4(16)(-3.5)}}{2(16)} = \frac{-15 \pm \sqrt{449}}{32}$$
$$x = \sqrt{\frac{-15 + \sqrt{449}}{32}} \approx 0.4398$$

Left endpoint: $(0, 0)$

Critical point: $(0.4398, -1.0613)$ Minimum

Right endpoint: $(1, 4.7)$ Maximum

46. (a)



Maximum: $\left(2, \frac{8}{3}\right)$

(b) $f(x) = \frac{4}{3}x\sqrt{3 - x}, \quad [0, 3]$

$$f'(x) = \frac{4}{3}\left[x\left(\frac{1}{2}\right)(3 - x)^{-1/2}(-1) + (3 - x)^{1/2}(1)\right] = \frac{4}{3}(3 - x)^{-1/2}\left(\frac{1}{2}\right)\left[-x + 2(3 - x)\right] = \frac{2(6 - 3x)}{3\sqrt{3 - x}} = \frac{6(2 - x)}{3\sqrt{3 - x}} = \frac{2(2 - x)}{\sqrt{3 - x}}$$

Left endpoint: $(0, 0)$ Minimum

Critical point: $\left(2, \frac{8}{3}\right)$ Maximum

Right endpoint: $(3, 0)$ Minimum

47. $f(x) = \left(1 + x^3\right)^{1/2}, \quad [0, 2]$

$$f'(x) = \frac{3}{2}x^2\left(1 + x^3\right)^{-1/2}$$
$$f''(x) = \frac{3}{4}\left(x^4 + 4x\right)\left(1 + x^3\right)^{-3/2}$$
$$f'''(x) = -\frac{3}{8}\left(x^6 + 20x^3 - 8\right)\left(1 + x^3\right)^{-5/2}$$

Setting $f''' = 0$, you have $x^6 + 20x^3 - 8 = 0$.

$$x^3 = \frac{-20 \pm \sqrt{400 - 4(1)(-8)}}{2}$$
$$x = \sqrt[3]{-10 \pm \sqrt{108}} = \sqrt{3} - 1$$

In the interval $[0, 2]$, choose $x = \sqrt[3]{-10 \pm \sqrt{108}} = \sqrt{3} - 1 \approx 0.732$.

$\left|f''\left(\sqrt[3]{-10 + \sqrt{108}}\right)\right| \approx 1.47$ is the maximum value.

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48. $f(x) = \frac{1}{x^2+1}, \quad \left[\frac{1}{2}, 3\right]$

$f'(x) = \frac{-2x}{(x^2+1)^2}$

$f''(x) = \frac{-2(1-3x^2)}{(x^2+1)^3}$

$f'''(x) = \frac{24x - 24x^3}{(x^2+1)^4}$

Setting $f''' = 0$, you have $x = 0, \pm 1$.

$|f''(1)| = \frac{1}{2}$ is the maximum value.

49. $f(x) = (x+1)^{2/3}, \quad [0, 2]$

$f'(x) = \frac{2}{3}(x+1)^{-1/3}$

$f''(x) = -\frac{2}{9}(x+1)^{-4/3}$

$f'''(x) = \frac{8}{27}(x+1)^{-7/3}$

$f^{(4)}(x) = -\frac{56}{81}(x+1)^{-10/3}$

$f^{(5)}(x) = \frac{560}{243}(x+1)^{-13/3}$

$\left|f^{(4)}(0)\right| = \frac{56}{81}$ is the maximum value.

50. $f(x) = \frac{1}{x^2+1}, \quad [-1, 1]$

$f'''(x) = \frac{24x - 24x^3}{(x^2+1)^4}$

$f^{(4)}(x) = \frac{24(5x^4 - 10x^2 + 1)}{(x^2+1)^5}$

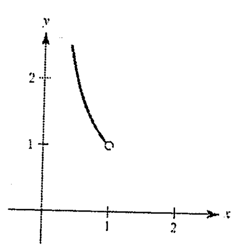
$f^{(5)}(x) = \frac{-240x(3x^4 - 10x^2 + 3)}{(x^2+1)^6}$

$\left|f^{(4)}(0)\right| = 24$ is the maximum value.

51. Answers will vary. *Sample answer*:

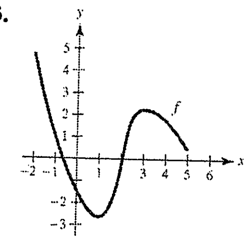
$y = \frac{1}{x}$ on the interval $(0, 1)$

There is no maximum or minimum value.

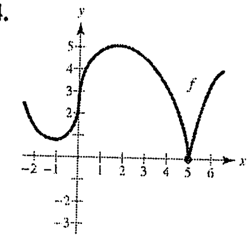


52. A: absolute minimum
B: relative maximum
C: neither
D: relative minimum
E: relative maximum
F: relative minimum
G: neither

53.



54.



55. (a) Yes
(b) No

56. (a) No
(b) Yes

57. (a) No
(b) Yes

58. (a) No
(b) Yes

59. $P = VI - RI^2 = 12I - 0.5I^2, 0 \le I \le 15$

$P = 0$ when $I = 0$.

$P = 67.5$ when $I = 15$.

$P' = 12 - I = 0$

Critical number: $I = 12$ amps

When $I = 12$ amps, $P = 72$, the maximum output.

No, a 20-amp fuse would not increase the power output. P is decreasing for $I > 12$.

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60. $x = \dfrac{v^2 \sin 2\theta}{32}, \dfrac{\pi}{4} \le \theta \le \dfrac{3\pi}{4}$

$\dfrac{d\theta}{dt}$ is constant.

$\dfrac{dx}{dt} = \dfrac{dx}{d\theta}\dfrac{d\theta}{dt}$ (by the Chain Rule) $= \dfrac{v^2 \cos 2\theta}{16}\dfrac{d\theta}{dt}$

In the interval $[\pi/4, 3\pi/4]$, $\theta = \pi/4, 3\pi/4$ indicate minimums for dx/dt and $\theta = \pi/2$ indicates a maximum for dx/dt. This implies that the sprinkler waters longest when $\theta = \pi/4$ and $3\pi/4$. So, the lawn farthest from the sprinkler gets the most water.

61. $S = 6hs + \dfrac{3s^2}{2}\left(\dfrac{\sqrt{3} - \cos\theta}{\sin\theta}\right), \dfrac{\pi}{6} \le \theta \le \dfrac{\pi}{2}$

$$\frac{dS}{d\theta} = \frac{3s^2}{2}\left(-\sqrt{3}\csc\theta\cot\theta + \csc^2\theta\right)$$
$$= \frac{3s^2}{2}\csc\theta\left(-\sqrt{3}\cot\theta + \csc\theta\right) = 0$$

$\csc\theta = \sqrt{3}\cot\theta$

$\sec\theta = \sqrt{3}$

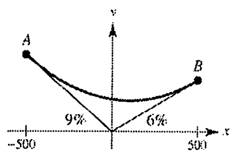
$\theta = \text{arcsec}\sqrt{3} \approx 0.9553$ radians

$$S\left(\frac{\pi}{6}\right) = 6hs + \frac{3s^2}{2}\left(\sqrt{3}\right)$$
$$S\left(\frac{\pi}{6}\right) = 6hs + \frac{3s^2}{2}\left(\sqrt{3}\right)$$
$$S\left(\text{arcsec}\sqrt{3}\right) = 6hs + \frac{3s^2}{2}\left(\sqrt{2}\right)$$

S is minimum when $\theta = \text{arcsec}\sqrt{3} \approx 0.9553$ radian.

62. (a) Because the grade at A is 9%, $A(-500, 45)$

The grade at B is 6%, $B(500, 30)$.



(b) $y = ax^2 + bx + c$

$y' = 2ax + b$

At A: $2a(-500) + b = -0.09$

At B: $2a(500) + b = 0.06$

Solving these two equations, you obtain

$a = \dfrac{3}{40{,}000}$ and $b = -\dfrac{3}{200}$.

Using the points $A(-500, 45)$ and $B(500, 30)$, you obtain

$$45 = \frac{3}{40{,}000}(-500)^2 + \left(-\frac{3}{200}\right)(-500) + C$$
$$30 = \frac{3}{40{,}000}(500)^2 + \left(-\frac{3}{200}\right)(500) + C.$$

In both cases, $C = 18.75 = \dfrac{75}{4}$. So, $y = \dfrac{3}{40{,}000}x^2 - \dfrac{3}{200}x + \dfrac{75}{4}$

(c)

x	−500	−400	−300	−200	−100	0	100	200	300	400	500
d	0	0.75	3	6.75	12	18.75	12	6.75	3	0.75	0

For $-500 \le x \le 0$, $d = (ax^2 + bx + c) - (-0.09x)$.

For $0 \le x \le 500$, $d = (ax^2 + bx + c) - (0.06x)$.

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(d) $y' = \frac{3}{20{,}000}x - \frac{3}{200} = 0$

$x = \frac{3}{200} \cdot \frac{20{,}000}{3} = 100$

The lowest point on the highway is $(100, 18)$, is not directly over the origin.

63. True. See Exercise 25.

64. True. This is stated in the Extreme Value Theorem.

65. True

66. False. Let $f(x) = x^2$. $x = 0$ is a critical number of f.

$g(x) = f(x - k) = (x - k)^2$

$x = k$ is a critical number of g.

67. If f has a maximum value at $x = c$, then $f(c) \ge f(x)$ for all x in I. So, $-f(c) \le -f(x)$ for all x in I. So, $-f$ has a minimum value at $x = c$.

68. $f(x) = ax^3 + bx^2 + cx + d, \quad a \ne 0$

$f'(x) = 3ax^2 + 2bx + c$

The quadratic polynomial can have zero, one, or two zeros.

$$x = \frac{-2b \pm \sqrt{4b^2 - 12ac}}{6a} = \frac{-b \pm \sqrt{b^2 - 3ac}}{3a}$$

Zero critical numbers: $b^2 < 3ac$.

Example: $(a = b = c = 1, d = 0)$ $f(x) = x^3 + x^2 + x$ has no critical numbers.

One critical number: $b^2 = 3ac$.

Example: $(a = 1, b = c = d = 0)$ $f(x) = x^3$ has one critical number, $x = 0$.

Two critical numbers: $b^2 > 3ac$.

Example:

$(a = c = 1, b = 2, d = 0)$ $f(x) = x^3 + 2x^2 + x$ has two critical numbers: $x = -1, -\frac{1}{3}$.

69. First do an example: Let $a = 4$ and $f(x) = 4$. Then R is the square $0 \le x \le 4, 0 \le y \le 4$. Its area and perimeter are both $k = 16$.

Claim that all real numbers $a > 2$ work. On the one hand, if $a > 2$ is given, then let $f(x) = 2a/(a - 2)$. Then the rectangle

$$R = \left\{(x, y)\colon 0 \le x \le a, 0 \le y \le \frac{2a}{a - 2}\right\}$$

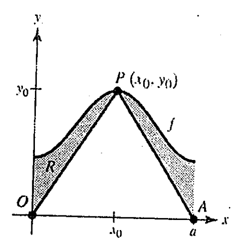
has $k = \frac{2a^2}{a - 2}$:

$$\text{Area} = a\left(\frac{2a}{a - 2}\right) = \frac{2a^2}{a - 2}$$

$$\text{Perimeter} = 2a + 2\left(\frac{2a}{a - 2}\right) = \frac{2a(a - 2) + 2(2a)}{a - 2} = \frac{2a^2}{a - 2}.$$

To see that a must be greater than 2, consider $R = \{(x, y)\colon 0 \le x \le a, 0 \le y \le f(x)\}$.

f attains its maximum value on $[0, a]$ at some point $P(x_0, y_0)$, as indicated in the figure.



Draw segments $\overline{OP}$ and $\overline{PA}$. The region R is bounded by the rectangle $0 \le x \le a, 0 \le y \le y_0$, so area$(R) = k \le ay_0$. Furthermore, from the figure, $y_0 < \overline{OP}$ and $y_0 < \overline{PA}$. So, $k = \text{Perimeter}(R) > \overline{OP} + \overline{PA} > 2y_0$. Combining, $2y_0 < k \le ay_0 \Rightarrow a > 2$.

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