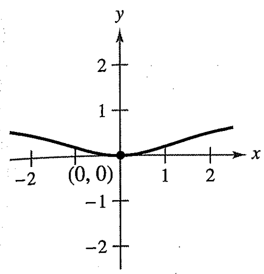
3.1 Exercises

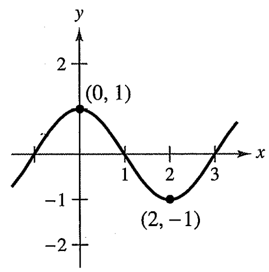
See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Finding the Value of the Derivative at Relative Extrema In Exercises 1–6, find the value of the derivative (if it exists) at each indicated extremum.

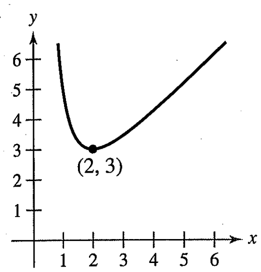
1. $f(x) = \dfrac{x^2}{x^2+4}$



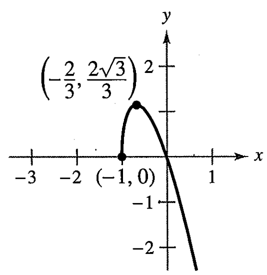
2. $f(x) = \cos \dfrac{\pi x}{2}$



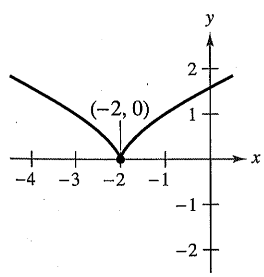
3. $g(x) = x + \dfrac{4}{x^2}$



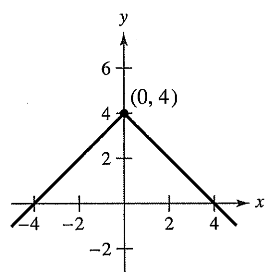
4. $f(x) = -3x\sqrt{x+1}$



5. $f(x) = (x+2)^{2/3}$



6. $f(x) = 4 - |x|$



Approximating Critical Numbers In Exercises 7–10, approximate the critical numbers of the function shown in the graph. Determine whether the function has a relative maximum, a relative minimum, an absolute maximum, an absolute minimum, or none of these at each critical number on the interval shown.

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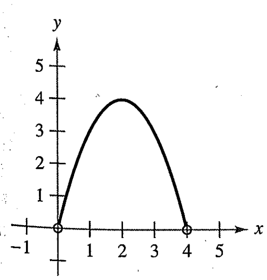

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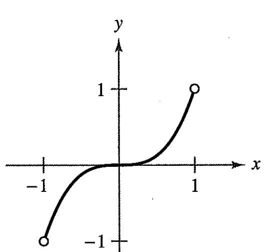

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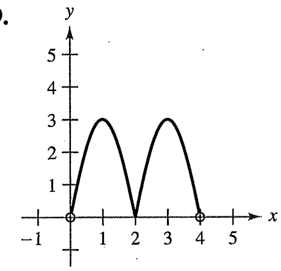

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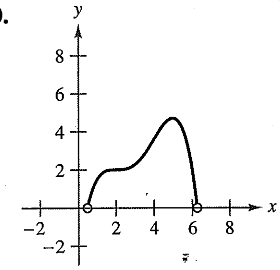


Finding Critical Numbers In Exercises 11–16, find the critical numbers of the function.

11. $f(x) = x^3 - 3x^2$

12. $g(x) = x^4 - 8x^2$

13. $g(t) = t\sqrt{4-t},\ t < 3$

14. $f(x) = \dfrac{4x}{x^2+1}$

15. $h(x) = \sin^2 x + \cos x$
$0 < x < 2\pi$

16. $f(\theta) = 2\sec\theta + \tan\theta$
$0 < \theta < 2\pi$

Finding Extrema on a Closed Interval In Exercises 17–36, find the absolute extrema of the function on the closed interval.

17. $f(x) = 3 - x,\ [-1, 2]$

18. $f(x) = \dfrac{3}{4}x + 2,\ [0, 4]$

19. $g(x) = 2x^2 - 8x,\ [0, 6]$

20. $h(x) = 5 - x^2,\ [-3, 1]$

21. $f(x) = x^3 - \dfrac{3}{2}x^2,\ [-1, 2]$

22. $f(x) = 2x^3 - 6x,\ [0, 3]$

23. $y = 3x^{2/3} - 2x,\ [-1, 1]$

24. $g(x) = \sqrt[3]{x},\ [-8, 8]$

25. $g(t) = \dfrac{t^2}{t^2+3},\ [-1, 1]$

26. $f(x) = \dfrac{2x}{x^2+1},\ [-2, 2]$

27. $h(s) = \dfrac{1}{s-2},\ [0, 1]$

28. $h(t) = \dfrac{t}{t+3},\ [-1, 6]$

29. $y = 3 - |t - 3|, [-1, 5]$

30. $g(x) = |x + 4|,\ [-7, 1]$

31. $f(x) = [\![x]\!],\ [-2, 2]$

32. $h(x) = [\![2 - x]\!],\ [-2, 2]$

33. $f(x) = \sin x,\ \left[\dfrac{5\pi}{6}, \dfrac{11\pi}{6}\right]$

34. $g(x) = \sec x,\ \left[-\dfrac{\pi}{6}, \dfrac{\pi}{3}\right]$

35. $y = 3\cos x,\ [0, 2\pi]$

36. $y = \tan\left(\dfrac{\pi x}{8}\right),\ [0, 2]$

Finding Extrema on an Interval In Exercises 37–40, find the absolute extrema of the function (if any exist) on each interval.

37. $f(x) = 2x - 3$
(a) $[0, 2]$ (b) $[0, 2)$
(c) $(0, 2]$ (d) $(0, 2)$

38. $f(x) = 5 - x$
(a) $[1, 4]$ (b) $[1, 4)$
(c) $(1, 4]$ (d) $(1, 4)$

39. $f(x) = x^2 - 2x$
(a) $[-1, 2]$ (b) $(1, 3]$
(c) $(0, 2)$ (d) $[1, 4)$

40. $f(x) = \sqrt{4 - x^2}$
(a) $[-2, 2]$ (b) $[-2, 0)$
(c) $(-2, 2)$ (d) $[1, 2)$

Finding Absolute Extrema In Exercises 41–44, use a graphing utility to graph the function and find the absolute extrema of the function on the given interval.

41. $f(x) = \dfrac{3}{x-1}, \quad (1, 4]$ **42.** $f(x) = \dfrac{2}{2-x}, \quad [0, 2)$

43. $f(x) = x^4 - 2x^3 + x + 1, \quad [-1, 3]$

44. $f(x) = \sqrt{x} + \cos\dfrac{x}{2}, \quad [0, 2\pi]$

Finding Extrema Using Technology In Exercises 45 and 46, (a) use a computer algebra system to graph the function and approximate any absolute extrema on the given interval. (b) Use the utility to find any critical numbers, and use them to find any absolute extrema not located at the endpoints. Compare the results with those in part (a).

45. $f(x) = 3.2x^5 + 5x^3 - 3.5x, \quad [0, 1]$

46. $f(x) = \dfrac{4}{3}x\sqrt{3-x}, \quad [0, 3]$

Finding Maximum Values Using Technology In Exercises 47 and 48, use a computer algebra system to find the maximum value of $|f''(x)|$ on the closed interval. (This value is used in the error estimate for the Trapezoidal Rule, as discussed in Section 4.6.)

47. $f(x) = \sqrt{1 + x^3}, \quad [0, 2]$ **48.** $f(x) = \dfrac{1}{x^2+1}, \quad \left[\dfrac{1}{2}, 3\right]$

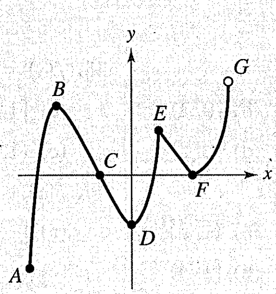
Finding Maximum Values Using Technology In Exercises 49 and 50, use a computer algebra system to find the maximum value of $|f^{(4)}(x)|$ on the closed interval. (This value is used in the error estimate for Simpson's Rule, as discussed in Section 4.6.)

49. $f(x) = (x+1)^{2/3}, \quad [0, 2]$

50. $f(x) = \dfrac{1}{x^2+1}, \quad [-1, 1]$

51. Writing Write a short paragraph explaining why a continuous function on an open interval may not have a maximum or minimum. Illustrate your explanation with a sketch of the graph of such a function.

52. HOW DO YOU SEE IT? Determine whether each labeled point is an absolute maximum or minimum, a relative maximum or minimum, or none of these.



WRITING ABOUT CONCEPTS

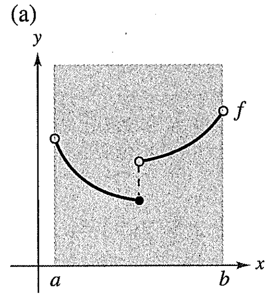
Creating the Graph of a Function In Exercises 53 and 54, graph a function on the interval $[-2, 5]$ having the given characteristics.

53. Absolute maximum at $x = -2$
Absolute minimum at $x = 1$
Relative maximum at $x = 3$

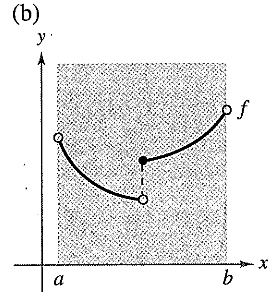
54. Relative minimum at $x = -1$
Critical number (but no extremum) at $x = 0$
Absolute maximum at $x = 2$
Absolute minimum at $x = 5$

Using Graphs In Exercises 55–58, determine from the graph whether f has a minimum in the open interval (a, b).

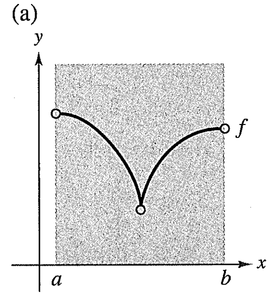
55. (a)



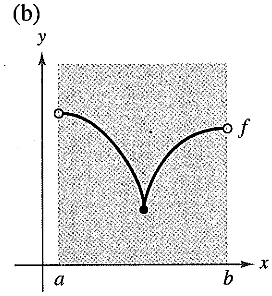
(b)



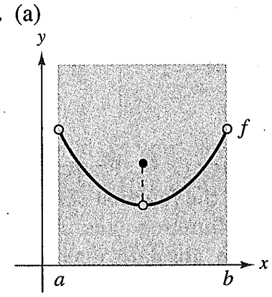
56. (a)



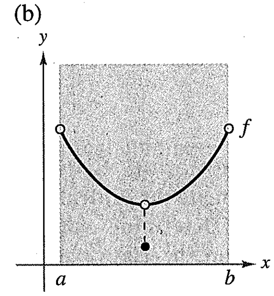
(b)



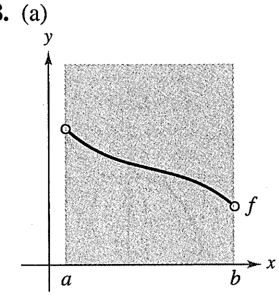
57. (a)



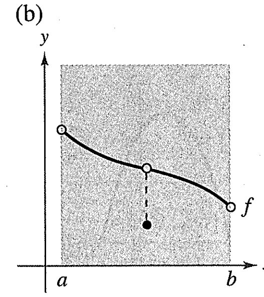
(b)



58. (a)



(b)



59. **Power** The formula for the power output P of a battery is

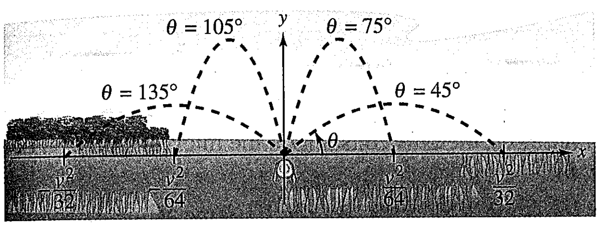
$$P = VI - RI^2$$

where V is the electromotive force in volts, R is the resistance in ohms, and I is the current in amperes. Find the current that corresponds to a maximum value of P in a battery for which $V = 12$ volts and $R = 0.5$ ohm. Assume that a 15-ampere fuse bounds the output in the interval $0 \le I \le 15$. Could the power output be increased by replacing the 15-ampere fuse with a 20-ampere fuse? Explain.

60. **Lawn Sprinkler** A lawn sprinkler is constructed in such a way that $d\theta/dt$ is constant, where θ ranges between 45° and 135° (see figure). The distance the water travels horizontally is

$$x = \frac{v^2 \sin 2\theta}{32}, \quad 45° \le \theta \le 135°$$

where v is the speed of the water. Find dx/dt and explain why this lawn sprinkler does not water evenly. What part of the lawn receives the most water?



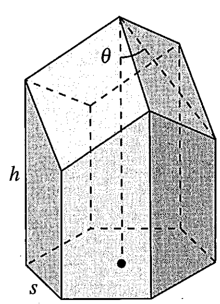
Water sprinkler: $45° \le \theta \le 135°$

■ **FOR FURTHER INFORMATION** For more information on the "calculus of lawn sprinklers," see the article "Design of an Oscillating Sprinkler" by Bart Braden in *Mathematics Magazine*. To view this article, go to *MathArticles.com*.

61. **Honeycomb** The surface area of a cell in a honeycomb is

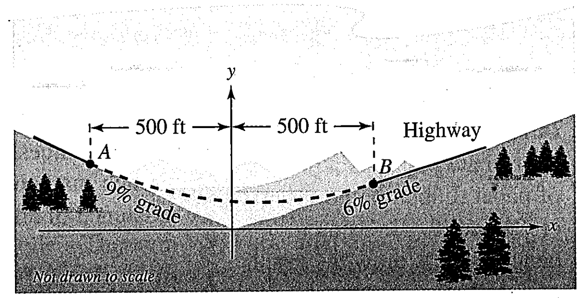
$$S = 6hs + \frac{3s^2}{2}\left(\frac{\sqrt{3} - \cos\theta}{\sin\theta}\right)$$

where h and s are positive constants and θ is the angle at which the upper faces meet the altitude of the cell (see figure). Find the angle θ $(\pi/6 \le \theta \le \pi/2)$ that minimizes the surface area S.



■ **FOR FURTHER INFORMATION** For more information on the geometric structure of a honeycomb cell, see the article "The Design of Honeycombs" by Anthony L. Peressini in UMAP Module 502, published by COMAP, Inc., Suite 210, 57 Bedford Street, Lexington, MA.

62. **Highway Design** In order to build a highway, it is necessary to fill a section of a valley where the grades (slopes) of the sides are 9% and 6% (see figure). The top of the filled region will have the shape of a parabolic arc that is tangent to the two slopes at the points A and B. The horizontal distances from A to the y-axis and from B to the y-axis are both 500 feet.



(a) Find the coordinates of A and B.

(b) Find a quadratic function $y = ax^2 + bx + c$ for $-500 \le x \le 500$ that describes the top of the filled region.

(c) Construct a table giving the depths d of the fill for $x = -500, -400, -300, -200, -100, 0, 100, 200, 300, 400$, and 500.

(d) What will be the lowest point on the completed highway? Will it be directly over the point where the two hillsides come together?

True or False? **In Exercises 63–66, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.**

63. The maximum of a function that is continuous on a closed interval can occur at two different values in the interval.

64. If a function is continuous on a closed interval, then it must have a minimum on the interval.

65. If $x = c$ is a critical number of the function f, then it is also a critical number of the function $g(x) = f(x) + k$, where k is a constant.

66. If $x = c$ is a critical number of the function f, then it is also a critical number of the function $g(x) = f(x - k)$, where k is a constant.

67. **Functions** Let the function f be differentiable on an interval I containing c. If f has a maximum value at $x = c$, show that $-f$ has a minimum value at $x = c$.

68. **Critical Numbers** Consider the cubic function $f(x) = ax^3 + bx^2 + cx + d$, where $a \ne 0$. Show that f can have zero, one, or two critical numbers and give an example of each case.

PUTNAM EXAM CHALLENGE

69. Determine all real numbers $a > 0$ for which there exists a nonnegative continuous function $f(x)$ defined on $[0, a]$ with the property that the region $R = \{(x, y); 0 \le x \le a, 0 \le y \le f(x)\}$ has perimeter k units and area k square units for some real number k.

This problem was composed by the Committee on the Putnam Prize Competition.