Ch. 3.2 Exercise Problems – Implicit Differentiation

p. 242-245 #9-17 odd, 23, 25, 31, 39, 47-51 odd

Find y' (or $\frac{dy}{dx}$) using implicit differentiation

9) $e^y = \sin x$

$$e^y\left(\frac{dy}{dx}\right) = \cos x$$

$$\boxed{\frac{dy}{dx} = \frac{\cos x}{e^y}}$$

11) $e^{x+y} = y$

$$e^{x+y}\cdot\left(1 + \frac{dy}{dx}\right) = 1\left(\frac{dy}{dx}\right)$$

$$e^{x+y} + e^{x+y}\left(\frac{dy}{dx}\right) = 1\left(\frac{dy}{dx}\right)$$

$$e^{x+y}\left(\frac{dy}{dx}\right) - 1\left(\frac{dy}{dx}\right) = -e^{x+y}$$

$$\frac{dy}{dx}\left(e^{x+y} - 1\right) = -e^{x+y}$$

$$\boxed{\frac{dy}{dx} = \frac{-e^{x+y}}{e^{x+y} - 1}}$$

13) $x^2 y = 5$

*product Rule

$$\overbrace{x^2}^{f}\cdot\overbrace{y}^{g} = 5$$

$$\overbrace{2x}^{f'}\cdot\overbrace{y}^{g} + \overbrace{x^2}^{f}\cdot\overbrace{1\left(\frac{dy}{dx}\right)}^{g'} = 0$$

$$x^2\left(\frac{dy}{dx}\right) = -2xy$$

$$\frac{dy}{dx} = \frac{-2xy}{x^2}$$

$$\boxed{\frac{dy}{dx} = \frac{-2y}{x}}$$

15) $x^2 - y^2 - xy = 2$

*product Rule

*watch out for negative

$$x^2 - y^2 - (\overbrace{x}^{f}\overbrace{y}^{g}) = 2$$

$$2x - 2y\left(\frac{dy}{dx}\right) - \left(\overbrace{(1)}^{f'}\cdot\overbrace{y}^{g} + \overbrace{x}^{f}\cdot\overbrace{\left(\frac{dy}{dx}\right)}^{g'}\right) = 0$$

$$2x - 2y\left(\frac{dy}{dx}\right) - 1y - x\left(\frac{dy}{dx}\right) = 0$$

$$-2y\left(\frac{dy}{dx}\right) - x\left(\frac{dy}{dx}\right) = 1y - 2x$$

$$\frac{dy}{dx}\left(-2y - x\right) = 1y - 2x$$

$$\boxed{\frac{dy}{dx} = \frac{y - 2x}{-2y - x} \quad \text{or} \quad \frac{dy}{dx} = \frac{2x - y}{x + 2y}}$$

$\boxed{3.2}$ Find $\frac{dy}{dx}$

17) $\frac{1}{x} + \frac{1}{y} = 1$

$(x)^{-1} + y^{-1} = 1$

$-1x^{-2} \cdot -1y^{-2}\left(\frac{dy}{dx}\right) = 0$

$\frac{-1}{x^2} - \frac{1}{y^2}\left(\frac{dy}{dx}\right) = 0$

$-\frac{1}{y^2}\left(\frac{dy}{dx}\right) = \frac{1}{x^2}$

$\frac{dy}{dx} = -y^2 \cdot \frac{1}{x^2}$

$\boxed{\frac{dy}{dx} = \frac{-y^2}{x^2}}$

23) $(x^2+y)^3 = y$

*chain Rule
*implicit

$3(x^2+y)^2\left(2x + 1\left(\frac{dy}{dx}\right)\right) = 1\left(\frac{dy}{dx}\right)$

$6x(x^2+y)^2 + 3(x^2+y)^2\left(\frac{dy}{dx}\right) = 1\left(\frac{dy}{dx}\right)$

$3(x^2+y)^2\left(\frac{dy}{dx}\right) - 1\left(\frac{dy}{dx}\right) = -6x(x^2+y)^2$

$\frac{dy}{dx}\left(3(x^2+y)^2 - 1\right) = -6x(x^2+y)^2$

$$\boxed{\frac{dy}{dx} = \frac{-6x(x^2+y)^2}{3(x^2+y)^2 - 1}}$$

25) $y = \tan(x-y)$

chain Rule:
out: tan( )
in: x-y

$1\left(\frac{dy}{dx}\right) = \sec^2(x-y)\left[1 - 1\left(\frac{dy}{dx}\right)\right]$

$1\left(\frac{dy}{dx}\right) = \sec^2(x-y) - \sec^2(x-y)\left(\frac{dy}{dx}\right)$

$1\left(\frac{dy}{dx}\right) + \sec^2(x-y)\left(\frac{dy}{dx}\right) = \sec^2(x-y)$

$\frac{dy}{dx}\left(1 + \sec^2(x-y)\right) = \sec^2(x-y)$

$$\boxed{\frac{dy}{dx} = \frac{\sec^2(x-y)}{1+\sec^2(x-y)}}$$

$\boxed{3.2}$ find $\frac{dy}{dx}$

31) $y = x^{2/3} + 4$

$$\frac{dy}{dx} = \frac{2}{3}x^{-1/3} + 0$$

$$\boxed{\frac{dy}{dx} = \frac{2}{3x^{1/3}}}$$

39) $y = x\sqrt{x^2-1}$

*product Rule
*implicit

$$y = \underbrace{x}_{f}\underbrace{(x^2-1)^{1/2}}_{g}$$

$$y' = \underbrace{(1)}_{f'}\cdot\underbrace{(x^2-1)^{1/2}}_{g} + \underbrace{x}_{f}\cdot\underbrace{\tfrac{1}{2}(x^2-1)^{-1/2}(2x)}_{g'}$$

$$\boxed{y' = (x^2-1)^{1/2} + \frac{x^2}{(x^2-1)^{1/2}} \quad \text{or} \quad y' = \sqrt{x^2-1} + \frac{x^2}{\sqrt{x^2-1}}}$$

Find y' and y''

47) $x^2 + y^2 = 4$

$$2x + 2y\left(\frac{dy}{dx}\right) = 0$$

$$2y\left(\frac{dy}{dx}\right) = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{2y}$$

$$\boxed{\frac{dy}{dx} = \frac{-x}{y}}$$

use quotient Rule

implicit diff.

$$\frac{d^2y}{dx^2} = \frac{\underbrace{-1}_{f'}\cdot\underbrace{y}_{g} - \underbrace{(-x)}_{f}\underbrace{(dy/dx)}_{g'}}{\underbrace{y^2}_{g^2}} \rightarrow \frac{d^2y}{dx^2} = \frac{-y + x\left(\frac{dy}{dx}\right)}{y^2} \rightarrow \frac{-y + x\left(\frac{-x}{y}\right)}{y^2}$$

Replace $\frac{dy}{dx}$ with $\frac{-x}{y}$

$$\frac{d^2y}{dx^2} = \frac{-y - \frac{x^2}{y}}{y^2} \rightarrow \frac{\frac{-y}{1} - \frac{x^2}{y}}{y^2} \rightarrow \frac{\frac{-y^2}{y} - \frac{x^2}{y}}{y^2} \rightarrow \frac{\frac{-y^2-x^2}{y}}{y^2} \rightarrow \frac{-y^2-x^2}{y}\cdot\frac{1}{y^2}$$

$$\frac{d^2y}{dx^2} = \frac{-y^2-x^2}{y^3}$$

← since $x^2+y^2=4$, replace $-y^2-x^2$ with -4

$$\boxed{\frac{d^2y}{dx^2} = \frac{-4}{y^3}}$$

$\boxed{3.2}$ Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$

49) $x^2 - y^2 = 4 + 5x$

$2x - 2y\left(\frac{dy}{dx}\right) = 0 + 5$

$-2y\left(\frac{dy}{dx}\right) = 5 - 2x$

$\frac{dy}{dx} = \frac{5-2x}{-2y}$ or $\boxed{\frac{dy}{dx} = \frac{2x-5}{2y}}$

replace $\frac{dy}{dx} = \frac{2x-5}{2y}$

$$\frac{d^2y}{dx^2} = \frac{\overbrace{(2)}^{f'}\overbrace{(2y)}^{g} - \overbrace{(2x-5)}^{f}\overbrace{(2\,dy/dx)}^{g'}}{\underbrace{(2y)^2}_{g^2}} \rightarrow \frac{4y - 2(2x-5)\left(\frac{dy}{dx}\right)}{4y^2} \rightarrow \frac{4y - 2(2x-5)\left(\frac{2x-5}{2y}\right)}{4y^2}$$

$$\frac{d^2y}{dx^2} = \frac{4y - \frac{(2x-5)^2}{y}}{4y^2} \rightarrow \frac{\frac{4y^2}{y} - \frac{(2x-5)^2}{y}}{4y^2} \rightarrow \frac{4y^2-(2x-5)^2}{y} \cdot \frac{1}{4y^2}$$

$$\boxed{\frac{d^2y}{dx^2} = \frac{4y^2-(2x-5)^2}{4y^3}}$$

51) $y = \sqrt{x^2+1}$

$y = (x^2+1)^{1/2}$

$\frac{dy}{dx} = \frac{1}{2}(x^2+1)^{-1/2}(2x)$

$$\boxed{\frac{dy}{dx} = \frac{x}{(x^2+1)^{1/2}}}$$

$$\frac{d^2y}{dx^2} = \frac{\overbrace{(1)}^{f'}\overbrace{(x^2+1)^{1/2}}^{g} - \overbrace{x}^{f} \cdot \overbrace{\tfrac{1}{2}(x^2+1)^{-1/2}(2x)}^{g'}}{\underbrace{[(x^2+1)^{1/2}]^2}_{g^2}}$$

$$\frac{d^2y}{dx^2} = \frac{(x^2+1)^{1/2} - \frac{x^2}{(x^2+1)^{1/2}}}{(x^2+1)} \rightarrow \frac{\frac{x^2+1}{(x^2+1)^{1/2}} - \frac{x^2}{(x^2+1)^{1/2}}}{x^2+1}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{x^2+1-x^2}{(x^2+1)^{1/2}}}{x^2+1} \rightarrow \frac{1}{(x^2+1)^{1/2}} \cdot \frac{1}{(x^2+1)}$$

$$\boxed{\frac{d^2y}{dx^2} = \frac{1}{(x^2+1)^{3/2}}}$$