3.2b Exercise Problems Implicit Differentiation (Day 2)

p. 242-245 #53-59 odd, 71, 77

(#53-57)

a) Find slope of tangent line to graph at point

b) Write equation of tangent line

c) Graph tangent line and the graph of curve.

53) $x^2+y^2=5$ at $(1,2)$

$2x+2y\left(\frac{dy}{dx}\right)=0$

$2y\left(\frac{dy}{dx}\right)=-2x$

$\frac{dy}{dx}=\frac{-2x}{2y}$

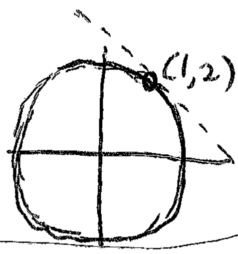
$\frac{dy}{dx}=\frac{-x}{y}$

$\left.\frac{dy}{dx}\right|_{(1,2)}=\frac{-1}{2}$

$\boxed{\text{a) slope: } m=-1/2}$

b) Tangent line equation at (1,2)

$y-y_1=m(x-x_1)$ $\boxed{y-2=\frac{-1}{2}(x-1)}$

c) 

55) $x^2-y^2=8$ at $(3,1)$

$2x-2y\left(\frac{dy}{dx}\right)=0$

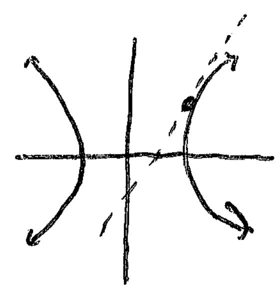
$-2y\left(dy/dx\right)=2x$

$\frac{dy}{dx}=\frac{x}{y}$

a) $\left.\frac{dy}{dx}\right|_{(3,1)}=\frac{3}{1}=3$

$\boxed{\text{a) slope: } m=3}$

$\boxed{\text{b) } y-1=3(x-3)}$

c) 

57) $\frac{x^2}{4}+\frac{y^2}{3}=1$ at $(-1,3/2)$

$\frac{1}{4}x^2+\frac{1}{3}y^2=1$

$\frac{1}{4}\cdot 2x+\frac{2}{3}y\left(\frac{dy}{dx}\right)=0$

$\frac{2}{3}y\left(\frac{dy}{dx}\right)=-\frac{1}{2}x$

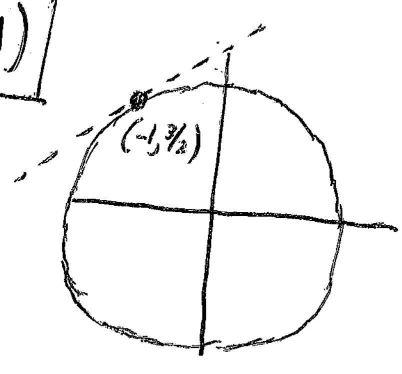
$\frac{dy}{dx}=\frac{3}{2y}\cdot\frac{-1x}{2}$

$\frac{dy}{dx}=\frac{-3x}{4y}$

$\left.\frac{dy}{dx}\right|_{(-1,3/2)}=\frac{-3(-1)}{4(3/2)}=\frac{+3}{6}=\frac{+1}{2}$

$\boxed{\text{a) slope: } m=+1/2}$

$\boxed{\text{b) } y-\frac{3}{2}=\frac{+1}{2}(x+1)}$



$\boxed{3.2b}$

59) Find y' and y'' at $(-1,1) \rightarrow$ $3x^2y + 2y^3 = 5x^2$

*product Rule
*implicit diff.

$$\overbrace{6x}^{f'} \cdot \overbrace{y}^{g} + \overbrace{3x^2}^{f} \cdot \overbrace{\left(dy/dx\right)}^{g'} + 6y^2\left(\frac{dy}{dx}\right) = 10x$$

$$\frac{dy}{dx}\left(3x^2+6y^2\right) = 10x - 6xy$$

$$\frac{dy}{dx} = \frac{10x-6xy}{3x^2+6y^2}$$

a) $$\left.\frac{dy}{dx}\right|_{(-1,1)} = \frac{10(-1)-6(-1)(1)}{3(-1)^2+6(1)^2} = \boxed{\frac{-4}{9}}$$

$$\frac{dy}{dx} = \frac{\overbrace{10x-6xy}^{f}}{\underbrace{3x^2+6y^2}_{g}}$$

$$\frac{d^2y}{dx^2} = \frac{\overbrace{10-(\overbrace{6}^{f'}\cdot\overbrace{y}^{g}+\overbrace{6x}^{f}\cdot\overbrace{dy/dx}^{g'})}^{f'}\cdot\overbrace{(3x^2+6y^2)}^{g} - \overbrace{(10x-6xy)}^{f}\overbrace{(6x+12y(\tfrac{dy}{dx}))}^{g'}}{\underbrace{(3x^2+6y^2)^2}_{g^2}}$$

b) $$\left.\frac{d^2y}{dx^2}\right|_{(-1,1)} = \frac{10-6-6(-1)(-4/9)(3+6) - (-10+6)(-6+12(-4/9))}{(3+6)^2} = \boxed{\frac{-100}{243}}$$

71) $x + \overbrace{x}^{f}\overbrace{y}^{g} + 2y^2 = 6$

*product Rule
*implicit

a) $$1 + \overbrace{(1)}^{f'}\overbrace{(y)}^{g} + \overbrace{(x)}^{f}\overbrace{(1)\left(\frac{dy}{dx}\right)}^{g'} + 4y\left(\frac{dy}{dx}\right) = 0$$

$$1 + y + x\left(\frac{dy}{dx}\right) + 4y\left(\frac{dy}{dx}\right) = 0$$

$$\frac{dy}{dx}(x+4y) = -1-y$$

a) $$\boxed{\frac{dy}{dx} = \frac{-1-y}{x+4y}}$$

b) $$\left.\frac{dy}{dx}\right|_{(2,1)} = \frac{-1-1}{2+4(1)} = \frac{-2}{6} = \frac{-1}{3}$$

point: $(2,1)$

slope: $m = -1/3$

$$\boxed{y-1 = -1/3(x-2)}$$

c) Find elsewhere on curve where slope of tangent is also $m = -1/3$ besides at $(2,1)$

$$\frac{-1-y}{x+4y} = \frac{-1}{3}$$

$$-3-3y = -x-4y$$

$$3+3y = x+4y$$

$$x = 3-y$$

substitute in

$$x + xy + 2y^2 = 6$$

$$(3-y)+(3-y)y+2y^2 = 6$$

$$3-y+3y-y^2+2y^2-6 = 0$$

$$y^2+2y-3 = 0$$

$$(y+3)(y-1) = 0$$

$y=-3$ and $y=1 \rightarrow (2,1)$

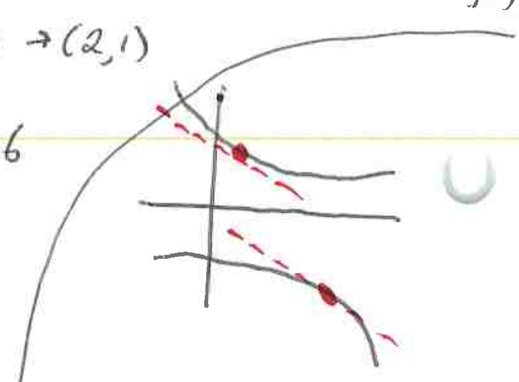
$$x + x(-3) + 2(-3)^2 = 6$$

$$x - 3x + 18 = 6$$

$$-2x = -12$$

$$x = 6$$

$\boxed{\text{point at } (6,-3)}$



$\boxed{3.2b}$

77) $x^3+y^3=\overbrace{2x}^{f}\overbrace{y}^{g}$

a) Find $\frac{dy}{dx}$

$$3x^2+3y^2\left(\frac{dy}{dx}\right)=\overbrace{(2)}^{f'}\overbrace{(y)}^{g}+\overbrace{2x}^{f}\cdot\overbrace{\left(dy/dx\right)}^{g'}$$

$$3y^2\left(dy/dx\right)-2x\left(\frac{dy}{dx}\right)=2y-3x^2$$

$$\frac{dy}{dx}\left(3y^2-2x\right)=2y-3x^2$$

$$\boxed{\frac{dy}{dx}=\frac{2y-3x^2}{3y^2-2x}\quad\text{or}\quad\frac{3x^2-2y}{2x-3y^2}}$$

b) Find equation of tangent line at $(1,1)$

$$\left.\frac{dy}{dx}\right|_{(1,1)}=\frac{2(1)-3(1)^2}{3(1)^2-2(1)}=\frac{-1}{1}=-1$$

point: $(1,1)$

slope: $m=-1$

$$\boxed{y-1=-1(x-1)}$$

c) Find points where tangent line to graph is horizontal (ignore the origin)

* set numerator of $dy/dx=0$
* plug into original equation to solve for a variable
* find ordered pair.

$$2y-3x^2=0$$

$$y=\frac{3x^2}{2}$$

$$x^3+y^3=2xy$$

$$x^3+\left(\frac{3}{2}x^2\right)^3=2x\left(\frac{3x^2}{2}\right)$$

$$x^3+\frac{27}{8}x^6=3x^3$$

$$\frac{27}{8}x^6-2x^3=0$$

$$x^3\left(\frac{27}{8}x^3-2\right)=0$$

$x^3=0$

$(0,0)$

(ignore origin)

$$\frac{27}{8}x^3-2=0$$

$$\frac{27}{8}x^3=2$$

$$x^3=\frac{16}{27}$$

$$x=\sqrt[3]{\frac{16}{27}}=\frac{\sqrt[3]{16}}{3}$$

$$y=\frac{3x^2}{2}$$

$$y=\frac{3\left(\frac{\sqrt[3]{16}}{3}\right)^2}{2}$$

$$\frac{\sqrt[3]{16}}{3}=\frac{2\left(\sqrt[3]{2}\right)}{3}\text{ or }\frac{2^{4/3}}{3}$$

$$y=\frac{\left(2^{4/3}\right)^2}{3\cdot 2}\rightarrow\frac{2^{8/3}}{3\cdot 2}\rightarrow\frac{2^{5/3}}{3}$$

$$\boxed{\text{point: }\left(\frac{2^{4/3}}{3},\frac{2^{5/3}}{3}\right)}$$

d)