AB Calculus Ch. 3.3 Select HW Problems

Applying the First Derivative Test In Exercises 17–40, (a) find the critical numbers of f (if any), (b) find the open interval(s) on which the function is increasing or decreasing, (c) apply the First Derivative Test to identify all relative extrema, and (d) use a graphing utility to confirm your results.

19. $f(x) = -2x^2 + 4x + 3$

21. $f(x) = 2x^3 + 3x^2 - 12x$

25. $f(x) = \dfrac{x^5 - 5x}{5}$

27. $f(x) = x^{1/3} + 1$

29. $f(x) = (x + 2)^{2/3}$

33. $f(x) = 2x + \dfrac{1}{x}$

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Finding Points of Inflection **In Exercises 15–30, find the points of inflection and discuss the concavity of the graph of the function.**

15. $f(x) = x^3 - 6x^2 + 12x$

17. $f(x) = \frac{1}{2}x^4 + 2x^3$

19. $f(x) = x(x - 4)^3$

21. $f(x) = x\sqrt{x + 3}$

23. $f(x) = \dfrac{4}{x^2 + 1}$

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Key

Applying the First Derivative Test In Exercises 17–40, (a) find the critical numbers of f (if any), (b) find the open interval(s) on which the function is increasing or decreasing, (c) apply the First Derivative Test to identify all relative extrema, and (d) use a graphing utility to confirm your results.

19. $f(x) = -2x^2 + 4x + 3$

$f'(x) = -4x + 4$

$0 = -4(x-1)$

$x = 1$

$\nearrow$ + | $\searrow$ −

1

Increasing on $(-\infty, 1)$ b/c $f'(x) > 0$

Decreasing on $(1, \infty)$ b/c $f'(x) < 0$

Relative max at $(1, 5)$ b/c $f'(x)$ changes from + to −

21. $f(x) = 2x^3 + 3x^2 - 12x$

$f'(x) = 6x^2 + 6x - 12$

$0 = 6(x^2 + x - 2)$

$0 = 6(x+2)(x-1)$

$x = 1, -2$

$\nearrow$ + | $\searrow$ − | $\nearrow$ +

−2 1

Increasing on $(-\infty, -2) \cup (1, \infty)$ b/c $f'(x) > 0$

Decreasing on $(-2, 1)$ b/c $f'(x) < 0$

Rel. max $(-2, 20)$ b/c $f'(x)$ changes from + to −

Rel. min $(1, -7)$ b/c $f'(x)$ changes from − to +

25. $f(x) = \dfrac{x^5 - 5x}{5} = \frac{1}{5}(x^5 - 5x)$

$f(x) = \frac{1}{5}x^5 - \frac{5}{5}x = \frac{1}{5}x^5 - x$

$f'(x) = 5 \cdot \frac{1}{5}x^4 - 1$

$f'(x) = x^4 - 1$

$0 = (x^2+1)(x^2-1)$

$0 = (x^2+1)(x-1)(x+1)$

$x = 1, -1$

$\nearrow$ + | $\searrow$ − | $\nearrow$ +

−1 1

Inc: $(-\infty, -1) \cup (1, \infty)$

Dec: $(-1, 1)$

Rel. max: $(-1, 4/5)$

Rel. min: $(1, -4/5)$

27. $f(x) = x^{1/3} + 1$

$f'(x) = \frac{1}{3}x^{-2/3} + 0$

$0 = \dfrac{1}{3x^{2/3}}$

$x = 0$

$\nearrow$ + | $\nearrow$ +

0

Increasing: $(-\infty, 0] \cup (0, \infty)$ b/c $f'(x) > 0$

No relative extrema

29. $f(x) = (x+2)^{2/3}$ * Apply chain rule

$f'(x) = \frac{2}{3}(x+2)^{-1/3}(1)$

$0 = \dfrac{2}{3(x+2)^{1/3}}$

$x = -2$

$\searrow$ − | $\nearrow$ +

−2

Dec: $(-\infty, -2)$ b/c $f'(x) < 0$

Inc: $(-2, \infty)$ b/c $f'(x) > 0$

Rel. min $(-2, 0)$ b/c $f'(x)$ changes from − to +

33. $f(x) = 2x + \dfrac{1}{x}$

$f(x) = 2x + x^{-1}$

$f'(x) = 2 - 1x^{-2}$

$f'(x) = 2 - \dfrac{1}{x^2}$

$f'(x) = \dfrac{2x^2 - 1}{x^2}$

$2x^2 - 1 = 0$ | $x^2 = 0$

$x^2 = 1/2$

$x = \pm\sqrt{1/2} = \frac{\sqrt{2}}{2}$

$\nearrow$ + | $\searrow$ − | $\searrow$ − | $\nearrow$ +

$-\sqrt{2}/2$ 0 $\sqrt{2}/2$

Inc: $(-\infty, -\sqrt{2}/2) \cup (\sqrt{2}/2, \infty)$ b/c $f'(x) > 0$

Dec: $(-\sqrt{2}/2, 0) \cup (0, \sqrt{2}/2)$ b/c $f'(x) < 0$

Rel. max: $(-\sqrt{2}/2, -2\sqrt{2})$

Rel. min: $(\sqrt{2}/2, 2\sqrt{2})$

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Steps:
1) find $f''(x)$ (second derivative)
2) set $f''(x)=0$, find critical pts.
3) create sign line, evaluate concavity in intervals

Finding Points of Inflection In Exercises 15–30, find the points of inflection and discuss the concavity of the graph of the function.

15. $f(x) = x^3 - 6x^2 + 12x$

$f'(x) = 3x^2 - 12x + 12$

$f''(x) = 6x - 12$

$0 = 6(x-2)$

$x = 2$

$\cap$ $-$ | $+$ $\cup$ at 2

concave up: $(2,\infty)$ b/c $f''(x) > 0$

concave down: $(-\infty, 2)$ b/c $f''(x) < 0$

POI: $(2, 8)$ b/c $f''(x)$ change signs

17. $f(x) = \frac{1}{2}x^4 + 2x^3$

$f'(x) = 4\cdot\frac{1}{2}x^3 + 6x^2 = 2x^3 + 6x^2$

$f''(x) = 6x^2 + 12x$

$0 = 6x(x+2)$

$x = 0, -2$

$\cup$ $+$ | $-$ $\cap$ | $+$ $\cup$ at -2, 0

Concave up $(-\infty,-2)\cup(0,\infty)$ b/c $f''(x) > 0$

Concave down $(-2, 0)$ b/c $f''(x) < 0$

POI: $(-2, -8)$ and $(0, 0)$ b/c $f''(x)$ change signs

19. $f(x) = x(x-4)^3$ * apply product, chain rule

$f'(x) = 1\cdot(x-4)^3 + x\cdot 3(x-4)^2(1)$

$= (x-4)^2[x-4+3x] = (x-4)^2(4x-4)$

$f''(x) = 2(x-4)\cdot(4x-4) + (x-4)^2(4)$

$= 2(x-4)\cdot 4(x-1) + (x-4)^2\cdot 4$

$= 4(x-4)\left[\overset{2x-2+x-4}{2(x-1)+x-4}\right] = 4(x-4)(3x-6)$

$f''(x) = 12(x-4)(x-2)$

$0 = 12(x-4)(x-2)$

$x = 2, 4$

$\cup$ $+$ | $-$ $\cap$ | $+$ $\cup$ at 2, 4

concave up $(-\infty, 2)\cup(4,\infty)$

concave down $(2, 4)$

POI: $(2, -16)$ $(4, 0)$

Domain: $[-3, \infty)$

21. $f(x) = x\sqrt{x+3}$ * Apply product, chain rule

$f(x) = x(x+3)^{1/2}$

$f'(x) = 1(x+3)^{1/2} + x\cdot\frac{1}{2}(x+3)^{-1/2}(1)$

$f'(x) = \sqrt{x+3} + \dfrac{x}{2\sqrt{x+3}} = \dfrac{2(x+3)+x}{2\sqrt{x+3}} = \dfrac{3x+6}{2\sqrt{x+3}}$

$f''(x) = \dfrac{3\cdot 2\sqrt{x+3} - (3x+6)\cdot 2\cdot\frac{1}{2}(x+3)^{-1/2}}{[2\sqrt{x+3}]^2}$

$= \dfrac{6\sqrt{x+3} - \frac{3x+6}{\sqrt{x+3}}}{4(x+3)}\cdot\dfrac{\sqrt{x+3}}{\sqrt{x+3}} = \dfrac{\overset{6x+18-3x-6}{6(x+3)-(3x+6)}}{4(x+3)^{3/2}}$

$f''(x) = \dfrac{3x+12}{4(x+3)^{3/2}}$

$x = -4, -3$

not in domain

$\times$ | $+$ $\cup$ at -3

Concave up: $(-3, \infty)$

No POI.

23. $f(x) = \dfrac{4}{x^2+1} = 4(x^2+1)^{-1}$

$f'(x) = -4(x^2+1)^{-2}(2x) = \dfrac{-8x}{(x^2+1)^2}$

$f''(x) = \dfrac{-8(x^2+1)^2 - 8x\cdot 2(x^2+1)(2x)}{(x^2+1)^4}$

$= \dfrac{-8(x^2+1)[x^2+1-4x^2]}{(x^2+1)^4}$

$f''(x) = \dfrac{-8(1-3x^2)}{(x^2+1)^3}$

$x = \pm\frac{\sqrt{3}}{3}$

$\cup$ $+$ | $-$ $\cap$ | $+$ $\cup$ at $-\sqrt{3}/3$, $\sqrt{3}/3$

Concave up: $(-\infty, -\sqrt{3}/3)\cup(\sqrt{3}/3, \infty)$

Concave down: $(-\sqrt{3}/3, \sqrt{3}/3)$

POI: $(-\frac{\sqrt{3}}{3}, 3)$ and $(\frac{\sqrt{3}}{3}, 3)$