

AB Calculus Ch. 3.3 Select HW Problems

Applying the First Derivative Test In Exercises 17–40,

(a) find the critical numbers of f (if any), (b) find the open interval(s) on which the function is increasing or decreasing, (c) apply the First Derivative Test to identify all relative extrema, and (d) use a graphing utility to confirm your results.

19. $f(x) = -2x^2 + 4x + 3$

21. $f(x) = 2x^3 + 3x^2 - 12x$

25. $f(x) = \frac{x^5 - 5x}{5}$

27. $f(x) = x^{1/3} + 1$

29. $f(x) = (x + 2)^{2/3}$

33. $f(x) = 2x + \frac{1}{x}$

AB Calculus Ch. 3.4 Select HW Problems

Finding Points of Inflection In Exercises 15–30, find the points of inflection and discuss the concavity of the graph of the function.

15. $f(x) = x^3 - 6x^2 + 12x$

17. $f(x) = \frac{1}{2}x^4 + 2x^3$

19. $f(x) = x(x - 4)^3$

21. $f(x) = x\sqrt{x + 3}$

23. $f(x) = \frac{4}{x^2 + 1}$

AB Calculus Ch. 3.3 Select HW Problems

Key

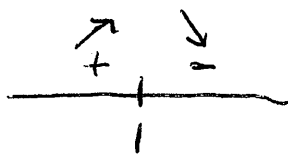
Applying the First Derivative Test In Exercises 17-40,
 (a) find the critical numbers of f (if any), (b) find the open interval(s) on which the function is increasing or decreasing,
 (c) apply the First Derivative Test to identify all relative extrema,
 and (d) use a graphing utility to confirm your results.

19. $f(x) = -2x^2 + 4x + 3$

$$f'(x) = -4x + 4$$

$$0 = -4(x-1)$$

$$x = 1$$



Increasing on $(-\infty, 1)$
 b/c $f'(x) > 0$
 Decreasing on $(1, \infty)$
 b/c $f'(x) < 0$
 Relative max at $(1, 5)$
 b/c $f'(x)$ changes from + to -

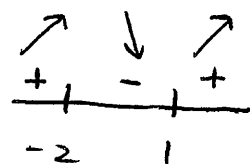
21. $f(x) = 2x^3 + 3x^2 - 12x$

$$f'(x) = 6x^2 + 6x - 12$$

$$0 = 6(x^2 + x - 2)$$

$$0 = 6(x+2)(x-1)$$

$$x = -2, 1$$



Increasing on $(-\infty, -2)$
 $\cup (1, \infty)$ b/c $f'(x) > 0$
 Decreasing on $(-2, 1)$
 b/c $f'(x) < 0$
 Rel. max $(-2, 20)$ b/c
 $f'(x)$ changes from + to -
 Rel. min $(1, -7)$ b/c
 $f'(x)$ changes from - to +

25. $f(x) = \frac{x^5 - 5x}{5} = \frac{1}{5}(x^5 - 5x)$

$$f(x) = \frac{1}{5}x^5 - \frac{5}{5}x = \frac{1}{5}x^5 - x$$

$$f'(x) = 5 \cdot \frac{1}{5}x^4 - 1$$

$$f'(x) = x^4 - 1$$

$$0 = (x^2 + 1)(x^2 - 1)$$

$$0 = (x^2 + 1)(x-1)(x+1)$$

$$x = -1, 1$$

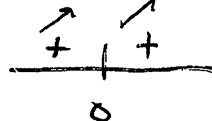
Inc: $(-\infty, -1) \cup (1, \infty)$
 Dec: $(-1, 1)$
 Rel. max: $(-1, 4/5)$
 Rel. min: $(1, -4/5)$

27. $f(x) = x^{1/3} + 1$

$$f'(x) = \frac{1}{3}x^{-2/3} + 0$$

$$0 = \frac{1}{3x^{2/3}}$$

$$x = 0$$



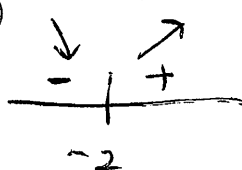
Increasing: $(-\infty, 0) \cup (0, \infty)$
 b/c $f'(x) > 0$
 No relative extrema

29. $f(x) = (x+2)^{2/3}$ * Apply chain rule

$$f'(x) = \frac{2}{3}(x+2)^{-1/3} (1)$$

$$0 = \frac{2}{3(x+2)^{1/3}}$$

$$x = -2$$



Dec: $(-\infty, -2)$
 b/c $f'(x) < 0$
 Inc: $(-2, \infty)$
 b/c $f'(x) > 0$
 Rel. min $(-2, 0)$
 b/c $f'(x)$ changes from - to +

33. $f(x) = 2x + \frac{1}{x}$

$$f(x) = 2x + x^{-1}$$

$$f'(x) = 2 - 1x^{-2}$$

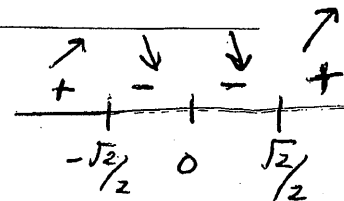
$$f'(x) = 2 - \frac{1}{x^2}$$

$$f'(x) = \frac{2x^2 - 1}{x^2}$$

$$2x^2 - 1 = 0 \quad | \quad x^2 = 0$$

$$x^2 = \frac{1}{2}$$

$$x = \pm\sqrt{\frac{1}{2}} = \pm\frac{\sqrt{2}}{2}$$



Inc: $(-\infty, -\frac{\sqrt{2}}{2}) \cup (\frac{\sqrt{2}}{2}, \infty)$
 b/c $f'(x) > 0$
 Dec: $(-\frac{\sqrt{2}}{2}, 0) \cup (0, \frac{\sqrt{2}}{2})$
 b/c $f'(x) < 0$
 Rel. max: $(-\frac{\sqrt{2}}{2}, -2\sqrt{2})$
 Rel. min: $(\frac{\sqrt{2}}{2}, 2\sqrt{2})$

AB Calculus Ch. 3.4 Select HW Problems

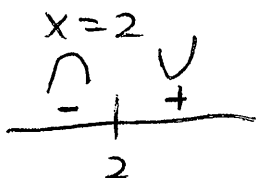
key

Steps:

- 1) find $f''(x)$ (second derivative)
- 2) set $f''(x) = 0$, find critical pts.
- 3) create sign line, evaluate concavity in intervals

Finding Points of Inflection In Exercises 15-30, find the points of inflection and discuss the concavity of the graph of the function.

15. $f(x) = x^3 - 6x^2 + 12x$
 $f'(x) = 3x^2 - 12x + 12$
 $f''(x) = 6x - 12$
 $0 = 6(x - 2)$

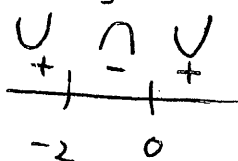


Concave up: $(2, \infty)$ b/c $f''(x) > 0$
 Concave down: $(-\infty, 2)$ b/c $f''(x) < 0$
 POI: $(2, 8)$ b/c $f''(x)$ change signs

17. $f(x) = \frac{1}{2}x^4 + 2x^3$

$f'(x) = 4 \cdot \frac{1}{2}x^3 + 6x^2 = 2x^3 + 6x^2$
 $f''(x) = 6x^2 + 12x$

$0 = 6x(x + 2)$
 $x = 0, -2$



Concave up $(-\infty, -2) \cup (0, \infty)$
 b/c $f''(x) > 0$
 Concave down $(-2, 0)$
 b/c $f''(x) < 0$
 POI: $(-2, -8)$ and $(0, 0)$
 b/c $f''(x)$ change signs

Domain: $[-3, \infty)$

19. $f(x) = x(x - 4)^3$ * apply product, chain rule

$f'(x) = 1 \cdot (x - 4)^3 + x \cdot 3(x - 4)^2(1)$
 $= (x - 4)^2 [x - 4 + 3x] = (x - 4)^2 (4x - 4)$

$f''(x) = 2(x - 4) \cdot (4x - 4) + (x - 4)^2(4)$
 $= 2(x - 4) \cdot 4(x - 1) + (x - 4)^2 \cdot 4$
 $= 4(x - 4) [2(x - 1) + x - 4] = 4(x - 4)(3x - 6)$

$f'''(x) = 12(x - 4)(x - 2)$
 $0 = 12(x - 4)(x - 2)$
 $x = 2, 4$

Concave up $(-\infty, 2) \cup (4, \infty)$
 Concave down $(2, 4)$
 POI: $(2, -16)$
 $(4, 0)$

21. $f(x) = x\sqrt{x+3}$ * Apply product, chain rule

$f(x) = x(x + 3)^{1/2}$
 $f'(x) = 1(x + 3)^{1/2} + x \cdot \frac{1}{2}(x + 3)^{-1/2}(1)$

$f''(x) = \frac{1}{2\sqrt{x+3}} + \frac{x}{2\sqrt{x+3}} = \frac{2(x+3) + x}{2\sqrt{x+3}} = \frac{3x+6}{2\sqrt{x+3}}$

$f''(x) = \frac{3 \cdot 2\sqrt{x+3} - (3x+6) \cdot 2 \cdot \frac{1}{2}(x+3)^{-1/2}}{[2\sqrt{x+3}]^2}$
 $= \frac{6\sqrt{x+3} - \frac{3x+6}{\sqrt{x+3}}}{4(x+3)} = \frac{6(x+3) - (3x+6)}{4(x+3)^{3/2}}$

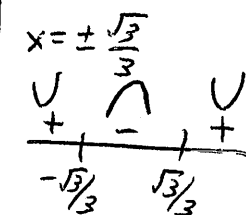
$f''(x) = \frac{3x+12}{4(x+3)^{3/2}}$
 $x = -4, -3$
 not in domain

Concave up: $(-3, \infty)$
 No POI.

23. $f(x) = \frac{4}{x^2+1} = 4(x^2+1)^{-1}$

$f'(x) = -4(x^2+1)^{-2}(2x) = \frac{-8x}{(x^2+1)^2}$

$f''(x) = \frac{-8(1-3x^2)}{(x^2+1)^3}$



$f'''(x) = \frac{-8(x^2+1)^2 - 8x \cdot 2(x^2+1)(2x)}{(x^2+1)^4}$
 $= \frac{-8(x^2+1)[x^2+1 - 4x^2]}{(x^2+1)^4}$

Concave up: $(-\infty, -\sqrt{3}/3) \cup (\sqrt{3}/3, \infty)$
 Concave down: $(-\sqrt{3}/3, \sqrt{3}/3)$
 POI: $(-\frac{\sqrt{3}}{3}, 3)$ and $(\frac{\sqrt{3}}{3}, 3)$