3.3 AP Practice Problems (p.251) – Derivatives of Inverse Trig Functions

perpendicular to tangent line

$$\frac{d}{dx}\tan^{-1}(u) = \frac{u'}{1+u^2}$$

1. What is the slope of the normal line to the graph of $y = \tan^{-1}(2x)$ where $x = -1$?

   (A) $\frac{2}{5}$ (B) $-\frac{2}{5}$ (C) -5 $\boxed{\text{(D)}}$ $-\frac{5}{2}$

$$y' = \frac{2}{1+(2x)^2} \quad\Big|\quad y'(-1) = \frac{2}{1+(-2)^2} = \frac{2}{5} \longrightarrow$$

$\boxed{\text{slope of normal line (perpendicular slope)} \quad m_2 = \frac{-5}{2}}$

2. $\frac{d}{dx}\sin^{-1}(e^{2x}) =$

   (A) $-\frac{2e^{2x}}{\sqrt{1-e^{2x}}}$ $\boxed{\text{(B)}}$ $\frac{2e^{2x}}{\sqrt{1-e^{4x}}}$

   (C) $\frac{2e^{2x}}{\sqrt{1-e^{4x^2}}}$ (D) $\frac{2e^{2x}}{\sqrt{1-e^{2x}}}$

$$y' = \frac{e^{2x}(2)}{\sqrt{1-(e^{2x})^2}} = \boxed{\frac{2e^{2x}}{\sqrt{1-e^{4x}}}}$$

$$* \frac{d}{dx}\sin^{-1}(u) = \frac{u'}{\sqrt{1-u^2}}$$

$f(a)=b$	$g(b)=a$
$f'(a)=n$	$g'(b)=\frac{1}{n}$

$f(2)=-3$	$g(-3)=2$
$f'(2)=\frac{-1}{3}$	$g'(-3)=\boxed{-3}$

3. The functions f and g are differentiable and g is the inverse of f. If $g(-3) = 2$ and $f'(2) = -\frac{1}{3}$, then $g'(-3)$ is

   (A) 3 (B) $\frac{1}{2}$ $\boxed{\text{(C)}}$ -3 (D) $\frac{1}{3}$

$f(\ )=-3$	$g(-3)=$
	$g'(-3)=$___

4. If $y = \tan^{-1}(\cos x)$, then $y' =$

   (A) $\frac{\sin x}{1+\cos^2 x}$ (B) $-\frac{\cos x}{1+\sin^2 x}$

   $\boxed{\text{(C)}}$ $-\frac{\sin x}{1+\cos^2 x}$ (D) $\frac{\cos x}{1+\sin^2 x}$

$$y' = \frac{-\sin x}{1+(\cos x)^2}$$

$$\boxed{y' = \frac{-\sin x}{1+\cos^2 x}}$$

$$* \frac{d}{dx}\tan^{-1}(u) = \frac{u'}{1+u^2}$$

5. The function g is given by $g(x) = x^3 + x^2 - 3$, $x \geq 0$, so $g(2) = 9$. If the function f is the inverse function of g, find $f'(9)$.

(A) $\frac{1}{261}$ (B) -261 (C) 16 $\boxed{\text{(D)}}$ $\frac{1}{16}$

$g(\,) = 9$ | $f(9) = \_$; $f'(9) = \_$

$g(x) = x^3 + x^2 - 3$
$9 = x^3 + x^2 - 3$
$0 = x^3 + x^2 - 12$

*guess and check
$x = 2$

$g(2) = 9$ | $f(9) = 2$; $g'(2) = 16$ | $f'(9) = \frac{1}{16}$

$g'(x) = 3x^2 + 2x$
$g'(2) = 3(2)^2 + 2(2)$
$g'(2) = 12 + 4 = 16$
$g'(2) = 16$

$\boxed{f'(9) = \frac{1}{16}}$

6. An object is moving along the y-axis. Its position (in centimeters) at time $t > 0$ seconds is given by $y(t) = \tan^{-1} t$. What is the acceleration of the object at $t = 1$ second?

(A) $\frac{1}{2}$ cm/s^2 (B) $-\frac{1}{2}$ cm/s

(C) -2 cm/s^2 $\boxed{\text{(D)}}$ $-\frac{1}{2}$ cm/s^2

position: $y = \tan^{-1}(t)$
$V(t) = y'(t) = \frac{1}{1+t^2}$

$v(t) = (1+t^2)^{-1}$
$V'(t) = -1(1+t^2)^{-2}(2t)$

$a(t) = \frac{-2t}{(1+t^2)^2}$

$a(1) = \frac{-2(1)}{(1+1^2)^2} = \frac{-2}{4} = \frac{-1}{2}$

$\boxed{a(1) = \frac{-1}{2}}$

7. $F(x) = x^3 - x^2 + x - 5$ and g are inverse functions. Find an equation of the tangent line to the graph of g at the point $(-4, 1)$ on g.

(A) $y = \frac{1}{2}x - \frac{9}{2}$ (B) $y = \frac{1}{57}x - \frac{53}{57}$

$\boxed{\text{(C)}}$ $y = \frac{1}{2}x + 3$ (D) $y = 2x + 9$

$f(1) = -4$ | $g(-4) = 1$; $f'(1) = 2$ | $g'(-4) = \frac{1}{2}$

$F'(x) = 3x^2 - 2x + 1$
$F'(1) = 3(1)^2 - 2(1) + 1$
$F'(1) = 2$

point: $(-4, 1)$
slope: $m = 1/2$
$y - 1 = \frac{1}{2}(x+4)$

$y - 1 = \frac{1}{2}x + 2$
$\boxed{y = \frac{1}{2}x + 3}$

8. The functions f and g are differentiable; $f(x) = g^{-1}(x)$. Find $f'(4)$ if $g(4) = 3$, $g(-1) = 4$, $g'(-1) = 5$, and $g'(4) = -1$.

(A) -1 (B) $-\frac{1}{5}$ $\boxed{\text{(C)}}$ $\frac{1}{5}$ (D) $\frac{1}{4}$

$g(\,) = 4$ | $f(4) = \_$; $f'(4) = \_$

$\rightarrow$ $g(-1) = 4$ | $f(4) = -1$; $g'(-1) = 5$ | $f'(4) = 1/5$

$\rightarrow$ $\boxed{f'(4) = \frac{1}{5}}$