Key

AP Calculus – 3.3a Notes – Derivative of Inverse at a Point

Inverse Function: A function's inverse is found by swapping the input (x) and output (y) values.

(Domains and Ranges are also swapped between a function and its inverse)

Example A:

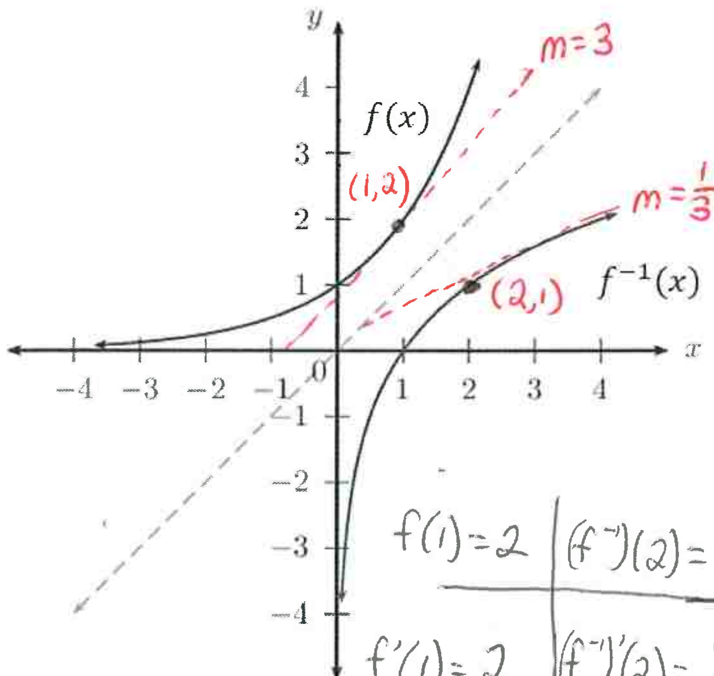
Find the inverse of $f(x) = 6x + 2$

$y = 6x + 2$	$f^{-1}(x) = \frac{x-2}{6}$
$x = 6y + 2$	
$x - 2 = 6y$	If $f(1) = 8$, then
$\frac{x-2}{6} = y$	$(f^{-1})(8) = 1$

Three ways to say the same thing:

1. $g(x)$ is the inverse of $f(x)$
2. $g(x) = f^{-1}(x)$
3. $f(g(x)) = x$ and $g(f(x)) = x$

Ex: Graphical example of function $f(x)$ & its inverse $f^{-1}(x)$



$f(1) = 2$	$(f^{-1})(2) = 1$
$f'(1) = 2$	$(f^{-1})'(2) = \underline{1/3}$

Evaluate derivative of inverse at a point: (find $(f^{-1})'(a)$

$f(b) = a$	$(f^{-1})(a) = b$
$f'(b) = n$	$(f^{-1})'(a) = \frac{1}{n}$

Derivative of an Inverse Function:

$$\frac{d}{dx}[f^{-1}(x)] = \frac{1}{f'[f^{-1}(x)]}$$

test $x = 0, \pm1, \pm2, \pm3$

Example B: $f(x) = x^3 + 4x + 2$ $\quad$ *find* $(f^{-1})'(-3)$

$f(-1) = -3$	$(f^{-1})(-3) =$
$f'(-1) = 7$	$(f^{-1})'(-3) = \boxed{\frac{1}{7}}$

$-3 = x^3 + 4x + 2$

$0 = x^3 + 4x + 5$

$x = \cancel{1}, -1$ ✓

$f'(x) = 3x^2 + 4$

$f'(-1) = 3(-1)^2 + 4 = 7$

Example C: $f(x) = \sqrt{x^3 - 7}$ $\quad$ *find* $(f^{-1})'(1)$

$f(2) = 1$	$(f^{-1})(1) = 2$
$f'(2) = 6$	$(f^{-1})'(1) = \boxed{1/6}$

$1 = \sqrt{x^3 - 7}$

$1 = x^3 - 7$

$8 = x^3$

$x = 2$

$f'(x) = \frac{1}{2}(x^3 - 7)^{-1/2}(3x^2) = \frac{3x^2}{2\sqrt{x^3 - 7}}$

$f'(2) = \frac{3(2)^2}{2\sqrt{2^3 - 7}} = \frac{12}{2(1)} = 6$

Example D:

The table below gives values of the differentiable functions f, g, and f' at selected values of x. Let $g(x) = f^{-1}(x)$.

x	$f(x)$	$f'(x)$
1	3	−3
2	1	−2
3	−5	−5
4	0	−6

Evaluate derivative of inverse at a point: (find $(f^{-1})'(a)$

$f(b) = a$	$(f^{-1})(a) = b$
$f'(b) = n$	$(f^{-1})'(a) = \frac{1}{n}$

1. What is the value of $g'(1)$?

$f(2) = 1 \mid g(1) = 2$

$f'(2) = -2 \mid g'(1) = -1/2$

$\boxed{g'(1) = -\frac{1}{2}}$

2. Write an equation for the line tangent to f^{-1} at $x = 1$.

point: $(1, 2)$

$(f^{-1})'(1) = -\frac{1}{2}$

$y - y_1 = m(x - x_1)$

$\boxed{y - 2 = -\frac{1}{2}(x - 1)}$

3. Let g be a differentiable function such that $g(12) = 4$, $g(3) = 6$, $g'(12) = -5$, and $g'(3) = -2$. The function h is differentiable and $h(x) = g^{-1}(x)$ for all x. What is the value of $h'(6)$?

$\boxed{h'(6) = -\frac{1}{2}}$

$g(3) = 6 \mid h(6) = 3$

$g'(3) = -2 \mid h'(6) = -1/2$

4. If $f(x) = 3x^3 + 1$ and g is the inverse function of f, what is the value of $g'(25)$?

$25 = 3x^3 + 1$

$24 = 3x^3$

$8 = x^3$

$2 = x$

$f(2) = 25 \mid g(25) = 2$

$f'(2) = 36 \mid g'(25) = \boxed{1/36}$

$f'(x) = 9x^2$

$f'(2) = 9(2)^2 = 36$

For each function $g(x)$, its inverse $g^{-1}(x) = f(x)$. Evaluate the given derivative.

13. $g(x) = \cos(x) + 3x^2$

$g\left(\frac{\pi}{2}\right) = \frac{3\pi}{4}$. Find $f'\left(\frac{3\pi}{4}\right)$

$g'(x) = -\sin x + 6x$

$g'\left(\frac{\pi}{2}\right) = -\sin\left(\frac{\pi}{2}\right) + 6\left(\frac{\pi}{2}\right)$

$g'(\pi/2) = -1 + 3\pi$

$g'(\pi/2 = 3\pi - 1$

$g\left(\frac{\pi}{2}\right) = \frac{3\pi}{4} \mid f\left(\frac{3\pi}{4}\right) =$

$g'\left(\frac{\pi}{2}\right) = 3\pi - 1 \mid f'\left(\frac{3\pi}{4}\right) = \boxed{\frac{1}{3\pi - 1}}$

14. $g(x) = 2x^3 - x^2 - 5x$

$g(-2) = -10$. Find $f'(-10)$

$g'(x) = 6x^2 - 2x - 5$

$g'(-2) = 6(-2)^2 - 2(-2) - 5$

$g'(-2) = 24 + 4 - 5$

$= 28 - 5 = 23$

$g(-2) = -10 \mid f(-10) = -2$

$g'(-2) = 23 \mid f'(-10) = \boxed{\frac{1}{23}}$

19. A function h satisfies $h(3) = 5$ and $h'(3) = 7$. Which of the following statements about the inverse of h must be true?

(A) $(h^{-1})'(5) = 3$ (B) $(h^{-1})'(7) = 3$ (C) $(h^{-1})'(5) = 7$

$\boxed{\text{(D) } (h^{-1})'(5) = \frac{1}{7}}$ (E) $(h^{-1})'(7) = \frac{1}{5}$

$h(3) = 5 \mid (h^{-1})(5) = 3$

$h'(3) = 7 \mid \boxed{(h^{-1})'(5) = \frac{1}{7}}$