79. If $p(x) = Ax^2 + Bx + C$, then

$$p'(x) = 2Ax + B = \frac{f(b) - f(a)}{b - a}$$
$$= \frac{(Ab^2 + Bb + C) - (Aa^2 + Ba + C)}{b - a}$$
$$= \frac{A(b^2 - a^2) + B(b - a)}{b - a}$$
$$= \frac{(b - a)[A(b + a) + B]}{b - a}$$
$$= A(b + a) + B.$$

So, $2Ax = A(b + a)$ and $x = (b + a)/2$ which is the midpoint of $[a, b]$.

80. (a) $f(x) = x^2, g(x) = -x^3 + x^2 + 3x + 2$

$f(-1) = g(-1) = 1, f(2) = g(2) = 4$

Let $h(x) = f(x) - g(x)$. Then,

$h(-1) = h(2) = 0$. So, by Rolle's Theorem these exists $c \in (-1, 2)$ such that

$h'(c) = f'(c) - g'(c) = 0.$

So, at $x = c$, the tangent line to f is parallel to the tangent line to g.

$h(x) = x^3 - 3x - 2, h'(x)$

$= 3x^2 - 3 = 0 \Rightarrow x = c = 1$

(b) Let $h(x) = f(x) - g(x)$. Then $h(a) = h(b) = 0$ by Rolle's Theorem, there exists c in (a, b) such that

$h'(c) = f'(c) - g'(c) = 0.$

So, at $x = c$, the tangent line to f is parallel to the tangent line to g.

81. Suppose $f(x)$ has two fixed points c_1 and c_2. Then, by the Mean Value Theorem, there exists c such that

$$f'(c) = \frac{f(c_2) - f(c_1)}{c_2 - c_1} = \frac{c_2 - c_1}{c_2 - c_1} = 1.$$

This contradicts the fact that $f'(x) < 1$ for all x.

82. $f(x) = \frac{1}{2}\cos x$ differentiable on $(-\infty, \infty)$.

$f'(x) = -\frac{1}{2}\sin x$

$-\frac{1}{2} \le f'(x) \le \frac{1}{2} \Rightarrow f'(x) < 1$ for all real numbers.

So, from Exercise 62, f has, at most, one fixed point. $(x \approx 0.4502)$

83. Let $f(x) = \cos x$. f is continuous and differentiable for all real numbers. By the Mean Value Theorem, for any interval $[a, b]$, there exists c in (a, b) such that

$$\frac{f(b) - f(a)}{b - a} = f'(c)$$
$$\frac{\cos b - \cos a}{b - a} = -\sin c$$
$$\cos b - \cos a = (-\sin c)(b - a)$$
$$|\cos b - \cos a| = |-\sin c||b - a|$$
$$|\cos b - \cos a| \le |b - a| \text{ since } |-\sin c| \le 1.$$

84. Let $f(x) = \sin x$. f is continuous and differentiable for all real numbers. By the Mean Value Theorem, for any interval $[a, b]$, there exists c in (a, b) such that

$$\frac{f(b) - f(a)}{b - a} = f'(c)$$
$$\sin(b) - \sin(a) = (b - a)\cos(c)$$
$$|\sin(b) - \sin(a)| = |b - a||\cos(c)|$$
$$|\sin a - \sin b| \le |a - b|.$$

85. Let $0 < a < b$. $f(x) = \sqrt{x}$ satisfies the hypotheses of the Mean Value Theorem on $[a, b]$. Hence, there exists c in (a, b) such that

$$f'(c) = \frac{1}{2\sqrt{c}} = \frac{f(b) - f(a)}{b - a} = \frac{\sqrt{b} - \sqrt{a}}{b - a}.$$

So, $\sqrt{b} - \sqrt{a} = (b - a)\frac{1}{2\sqrt{c}} < \frac{b - a}{2\sqrt{a}}$.

Section 3.3 Increasing and Decreasing Functions and the First Derivative Test

1. (a) Increasing: $(0, 6)$ and $(8, 9)$. Largest: $(0, 6)$

(b) Decreasing: $(6, 8)$ and $(9, 10)$. Largest: $(6, 8)$

2. (a) Increasing: $(4, 5), (6, 7)$. Largest: $(4, 5), (6, 7)$

(b) Decreasing: $(-3, 1), (1, 4), (5, 6)$. Largest: $(-3, 1)$

3. $f(x) = x^2 - 6x + 8$

From the graph, f is decreasing on $(-\infty, 3)$ and increasing on $(3, \infty)$.

Analytically, $f'(x) = 2x - 6$.

Critical number: $x = 3$

Test intervals:	$-\infty < x < 3$	$3 < x < \infty$
Sign of $f'(x)$:	$f' < 0$	$f' > 0$
Conclusion:	Decreasing	Increasing

4. $y = -(x + 1)^2$

From the graph, f is increasing on $(-\infty, -1)$ and decreasing on $(-1, \infty)$.

Analytically, $y' = -2(x + 1)$.

Critical number: $x = -1$

Test intervals:	$-\infty < x < -1$	$-1 < x < \infty$
Sign of y':	$y' > 0$	$y' < 0$
Conclusion:	Increasing	Decreasing

5. $y = \dfrac{x^3}{4} - 3x$

From the graph, y is increasing on $(-\infty, -2)$ and $(2, \infty)$, and decreasing on $(-2, 2)$.

Analytically, $y' = \dfrac{3x^2}{4} - 3 = \dfrac{3}{4}(x^2 - 4) = \dfrac{3}{4}(x - 2)(x + 2)$

Critical numbers: $x = \pm 2$

Test intervals:	$-\infty < x < -2$	$-2 < x < 2$	$2 < x < \infty$
Sign of y':	$y' > 0$	$y' < 0$	$y' > 0$
Conclusion:	Increasing	Decreasing	Increasing

6. $f(x) = x^4 - 2x^2$

From the graph, f is decreasing on $(-\infty, -1)$ and $(0, 1)$, and increasing on $(-1, 0)$ and $(1, \infty)$.

Analytically, $f'(x) = 4x^3 - 4x = 4x(x - 1)(x + 1)$.

Critical numbers: $x = 0, \pm 1$.

Test intervals:	$-\infty < x < -1$	$-1 < x < 0$	$0 < x < 1$	$1 < x < \infty$
Sign of f':	$f' < 0$	$f' > 0$	$f' < 0$	$f' > 0$
Conclusion:	Decreasing	Increasing	Decreasing	Increasing

7. $f(x) = \dfrac{1}{(x + 1)^2}$

From the graph, f is increasing on $(-\infty, -1)$ and decreasing on $(-1, \infty)$.

Analytically, $f'(x) = \dfrac{-2}{(x + 1)^3}$.

No critical numbers. Discontinuity: $x = -1$

Test intervals:	$-\infty < x < -1$	$-1 < x < \infty$
Sign of $f'(x)$:	$f' > 0$	$f' < 0$
Conclusion:	Increasing	Decreasing

8. $y = \dfrac{x^2}{2x - 1}$

From the graph, y is increasing on $(-\infty, 0)$ and $(1, \infty)$, and decreasing on $(0, 1/2)$ and $(1/2, 1)$.

Analytically, $y' = \dfrac{(2x - 1)2x - x^2(2)}{(2x - 1)^2} = \dfrac{2x^2 - 2x}{(2x - 1)^2} = \dfrac{2x(x - 1)}{(2x - 1)^2}$

Critical numbers: $x = 0, 1$

Discontinuity: $x = 1/2$

Test intervals:	$-\infty < x < 0$	$0 < x < 1/2$	$1/2 < x < 1$	$1 < x < \infty$
Sign of y':	$y' > 0$	$y' < 0$	$y' < 0$	$y' > 0$
Conclusion:	Increasing	Decreasing	Decreasing	Increasing

9. $g(x) = x^2 - 2x - 8$

$g'(x) = 2x - 2$

Critical number: $x = 1$

Test intervals:	$-\infty < x < 1$	$1 < x < \infty$
Sign of $g'(x)$:	$g' < 0$	$g' > 0$
Conclusion:	Decreasing	Increasing

Increasing on: $(1, \infty)$

Decreasing on: $(-\infty, 1)$

10. $h(x) = 12x - x^3$

$h'(x) = 12 - 3x^2 = 3(4 - x^2) = 3(2 - x)(2 + x)$

Critical numbers: $x = \pm 2$

Test intervals:	$-\infty < x < -2$	$-2 < x < 2$	$2 < x < \infty$
Sign of $h'(x)$:	$h' < 0$	$h' > 0$	$h' < 0$
Conclusion:	Decreasing	Increasing	Decreasing

Increasing on: $(-2, 2)$

Decreasing on: $(-\infty, -2), (2, \infty)$

11. $y = x\sqrt{16 - x^2}$ Domain: $[-4, 4]$

$y' = \dfrac{-2(x^2 - 8)}{\sqrt{16 - x^2}} = \dfrac{-2}{\sqrt{16 - x^2}}(x - 2\sqrt{2})(x + 2\sqrt{2})$

Critical numbers: $x = \pm 2\sqrt{2}$

Test intervals:	$-4 < x < -2\sqrt{2}$	$-2\sqrt{2} < x < 2\sqrt{2}$	$2\sqrt{2} < x < 4$
Sign of y':	$y' < 0$	$y' > 0$	$y' < 0$
Conclusion:	Decreasing	Increasing	Decreasing

Increasing on: $(-2\sqrt{2}, 2\sqrt{2})$

Decreasing on: $(-4, -2\sqrt{2}), (2\sqrt{2}, 4)$

12. $y = x + \dfrac{9}{x}$

$y' = \dfrac{1 - 9}{x^2} = \dfrac{x^2 - 9}{x^2} = \dfrac{(x - 3)(x + 3)}{x^2}$

Critical numbers: $x = \pm 3$

Discontinuity: $x = 0$

Test intervals:	$-\infty < x < -3$	$-3 < x < 0$	$0 < x < 3$	$3 < x < \infty$
Sign of y':	$y' > 0$	$y' < 0$	$y' < 0$	$y' > 0$
Conclusion:	Increasing	Decreasing	Decreasing	Increasing

Increasing on: $(-\infty, -3), (3, \infty)$

Decreasing on: $(-3, 0), (0, 3)$

13. $f(x) = \sin x - 1, \quad 0 < x < 2\pi$

$f'(x) = \cos x$

Critical numbers: $x = \dfrac{\pi}{2}, \dfrac{3\pi}{2}$

Test intervals:	$0 < x < \dfrac{\pi}{2}$	$\dfrac{\pi}{2} < x < \dfrac{3\pi}{2}$	$\dfrac{3\pi}{2} < x < 2\pi$
Sign of $f'(x)$:	$f' > 0$	$f' < 0$	$f' > 0$
Conclusion:	Increasing	Decreasing	Increasing

Increasing on: $\left(0, \dfrac{\pi}{2}\right), \left(\dfrac{3\pi}{2}, 2\pi\right)$

Decreasing on: $\left(\dfrac{\pi}{2}, \dfrac{3\pi}{2}\right)$

14. $h(x) = \cos\dfrac{x}{2}, \quad 0 < x < 2\pi$

$h'(x) = -\dfrac{1}{2}\sin\dfrac{x}{2}$

Critical numbers: none

Test interval:	$0 < x < 2\pi$
Sign of $h'(x)$:	$h' < 0$
Conclusion:	Decreasing

Decreasing on $0 < x < 2\pi$

15. $y = x - 2\cos x, \quad 0 < x < 2\pi$

$y' = 1 + 2\sin x$

$y' = 0\text{: } \sin x = -\dfrac{1}{2}$

Critical numbers: $x = \dfrac{7\pi}{6}, \dfrac{11\pi}{6}$

Test intervals:	$0 < x < \frac{7\pi}{6}$	$\frac{7\pi}{6} < x < \frac{11\pi}{6}$	$\frac{11\pi}{6} < x < 2\pi$
Sign of y':	$y' > 0$	$y' < 0$	$y' > 0$
Conclusion:	Increasing	Decreasing	Increasing

Increasing on: $\left(0, \dfrac{7\pi}{6}\right), \left(\dfrac{11\pi}{6}, 2\pi\right)$

Decreasing on: $\left(\dfrac{7\pi}{6}, \dfrac{11\pi}{6}\right)$

16. $f(x) = \sin^2 x + \sin x, \ 0 < x < 2\pi$

$f'(x) = 2\sin x\cos x + \cos x = \cos x(2\sin x + 1)$

$2\sin x + 1 = 0 \Rightarrow \sin x = -\dfrac{1}{2} \Rightarrow x = \dfrac{7\pi}{6}, \dfrac{11\pi}{6}$

$\cos x = 0 \Rightarrow x = \dfrac{\pi}{2}, \dfrac{3\pi}{2}$

Critical numbers: $\dfrac{\pi}{2}, \dfrac{7\pi}{6}, \dfrac{3\pi}{2}, \dfrac{11\pi}{6}$

Test intervals:	$0 < x < \frac{\pi}{2}$	$\frac{\pi}{2} < x < \frac{7\pi}{6}$	$\frac{7\pi}{6} < x < \frac{3\pi}{2}$	$\frac{3\pi}{2} < x < \frac{11\pi}{6}$	$\frac{11\pi}{6} < x < 2\pi$
Sign of $f'(x)$:	$f' > 0$	$f' < 0$	$f' > 0$	$f' < 0$	$f' > 0$
Conclusion:	Increasing	Decreasing	Increasing	Decreasing	Increasing

Increasing on: $\left(0, \dfrac{\pi}{2}\right), \left(\dfrac{7\pi}{6}, \dfrac{3\pi}{2}\right), \left(\dfrac{11\pi}{6}, 2\pi\right)$

Decreasing on: $\left(\dfrac{\pi}{2}, \dfrac{7\pi}{6}\right), \left(\dfrac{3\pi}{2}, \dfrac{11\pi}{6}\right)$

17. (a) $f(x) = x^2 - 4x$

$f'(x) = 2x - 4$

Critical number: $x = 2$

(b)

Test intervals:	$-\infty < x < 2$	$2 < x < \infty$
Sign of f':	$f' < 0$	$f' > 0$
Conclusion:	Decreasing	Increasing

Decreasing on: $(-\infty, 2)$

Increasing on: $(2, \infty)$

(c) Relative minimum: $(2, -4)$

18. (a) $f(x) = x^2 + 6x + 10$

$f'(x) = 2x + 6$

Critical number: $x = -3$

(b)

Test intervals:	$-\infty < x < -3$	$-3 < x < \infty$
Sign of f':	$f' < 0$	$f' > 0$
Conclusion:	Decreasing	Increasing

Decreasing on: $(-\infty, -3)$

Increasing on: $(-3, \infty)$

(c) Relative minimum: $(-3, 1)$

19. (a) $f(x) = -2x^2 + 4x + 3$

$f'(x) = -4x + 4 = 0$

Critical number: $x = 1$

(b)

Test intervals:	$-\infty < x < 1$	$1 < x < \infty$
Sign of $f'(x)$:	$f' > 0$	$f' < 0$
Conclusion:	Increasing	Decreasing

Increasing on: $(-\infty, 1)$

Decreasing on: $(1, \infty)$

(c) Relative maximum: $(1, 5)$

20. (a) $f(x) = -3x^2 - 4x - 2$

$f'(x) = -6x - 4 = 0$

Critical number: $x = -\frac{2}{3}$

(b)

Test intervals:	$-\infty < x < -\frac{2}{3}$	$-\frac{2}{3} < x < \infty$
Sign of $f'(x)$:	$f' > 0$	$f' < 0$
Conclusion:	Increasing	Decreasing

Increasing on: $\left(-\infty, -\frac{2}{3}\right)$

Decreasing on: $\left(-\frac{2}{3}, \infty\right)$

(c) Relative maximum: $\left(-\frac{2}{3}, -\frac{2}{3}\right)$

21. (a) $f(x) = 2x^3 + 3x^2 - 12x$

$f'(x) = 6x^2 + 6x - 12 = 6(x + 2)(x - 1) = 0$

Critical numbers: $x = -2, 1$

(b)

Test intervals:	$-\infty < x < -2$	$-2 < x < 1$	$1 < x < \infty$
Sign of $f'(x)$:	$f' > 0$	$f' < 0$	$f' > 0$
Conclusion:	Increasing	Decreasing	Increasing

Increasing on: $(-\infty, -2), (1, \infty)$

Decreasing on: $(-2, 1)$

(c) Relative maximum: $(-2, 20)$

Relative minimum: $(1, -7)$

22. (a) $f(x) = x^3 - 6x^2 + 15$

$f'(x) = 3x^2 - 12x = 3x(x - 4)$

Critical numbers: $x = 0, 4$

(b)

Test intervals:	$-\infty < x < 0$	$0 < x < 4$	$4 < x < \infty$
Sign of $f'(x)$:	$f' > 0$	$f' < 0$	$f' > 0$
Conclusion:	Increasing	Decreasing	Increasing

Increasing on: $(-\infty, 0), (4, \infty)$

Decreasing on: $(0, 4)$

(c) Relative maximum: $(0, 15)$

Relative minimum: $(4, -17)$

23. (a) $f(x) = (x - 1)^2(x + 3) = x^3 + x^2 - 5x + 3$

$f'(x) = 3x^2 + 2x - 5 = (x - 1)(3x + 5)$

Critical numbers: $x = 1, -\frac{5}{3}$

(b)

Test intervals:	$-\infty < x < -\frac{5}{3}$	$-5/3 < x < 1$	$1 < x < \infty$
Sign of f':	$f' > 0$	$f' < 0$	$f' > 0$
Conclusion:	Increasing	Decreasing	Increasing

Increasing on: $\left(-\infty, -\frac{5}{3}\right)$ and $(1, \infty)$

Decreasing on: $\left(-\frac{5}{3}, 1\right)$

(c) Relative maximum: $\left(-\frac{5}{3}, \frac{256}{27}\right)$

Relative minimum: $(1, 0)$

24. (a) $f(x) = (x+2)^2(x-1)$

$f'(x) = 3x(x+2)$

Critical numbers: $x = -2, 0$

(b)

Test intervals:	$-\infty < x < -2$	$-2 < x < 0$	$0 < x < \infty$
Sign of $f'(x)$:	$f' > 0$	$f' < 0$	$f' > 0$
Conclusion:	Increasing	Decreasing	Increasing

Increasing on: $(-\infty, -2), (0, \infty)$

Decreasing on: $(-2, 0)$

(c) Relative maximum: $(-2, 0)$

Relative minimum: $(0, -4)$

25. (a) $f(x) = \dfrac{x^5 - 5x}{5}$

$f'(x) = x^4 - 1$

Critical numbers: $x = -1, 1$

(b)

Test intervals:	$-\infty < x < -1$	$-1 < x < 1$	$1 < x < \infty$
Sign of $f'(x)$:	$f' > 0$	$f' < 0$	$f' > 0$
Conclusion:	Increasing	Decreasing	Increasing

Increasing on: $(-\infty, -1), (1, \infty)$

Decreasing on: $(-1, 1)$

(c) Relative maximum: $\left(-1, \dfrac{4}{5}\right)$

Relative minimum: $\left(1, -\dfrac{4}{5}\right)$

26. (a) $f(x) = x^4 - 32x + 4$

$f'(x) = 4x^3 - 32 = 4(x^3 - 8)$

Critical number: $x = 2$

(b)

Test intervals:	$-\infty < x < 2$	$2 < x < \infty$
Sign of $f'(x)$:	$f' < 0$	$f' > 0$
Conclusion:	Decreasing	Increasing

Increasing on: $(2, \infty)$

Decreasing on: $(-\infty, 2)$

(c) Relative minimum: $(2, -44)$

27. (a) $f(x) = x^{1/3} + 1$

$f'(x) = \dfrac{1}{3}x^{-2/3} = \dfrac{1}{3x^{2/3}}$

Critical number: $x = 0$

(b)

Test intervals:	$-\infty < x < 0$	$0 < x < \infty$
Sign of $f'(x)$:	$f' > 0$	$f' > 0$
Conclusion:	Increasing	Increasing

Increasing on: $(-\infty, \infty)$

(c) No relative extrema

28. (a) $f(x) = x^{2/3} - 4$

$f'(x) = \frac{2}{3}x^{-1/3} = \frac{2}{3x^{1/3}}$

Critical number: $x = 0$

(b)

Test intervals:	$-\infty < x < 0$	$0 < x < \infty$
Sign of $f'(x)$:	$f' < 0$	$f' > 0$
Conclusion:	Decreasing	Increasing

Increasing on: $(0, \infty)$

Decreasing on: $(-\infty, 0)$

(c) Relative minimum: $(0, -4)$

29. (a) $f(x) = (x + 2)^{2/3}$

$f'(x) = \frac{2}{3}(x + 2)^{-1/3} = \frac{2}{3(x + 2)^{1/3}}$

Critical number: $x = -2$

(b)

Test intervals:	$-\infty < x < -2$	$-2 < x < \infty$
Sign of f':	$f' < 0$	$f' > 0$
Conclusion:	Decreasing	Increasing

Decreasing on: $(-\infty, -2)$

Increasing on: $(-2, \infty)$

(c) Relative minimum: $(-2, 0)$

30. (a) $f(x) = (x - 3)^{1/3}$

$f'(x) = \frac{1}{3}(x - 3)^{-2/3} = \frac{1}{3(x - 3)^{2/3}}$

Critical number: $x = 3$

(b)

Test intervals:	$-\infty < x < 3$	$3 < x < \infty$
Sign of f':	$f' > 0$	$f' > 0$
Conclusion:	Increasing	Increasing

Increasing on: $(-\infty, \infty)$

(c) No relative extrema

31. (a) $f(x) = 5 - |x - 5|$

$f'(x) = -\frac{x - 5}{|x - 5|} = \begin{cases} 1, & x < 5 \\ -1, & x > 5 \end{cases}$

Critical number: $x = 5$

(b)

Test intervals:	$-\infty < x < 5$	$5 < x < \infty$
Sign of $f'(x)$:	$f' > 0$	$f' < 0$
Conclusion:	Increasing	Decreasing

Increasing on: $(-\infty, 5)$

Decreasing on: $(5, \infty)$

(c) Relative maximum: $(5, 5)$

32. (a) $f(x) = |x + 3| - 1$

$f'(x) = \frac{x + 3}{|x + 3|} = \begin{cases} 1, & x > -3 \\ -1, & x < -3 \end{cases}$

Critical number: $x = -3$

(b)

Test intervals:	$-\infty < x < -3$	$-3 < x < \infty$
Sign of $f'(x)$:	$f' < 0$	$f' > 0$
Conclusion:	Decreasing	Increasing

Increasing on: $(-3, \infty)$

Decreasing on: $(-\infty, -3)$

(c) Relative minimum: $(-3, -1)$

33. (a) $f(x) = 2x + \frac{1}{x}$

$f'(x) = 2 - \frac{1}{x^2} = \frac{2x^2 - 1}{x^2}$

Critical numbers: $x = \pm\frac{\sqrt{2}}{2}$

Discontinuity: $x = 0$

(b)

Test intervals:	$-\infty < x < -\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2} < x < 0$	$0 < x < \frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2} < x < \infty$
Sign of f':	$f' > 0$	$f' < 0$	$f' < 0$	$f' > 0$
Conclusion:	Increasing	Decreasing	Decreasing	Increasing

Increasing on: $\left(-\infty, -\frac{\sqrt{2}}{2}\right)$ and $\left(\frac{\sqrt{2}}{2}, \infty\right)$

Decreasing on: $\left(-\frac{\sqrt{2}}{2}, 0\right)$ and $\left(0, \frac{\sqrt{2}}{2}\right)$

(c) Relative maximum: $\left(-\frac{\sqrt{2}}{2}, -2\sqrt{2}\right)$

Relative minimum: $\left(\frac{\sqrt{2}}{2}, 2\sqrt{2}\right)$

34. (a) $f(x) = \frac{x}{x - 5}$

$f'(x) = \frac{(x - 5) - x}{(x - 5)^2} = \frac{-5}{(x - 5)^2}$

No critical numbers

Discontinuity: $x = 5$

(b)

Test intervals:	$-\infty < x < 5$	$5 < x < \infty$
Sign of $f'(x)$:	$f' < 0$	$f' < 0$
Conclusion:	Decreasing	Decreasing

Decreasing on: $(-\infty, 5)$, $(5, \infty)$

(c) No relative extrema

35. (a) $f(x) = \frac{x^2}{x^2 - 9}$

$f'(x) = \frac{(x^2 - 9)(2x) - (x^2)(2x)}{(x^2 - 9)^2} = \frac{-18x}{(x^2 - 9)^2}$

Critical number: $x = 0$

Discontinuities: $x = -3, 3$

(b)

Test intervals:	$-\infty < x < -3$	$-3 < x < 0$	$0 < x < 3$	$3 < x < \infty$
Sign of $f'(x)$:	$f' > 0$	$f' > 0$	$f' < 0$	$f' < 0$
Conclusion:	Increasing	Increasing	Decreasing	Decreasing

Increasing on: $(-\infty, -3), (-3, 0)$

Decreasing on: $(0, 3), (3, \infty)$

(c) Relative maximum: $(0, 0)$

36. (a) $f(x) = \dfrac{x^2 - 2x + 1}{x + 1}$

$$f'(x) = \frac{(x+1)(2x-2) - (x^2 - 2x + 1)(1)}{(x+1)^2} = \frac{x^2 + 2x - 3}{(x+1)^2} = \frac{(x+3)(x-1)}{(x+1)^2}$$

Critical numbers: $x = -3, 1$

Discontinuity: $x = -1$

(b)

Test intervals:	$-\infty < x < -3$	$-3 < x < -1$	$-1 < x < 1$	$1 < x < \infty$
Sign of $f'(x)$:	$f' > 0$	$f' < 0$	$f' < 0$	$f' > 0$
Conclusion:	Increasing	Decreasing	Decreasing	Increasing

Increasing on: $(-\infty, -3), (1, \infty)$

Decreasing on: $(-3, -1), (-1, 1)$

(c) Relative maximum: $(-3, -8)$

Relative minimum: $(1, 0)$

37. (a) $f(x) = \begin{cases} 4 - x^2, & x \le 0 \\ -2x, & x > 0 \end{cases}$

$f'(x) = \begin{cases} -2x, & x < 0 \\ -2, & x > 0 \end{cases}$

Critical number: $x = 0$

(b)

Test intervals:	$-\infty < x < 0$	$0 < x < \infty$
Sign of f':	$f' > 0$	$f' < 0$
Conclusion:	Increasing	Decreasing

Increasing on: $(-\infty, 0)$

Decreasing on: $(0, \infty)$

(c) Relative maximum: $(0, 4)$

38. (a) $f(x) = \begin{cases} 2x + 1, & x \le -1 \\ x^2 - 2, & x > -1 \end{cases}$

$f'(x) = \begin{cases} 2, & x < -1 \\ 2x, & x > -1 \end{cases}$

Critical numbers: $x = -1, 0$

(b)

Test intervals:	$-\infty < x < -1$	$-1 < x < 0$	$0 < x < \infty$
Sign of f':	$f' > 0$	$f' < 0$	$f' > 0$
Conclusion:	Increasing	Decreasing	Increasing

Increasing on: $(-\infty, -1)$ and $(0, \infty)$

Decreasing on: $(-1, 0)$

(c) Relative maximum: $(-1, -1)$

Relative minimum: $(0, -2)$

39. (a) $f(x) = \begin{cases} 3x + 1, & x \le 1 \\ 5 - x^2, & x > 1 \end{cases}$

$f'(x) = \begin{cases} 3, & x < 1 \\ -2x, & x > 1 \end{cases}$

Critical number: $x = 1$

(b)

Test intervals:	$-\infty < x < 1$	$1 < x < \infty$
Sign of f':	$f' > 0$	$f' < 0$
Conclusion:	Increasing	Decreasing

Increasing on: $(-\infty, 1)$

Decreasing on: $(1, \infty)$

(c) Relative maximum: $(1, 4)$

40. (a) $f(x) = \begin{cases} -x^3 + 1, & x \le 0 \\ -x^2 + 2x, & x > 0 \end{cases}$

$f'(x) = \begin{cases} -3x^2, & x < 0 \\ -2x + 2, & x > 0 \end{cases}$

Critical numbers: $x = 0, 1$

(b)

Test intervals:	$-\infty < x < 0$	$0 < x < 1$	$1 < x < \infty$
Sign of f':	$f' < 0$	$f' > 0$	$f' < 0$
Conclusion:	Decreasing	Increasing	Decreasing

Increasing on: $(0, 1)$

Decreasing on: $(-\infty, 0)$ and $(1, \infty)$

(c) Relative maximum: $(1, 1)$

Note: $(0, 1)$ is not a relative minimum