3.3b Exercise Problems Derivative of Inverse at a Point
Derivative of Inverse Trig Functions

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#23-47 odd, 48

Derivative Rules:

1) $\frac{d}{dx}\sin^{-1}(u) = \frac{u'}{\sqrt{1-u^2}}$ 4) $\frac{d}{dx}\cos^{-1}(u) = \frac{-u'}{\sqrt{1-u^2}}$

2) $\frac{d}{dx}\tan^{-1}(u) = \frac{u'}{1+u^2}$ 5) $\frac{d}{dx}\cot^{-1}(u) = \frac{-u'}{1+u^2}$

3) $\frac{d}{dx}\sec^{-1}(u) = \frac{u'}{|u|\sqrt{u^2-1}}$ 6) $\frac{d}{dx}\csc^{-1}(u) = \frac{-u'}{|u|\sqrt{u^2-1}}$

23) $f(x) = \tan^{-1}(1-2x^2)$

$$\boxed{f'(x) = \frac{-4x}{1+(1-2x^2)^2} \quad \text{or} \quad f'(x) = \frac{-2x}{2x^4-2x^2+1}}$$

25) $f(x) = \sec^{-1}(x^2+2)$

$$\boxed{f'(x) = \frac{2x}{|x^2+2|\sqrt{(x^2+2)^2-1}}}$$

27) $f(x) = \sin^{-1}(e^x)$

$$f'(x) = \frac{e^x}{\sqrt{1-(e^x)^2}}$$

$$\boxed{f'(x) = \frac{e^x}{\sqrt{1-e^{2x}}}}$$

29) $g(x) = \tan^{-1}\left(\frac{1}{x}\right)$

$g(x) = \tan^{-1}(x^{-1})$

$$g'(x) = \frac{-1x^{-2}}{1+\left(\frac{1}{x}\right)^2}$$

$$g'(x) = \frac{-\frac{1}{x^2}}{1+\frac{1}{x^2}} \rightarrow \frac{-\frac{1}{x^2}}{\frac{x^2}{x^2}+\frac{1}{x^2}} \rightarrow \frac{-\frac{1}{x^2}}{\frac{x^2+1}{x^2}}$$

$$g'(x) = \frac{-1}{x^2}\cdot\frac{x^2}{x^2+1} \rightarrow \boxed{\frac{-1}{x^2+1}}$$

31) $g(x) = \underbrace{x}_{f}\ \underbrace{\sin^{-1}(x)}_{g}$ *product Rule

$$g'(x) = \underbrace{1}_{f'}\cdot\underbrace{(\sin^{-1}(x))}_{g} + \underbrace{x}_{f}\cdot\underbrace{\left(\frac{1}{\sqrt{1-x^2}}\right)}_{g'}$$

$$\boxed{g'(x) = \sin^{-1}(x) + \frac{x}{\sqrt{1-x^2}}}$$

3.3b

33) $s(t) = \overbrace{t^2}^{f}\,\overbrace{\sec^{-1}(t^3)}^{g}$

$$s'(t) = \overbrace{2t}^{f'} \cdot \overbrace{\sec^{-1}(t^3)}^{g} + \overbrace{t^2}^{f} \cdot \overbrace{\frac{3t^2}{|t^3|\sqrt{(t^3)^2-1}}}^{g'}$$

$$s'(t) = 2t\sec^{-1}(t^3) + \frac{3t^4}{|t^3|\sqrt{t^6-1}} \rightarrow \boxed{2t\sec^{-1}(t^3) + \frac{3t}{\sqrt{t^6-1}}}$$

35) $f(x) = \tan^{-1}(\sin x)$ $\leftarrow$ $\frac{d}{dx}\tan^{-1}(u) = \frac{u'}{1+u^2}$

$$f'(x) = \frac{\cos x}{1+(\sin x)^2} \rightarrow \boxed{\frac{\cos x}{1+\sin^2 x}}$$

37) $G(x) = \sin(\tan^{-1}(x))$ *Chain Rule out: $\sin(\ )$ in: $\tan^{-1}(x)$

$$G'(x) = \cos(\tan^{-1}(x)) \cdot \frac{1}{1+x^2} \rightarrow \boxed{\frac{\cos(\tan^{-1}(x))}{1+x^2}}$$

39) $f(x) = e^{\tan^{-1}(3x)}$

$$f'(x) = e^{\tan^{-1}(3x)} \cdot \frac{3}{1+(3x)^2} \rightarrow \boxed{\frac{3e^{\tan^{-1}(3x)}}{1+9x^2}}$$

41) $g(x) = \frac{\sin^{-1}(x)}{\sqrt{1-x^2}} \rightarrow \frac{\sin^{-1}(x)}{(1-x^2)^{1/2}}$

$$g'(x) = \frac{\overbrace{\frac{1}{\sqrt{1-x^2}}}^{f'} \cdot \overbrace{\sqrt{1-x^2}}^{g} - \overbrace{\sin^{-1}(x)}^{f} \cdot \overbrace{\frac{1}{2}(1-x^2)^{-1/2}(-2x)}^{g'}}{\underbrace{(\sqrt{1-x^2})^2}_{g^2}}$$

$$g'(x) = \frac{1 + \frac{x\sin^{-1}(x)}{\sqrt{1-x^2}}}{1-x^2}$$

$$g'(x) = \frac{1}{1-x^2} + \frac{\frac{x\sin^{-1}(x)}{\sqrt{1-x^2}}}{1-x^2}$$

$$\boxed{g'(x) = \frac{1}{1-x^2} + \frac{x\sin^{-1}(x)}{(1-x^2)^{3/2}}}$$

$\boxed{3.3b}$ Find dy/dx

43) $y^2 + \sin^{-1}(y) = 2x$ *implicit differentiation

$$2y\left(\frac{dy}{dx}\right) + \frac{1}{\sqrt{1-y^2}}\left(\frac{dy}{dx}\right) = 2$$

$$\frac{dy}{dx}\left(2y + \frac{1}{\sqrt{1-y^2}}\right) = 2$$

$$\frac{dy}{dx} = \frac{2}{2y + \frac{1}{\sqrt{1-y^2}}} \rightarrow \frac{2}{\frac{2y\sqrt{1-y^2}}{\sqrt{1-y^2}} + \frac{1}{\sqrt{1-y^2}}}$$

$$\frac{dy}{dx} = \frac{2}{\frac{2y\sqrt{1-y^2}+1}{\sqrt{1-y^2}}} \rightarrow \boxed{\frac{2\sqrt{1-y^2}}{2y\sqrt{1-y^2}+1}}$$

45) $40\tan^{-1}(y^2) - \overbrace{\pi x^3}^{f}\overbrace{y}^{g} = 2\pi$

$$40\cdot\frac{2y}{1+(y^2)^2}\left(\frac{dy}{dx}\right) - \left(\overbrace{3\pi x^2}^{f'}\cdot\overbrace{y}^{g} + \overbrace{\pi x^3}^{f}\cdot\overbrace{\left(dy/dx\right)}^{g'}\right) = 0$$

$$\frac{80y}{1+y^4}\left(\frac{dy}{dx}\right) - \pi x^3\left(\frac{dy}{dx}\right) = 3\pi x^2 y$$

$$\frac{dy}{dx}\left(\frac{80y}{1+y^4} - \pi x^3\right) = 3\pi x^2 y$$

$$\frac{dy}{dx} = \frac{3\pi x^2 y}{\frac{80y}{1+y^4} - \pi x^3} = \frac{3\pi x^2 y}{\frac{80y}{1+y^4} - \frac{\pi x^3(1+y^4)}{1+y^4}}$$

$$\frac{dy}{dx} = \frac{3\pi x^2 y}{\frac{80y - \pi x^3(1+y^4)}{1+y^4}}$$

$$\boxed{\frac{dy}{dx} = \frac{3\pi x^2 y(1+y^4)}{80y - \pi x^3(1+y^4)}}$$

3.3b   Find derivative of the inverse at given point(s)

47) $f(x) = x^3 + 2x$ and inverse function is $g(x)$

Find $g'(0)$ and $g'(3)$

$f(a)=b$	$g(b)=a$
$f'(a)=n$	$g'(b)=\frac{1}{n}$

$f(\ )=0$	$g(0)=$
	$g'(0)=$ __

$0 = x^3 + 2x$

$\rightarrow 0 = x(x^2+2)$

$x = 0$

$\downarrow$

$f(0) = 0$

$f(0)=0$	$g(0)=0$
$f'(0)=2$	$g'(0)=\frac{1}{2}$

$f'(x) = 3x^2 + 2$

$f'(0) = 3(0)^2 + 2$

$\boxed{g'(0) = 1/2}$

47b) Find $g'(3)$

$3 = x^3 + 2x$

$0 = x^3 + 2x - 3$

guess and check

$x = 1$

$f(\ )=3$	$g(3)=$ __
$f'(\ )=$ __	$g'(3)=$ __

$f(1)=3$	$g(3)=1$
$f'(1)=5$	$g'(3)=1/5$

$f'(x) = 3x^2 + 2$

$f'(1) = 3(1)^2 + 2$

$f'(1) = 5$

$\boxed{g'(3) = \frac{1}{5}}$

3.3b Find derivative of inverse at given point

48a) $f(x) = 2x^3 + x - 3$ Find $g'(-3)$

$f(\ ) = -3$	$g(-3) = \_$
	$g'(-3) = \_$

$-3 = 2x^3 + x - 3$

$0 = 2x^3 + x + 0$

$0 = x(2x^2 + 1)$

$x = 0$

$f(0) = -3$

$f(0) = -3$	$g(-3) = 0$
$f'(0) = 1$	$g'(-3) = \frac{1}{1} = 1$

$f'(x) = 6x^2 + 1$

$f'(0) = 6(0)^2 + 1$

$f'(0) = 1$

$\boxed{g'(-3) = 1}$

b) Find $g'(0)$

$0 = 2x^3 + x - 3$

guess and check to solve for x

$x = 1$

$f(\ ) = 0$	$g(0) = \_$
	$g'(0) = \_$

$f(1) = 0$	$g(0) = 1$
$f'(1) = 7$	$g'(0) = 1/7$

$f'(x) = 6x^2 + 1$

$f'(1) = 6(1)^2 + 1$

$f'(1) = 7$

$\boxed{g'(0) = \frac{1}{7}}$