Key

AP Calculus – 3.3b Notes – Derivatives of Inverse Trig Functions

Quote from the AP Exam:
"Notation: The inverse of a trigonometric function x may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$)."

Inverse Trig Derivative Rules:

1) $\frac{d}{dx}\arcsin u = \frac{u'}{\sqrt{1-u^2}}$	2) $\frac{d}{dx}\arccos u = \frac{-u'}{\sqrt{1-u^2}}$
3) $\frac{d}{dx}\arctan u = \frac{u'}{1+u^2}$	4) $\frac{d}{dx}\text{arccot}\, u = \frac{-u'}{1+u^2}$
5) $\frac{d}{dx}\text{arcsec}\, u = \frac{u'}{\lvert u\rvert\sqrt{u^2-1}}$	6) $\frac{d}{dx}\text{arccsc}\, u = \frac{-u'}{\lvert u\rvert\sqrt{u^2-1}}$

Find the derivative.

1. $\frac{d}{dx}\sin^{-1}(3x)$ $\boxed{\frac{3}{\sqrt{1-9x^2}}}$

2. $\frac{d}{dx}\tan^{-1}(2x^2)$ $\boxed{\frac{4x}{1+4x^4}} \leftarrow \frac{4x}{1+(2x^2)^2}$

3. $\frac{d}{dx}\text{arcsec}(5x)$ $\boxed{\frac{5}{\lvert 5x\rvert\sqrt{25x^2-1}}}$

Simplifying $\sec^{-1} x$ Derivatives.

Simplify the following expressions.

4. $\frac{9x^2}{\lvert 3x^3\rvert\sqrt{9x^6-1}}$ $\boxed{\frac{3}{\lvert x\rvert\sqrt{9x^6-1}}}$

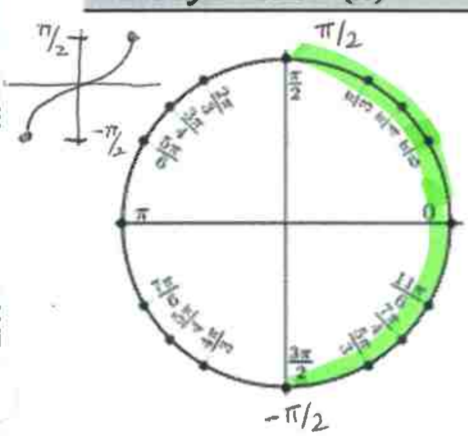
test
$x=-1$
$\frac{+}{+}=+$

5. $\frac{4x}{\lvert 2x^2\rvert\sqrt{4x^2-1}}$ $\boxed{\frac{2}{x\sqrt{4x^2-1}}}$

$\frac{-}{+}=-$

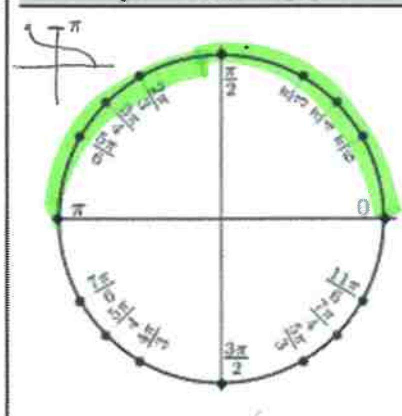
Domain of an inverse trig function.

$y = \sin^{-1}(x)$



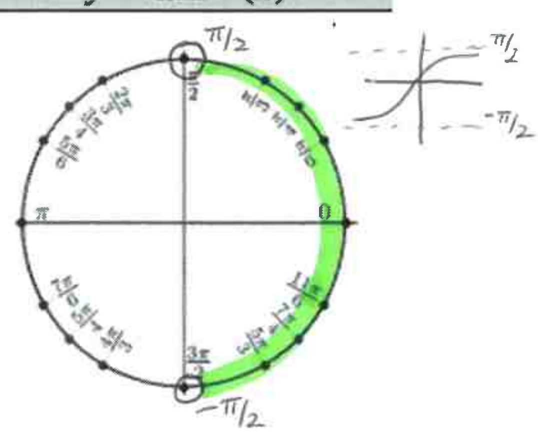
Domain: $-1 \le x \le 1$
Range: $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

$y = \cos^{-1}(x)$



Domain: $-1 \le x \le 1$
Range: $0 \le y \le \pi$

$y = \tan^{-1}(x)$



Domain: $-\infty \le x \le \infty$
Range: $-\frac{\pi}{2} < y < \frac{\pi}{2}$

Evaluate each function at the given x-value.

6. $f(x) = \arcsin(x)$ at $x = \frac{\sqrt{3}}{2}$

$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) \to \sin\theta = \frac{\sqrt{3}}{2}$

Q1 $\boxed{\theta = \frac{\pi}{3}}$

7. $f(x) = \cos^{-1}\left(\frac{x}{4}\right)$ at $x = -2$

$\cos^{-1}\left(\frac{-2}{4}\right) \to \cos\theta = -\frac{1}{2}$

$\boxed{\theta = \frac{2\pi}{3}}$

8. $f(x) = \arctan(x)$ at $x = \frac{1}{\sqrt{3}}$

$\arctan\left(\frac{1}{\sqrt{3}}\right) \to \tan\theta = \left(\frac{1}{\sqrt{3}}\right)$

$\boxed{\theta = \frac{\pi}{6}}$

Practice Problems:

Find the derivative of each expression.

1. $\frac{d}{dx}\sin^{-1}(5x)$

$\boxed{\frac{5}{\sqrt{1-25x^2}}}$

2. $\frac{d}{dx}\csc^{-1}(4x^5)$

$-\frac{20x^4}{|4x^5|\sqrt{16x^{10}-1}}$

$\boxed{\frac{-5}{|x|\sqrt{16x^{10}-1}}}$

3. $\frac{d}{dx}\arctan(2x)$

$\boxed{\frac{2}{1+4x^2}}$

4. $\frac{d}{dx}\sec^{-1}(x^3)$

$\frac{3x^2}{|x^3|\sqrt{x^6-1}}$

$\boxed{\frac{3}{|x|\sqrt{x^6-1}}}$

5. $\frac{d}{dx}\csc 6x$

$\frac{d}{dx}\csc u = -\csc u \cot u \cdot u'$

$\to -\csc(6x)\cot(6x)\cdot 6$

$\to \boxed{-6\csc(6x)\cot(6x)}$

6. $\frac{d}{dx}\arccos(3x^2)$

$\boxed{\frac{-6x}{\sqrt{1-9x^4}}}$

7. $\frac{d}{dx}\cot^{-1}(-x)$

$-\frac{(-1)}{1+x^2} \to \boxed{\frac{1}{1+x^2}}$

Find the tangent line equation of the curve at the given point.

11. $y = \arcsin(x)$ at the point where $x = \frac{\sqrt{2}}{2}$

$y' = \frac{1}{\sqrt{1-x^2}}$ $\quad y'\left(\frac{\sqrt{2}}{2}\right) = \frac{1}{\sqrt{1-\left(\frac{\sqrt{2}}{2}\right)^2}} \to$

$y'\left(\frac{\sqrt{2}}{2}\right) = \frac{1}{\sqrt{1-\frac{1}{2}}} \to \frac{1}{\sqrt{\frac{1}{2}}} = \sqrt{2}$

point: $\left(\frac{\sqrt{2}}{2}, \frac{\pi}{4}\right)$

slope: $m = \sqrt{2}$

$y\left(\frac{\sqrt{2}}{2}\right) = \arcsin\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$

$\boxed{y - \frac{\pi}{4} = \sqrt{2}\left(x - \frac{\sqrt{2}}{2}\right)}$

12. $y = \cos^{-1}(4x)$ at the point where $x = \frac{\sqrt{3}}{8}$

$y\left(\frac{\sqrt{3}}{8}\right) = \cos^{-1}\left(4\cdot\frac{\sqrt{3}}{8}\right) \to \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$

point: $\left(\frac{\sqrt{3}}{8}, \frac{\pi}{6}\right)$

$y' = \frac{-4}{\sqrt{1-16x^2}}$

slope: $m = -8$

$y'\left(\frac{\sqrt{3}}{8}\right) = \frac{-4}{\sqrt{1-16\left(\frac{\sqrt{3}}{8}\right)^2}} \to \frac{-4}{\sqrt{\frac{1}{4}}} = -8$

$\boxed{y - \frac{\pi}{6} = -8\left(x - \frac{\sqrt{3}}{8}\right)}$

19. If $\arctan y = \ln x$, then $\frac{dy}{dx} =$

(A) $\tan\left(\frac{1}{x}\right)$ (B) $\tan(\ln x)$ (C) $\frac{1+y^2}{xy}$ (D) $\frac{x}{1+y^2}$ $\boxed{\text{(E) } \frac{1+y^2}{x}}$

$\frac{1}{1+y^2}\left(\frac{dy}{dx}\right) = \frac{1}{x} \to \frac{dy}{dx} = \frac{1+y^2}{1}\cdot\frac{1}{x} \to \boxed{\frac{dy}{dx} = \frac{1+y^2}{x}}$