41. (a) $f(x) = \frac{x}{2} + \cos x, 0 < x < 2\pi$

$f'(x) = \frac{1}{2} - \sin x = 0$

Critical numbers: $x = \frac{\pi}{6}, \frac{5\pi}{6}$

Test intervals:	$0 < x < \frac{\pi}{6}$	$\frac{\pi}{6} < x < \frac{5\pi}{6}$	$\frac{5\pi}{6} < x < 2\pi$
Sign of $f'(x)$:	$f' > 0$	$f' < 0$	$f' > 0$
Conclusion:	Increasing	Decreasing	Increasing

Increasing on: $\left(0, \frac{\pi}{6}\right), \left(\frac{5\pi}{6}, 2\pi\right)$

Decreasing on: $\left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$

(b) Relative maximum: $\left(\frac{\pi}{6}, \frac{\pi + 6\sqrt{3}}{12}\right)$

Relative minimum: $\left(\frac{5\pi}{6}, \frac{5\pi - 6\sqrt{3}}{12}\right)$

(c) 

42. (a) $f(x) = \sin x \cos x + 5 = \frac{1}{2}\sin 2x + 5, 0 < x < 2\pi$

$f'(x) = \cos 2x$

Critical numbers: $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

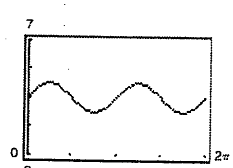
Test intervals:	$0 < x < \frac{\pi}{4}$	$\frac{\pi}{4} < x < \frac{3\pi}{4}$	$\frac{3\pi}{4} < x < \frac{5\pi}{4}$	$\frac{5\pi}{4} < x < \frac{7\pi}{4}$	$\frac{7\pi}{4} < x < 2\pi$
Sign of f':	$f' > 0$	$f' < 0$	$f' > 0$	$f' < 0$	$f' > 0$
Conclusion:	Increasing	Decreasing	Increasing	Decreasing	Increasing

Increasing on: $\left(0, \frac{\pi}{4}\right), \left(\frac{3\pi}{4}, \frac{5\pi}{4}\right), \left(\frac{7\pi}{4}, 2\pi\right)$

Decreasing on: $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right), \left(\frac{5\pi}{4}, \frac{7\pi}{4}\right)$

(b) Relative maxima: $\left(\frac{\pi}{4}, \frac{11}{2}\right), \left(\frac{5\pi}{4}, \frac{11}{2}\right)$

Relative minima: $\left(\frac{3\pi}{4}, \frac{9}{2}\right), \left(\frac{7\pi}{4}, \frac{9}{2}\right)$

(c) 

43. (a) $f(x) = \sin x + \cos x, \quad 0 < x < 2\pi$

$f'(x) = \cos x - \sin x = 0 \Rightarrow \sin x = \cos x$

Critical numbers: $x = \frac{\pi}{4}, \frac{5\pi}{4}$

Test intervals:	$0 < x < \frac{\pi}{4}$	$\frac{\pi}{4} < x < \frac{5\pi}{4}$	$\frac{5\pi}{4} < x < 2\pi$
Sign of $f'(x)$:	$f' > 0$	$f' < 0$	$f' > 0$
Conclusion:	Increasing	Decreasing	Increasing

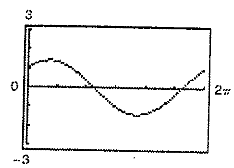
Increasing on: $\left(0, \frac{\pi}{4}\right), \left(\frac{5\pi}{4}, 2\pi\right)$

Decreasing on: $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$

(b) Relative maximum: $\left(\frac{\pi}{4}, \sqrt{2}\right)$

Relative minimum: $\left(\frac{5\pi}{4}, -\sqrt{2}\right)$

(c)



44. (a) $f(x) = x + 2\sin x, \quad 0 < x < 2\pi$

$f'(x) = 1 + 2\cos x = 0 \Rightarrow \cos x = -\frac{1}{2}$

Critical numbers: $\frac{2\pi}{3}, \frac{4\pi}{3}$

Test intervals:	$0 < x < \frac{2\pi}{3}$	$\frac{2\pi}{3} < x < \frac{4\pi}{3}$	$\frac{4\pi}{3} < x < 2\pi$
Sign of $f'(x)$:	$f' > 0$	$f' < 0$	$f' > 0$
Conclusion:	Increasing	Decreasing	Increasing

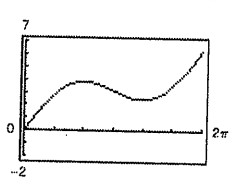
Increasing on: $\left(0, \frac{2\pi}{3}\right), \left(\frac{4\pi}{3}, 2\pi\right)$

Decreasing on: $\left(\frac{2\pi}{3}, \frac{4\pi}{3}\right)$

(b) Relative maximum: $\left(\frac{2\pi}{3}, \frac{2\pi}{3} + \sqrt{3}\right) \approx \left(\frac{2\pi}{3}, 3.826\right)$

Relative minimum: $\left(\frac{4\pi}{3}, \frac{4\pi}{3} - \sqrt{3}\right) \approx \left(\frac{4\pi}{3}, 2.457\right)$

(c)



45. (a) $f(x) = \cos^2(2x), \qquad 0 < x < 2\pi$

$f'(x) = -4\cos 2x \sin 2x = 0 \Rightarrow \cos 2x = 0 \text{ or } \sin 2x = 0$

Critical numbers: $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

Test intervals:	$0 < x < \frac{\pi}{4}$	$\frac{\pi}{4} < x < \frac{\pi}{2}$	$\frac{\pi}{2} < x < \frac{3\pi}{4}$	$\frac{3\pi}{4} < x < \pi$
Sign of $f'(x)$:	$f' < 0$	$f' > 0$	$f' < 0$	$f' > 0$
Conclusion:	Decreasing	Increasing	Decreasing	Increasing

Test intervals:	$\pi < x < \frac{5\pi}{4}$	$\frac{5\pi}{4} < x < \frac{3\pi}{2}$	$\frac{3\pi}{2} < x < \frac{7\pi}{4}$	$\frac{7\pi}{4} < x < 2\pi$
Sign of $f'(x)$:	$f' < 0$	$f' > 0$	$f' < 0$	$f' > 0$
Conclusion:	Decreasing	Increasing	Decreasing	Increasing

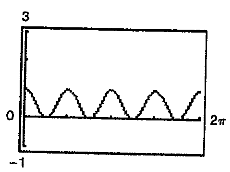
Increasing on: $\left(\frac{\pi}{4}, \frac{\pi}{2}\right), \left(\frac{3\pi}{4}, \pi\right), \left(\frac{5\pi}{4}, \frac{3\pi}{2}\right), \left(\frac{7\pi}{4}, 2\pi\right)$

Decreasing on: $\left(0, \frac{\pi}{4}\right), \left(\frac{\pi}{2}, \frac{3\pi}{4}\right), \left(\pi, \frac{5\pi}{4}\right), \left(\frac{3\pi}{2}, \frac{7\pi}{4}\right)$

(b) Relative maxima: $\left(\frac{\pi}{2}, 1\right), (\pi, 1), \left(\frac{3\pi}{2}, 1\right)$

Relative minima: $\left(\frac{\pi}{4}, 0\right), \left(\frac{3\pi}{4}, 0\right), \left(\frac{5\pi}{4}, 0\right), \left(\frac{7\pi}{4}, 0\right)$

(c)



46. (a) $f(x) = \sin x - \sqrt{3}\cos x, \; 0 < x < 2\pi$

$f'(x) = \cos x + \sqrt{3}\sin x = 0 \Rightarrow \sqrt{3}\sin x = -\cos x$

$\tan x = \frac{-1}{\sqrt{3}} = \frac{-\sqrt{3}}{3}$

Critical numbers: $x = \frac{5\pi}{6}, \frac{11\pi}{6}$

Test intervals:	$0 < x < \frac{5\pi}{6}$	$\frac{5\pi}{6} < x < \frac{11\pi}{6}$	$\frac{11\pi}{6} < x < 2\pi$
Sign of $f'(x)$:	$f' > 0$	$f' < 0$	$f' > 0$
Conclusion:	Increasing	Decreasing	Increasing

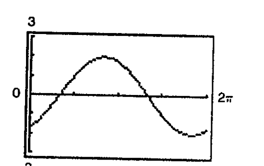
Increasing on: $\left(0, \frac{5\pi}{6}\right), \left(\frac{11\pi}{6}, 2\pi\right)$

Decreasing on: $\left(\frac{5\pi}{6}, \frac{11\pi}{6}\right)$

(b) Relative maximum: $\left(\frac{5\pi}{6}, 2\right)$

Relative minimum: $\left(\frac{11\pi}{6}, -2\right)$

(c)



47. (a) $f(x) = \sin^2 x + \sin x, \quad 0 < x < 2\pi$

$f'(x) = 2\sin x \cos x + \cos x = \cos x(2\sin x + 1) = 0$

Critical numbers: $x = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$

Test intervals:	$0 < x < \frac{\pi}{2}$	$\frac{\pi}{2} < x < \frac{7\pi}{6}$	$\frac{7\pi}{6} < x < \frac{3\pi}{2}$	$\frac{3\pi}{2} < x < \frac{11\pi}{6}$	$\frac{11\pi}{6} < x < 2\pi$
Sign of $f'(x)$:	$f' > 0$	$f' < 0$	$f' > 0$	$f' < 0$	$f' > 0$
Conclusion:	Increasing	Decreasing	Increasing	Decreasing	Increasing

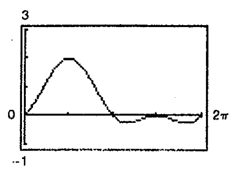
Increasing on: $\left(0, \frac{\pi}{2}\right), \left(\frac{7\pi}{6}, \frac{3\pi}{2}\right), \left(\frac{11\pi}{6}, 2\pi\right)$

Decreasing on: $\left(\frac{\pi}{2}, \frac{7\pi}{6}\right), \left(\frac{3\pi}{2}, \frac{11\pi}{6}\right)$

(b) Relative minima: $\left(\frac{7\pi}{6}, -\frac{1}{4}\right), \left(\frac{11\pi}{6}, -\frac{1}{4}\right)$

Relative maxima: $\left(\frac{\pi}{2}, 2\right), \left(\frac{3\pi}{2}, 0\right)$

(c)



48. (a) $f(x) = \frac{\sin x}{1 + \cos^2 x}, 0 < x < 2\pi$

$f'(x) = \frac{\cos x(2 + \sin^2 x)}{(1 + \cos^2 x)^2} = 0$

Critical numbers: $x = \frac{\pi}{2}, \frac{3\pi}{2}$

Test intervals:	$0 < x < \frac{\pi}{2}$	$\frac{\pi}{2} < x < \frac{3\pi}{2}$	$\frac{3\pi}{2} < x < 2\pi$
Sign of $f'(x)$:	$f' > 0$	$f' < 0$	$f' > 0$
Conclusion:	Increasing	Decreasing	Increasing

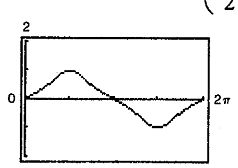
Increasing on: $\left(0, \frac{\pi}{2}\right), \left(\frac{3\pi}{2}, 2\pi\right)$

Decreasing on: $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

(b) Relative maximum: $\left(\frac{\pi}{2}, 1\right)$

Relative minimum: $\left(\frac{3\pi}{2}, -1\right)$

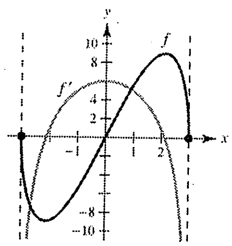
(c)



49. $f(x) = 2x\sqrt{9 - x^2}, [-3, 3]$

(a) $f'(x) = \dfrac{2(9 - 2x^2)}{\sqrt{9 - x^2}}$

(b)



(c) $\dfrac{2(9 - 2x^2)}{\sqrt{9 - x^2}} = 0$

Critical numbers: $x = \pm\dfrac{3}{\sqrt{2}} = \pm\dfrac{3\sqrt{2}}{2}$

(d) Intervals:

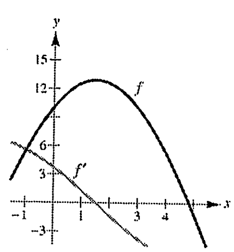
$\left(-3, -\frac{3\sqrt{2}}{2}\right)$	$\left(-\frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}\right)$	$\left(\frac{3\sqrt{2}}{2}, 3\right)$
$f'(x) < 0$	$f'(x) > 0$	$f'(x) < 0$
Decreasing	Increasing	Decreasing

f is increasing when f' is positive and decreasing when f' is negative.

50. $f(x) = 10\left(5 - \sqrt{x^2 - 3x + 16}\right), [0, 5]$

(a) $f'(x) = -\dfrac{5(2x - 3)}{\sqrt{x^2 - 3x + 16}}$

(b)



(c) $-\dfrac{5(2x - 3)}{\sqrt{x^2 - 3x + 16}} = 0$

Critical number: $x = \dfrac{3}{2}$

(d) Intervals:

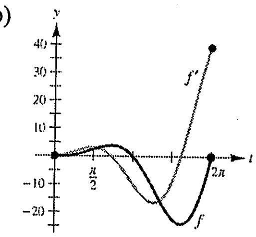
$\left(0, \frac{3}{2}\right)$	$\left(\frac{3}{2}, 5\right)$
$f'(x) > 0$	$f'(x) < 0$
Increasing	Decreasing

f is increasing when f' is positive and decreasing when f' is negative.

51. $f(t) = t^2 \sin t, [0, 2\pi]$

(a) $f'(t) = t^2 \cos t + 2t \sin t = t(t \cos t + 2 \sin t)$

(b)



(c) $t(t \cos t + 2 \sin t) = 0$

$t = 0 \text{ or } t = -2 \tan t$

$t \cot t = -2$

$t \approx 2.2889, 5.0870$ (graphing utility)

Critical numbers: $t = 2.2889, 5.0870$

(d) Intervals:

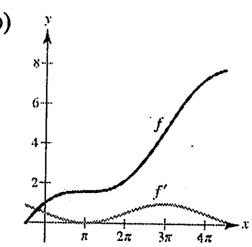
$(0, 2.2889)$	$(2.2889, 5.0870)$	$(5.0870, 2\pi)$
$f'(t) > 0$	$f'(t) < 0$	$f'(t) > 0$
Increasing	Decreasing	Increasing

f is increasing when f' is positive and decreasing when f' is negative.

52. $f(x) = \dfrac{x}{2} + \cos\dfrac{x}{2}, [0, 4\pi]$

(a) $f'(x) = \dfrac{1}{2} - \dfrac{1}{2}\sin\dfrac{x}{2}$

(b)



(c) $\dfrac{1}{2} - \dfrac{1}{2}\sin\dfrac{x}{2} = 0$

$\sin\dfrac{x}{2} = 1$

$\dfrac{x}{2} = \dfrac{\pi}{2}$

Critical number: $x = \pi$

(d) Intervals:

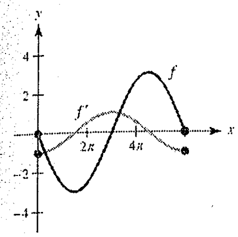
$(0, \pi)$	$(\pi, 4\pi)$
$f'(x) > 0$	$f'(x) > 0$
Increasing	Increasing

f is increasing when f' is positive.

53. (a) $f(x) = -3\sin\frac{x}{3}, [0, 6\pi]$

$f'(x) = -\cos\frac{x}{3}$

(b)



(c) Critical numbers: $x = \frac{3\pi}{2}, \frac{9\pi}{2}$

(d) Intervals:

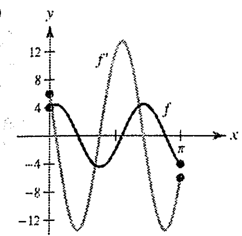
$\left(0, \frac{3\pi}{2}\right)$	$\left(\frac{3\pi}{2}, \frac{9\pi}{2}\right)$	$\left(\frac{9\pi}{2}, 6\pi\right)$
$f' < 0$	$f' > 0$	$f' < 0$
Decreasing	Increasing	Decreasing

f is increasing when f' is positive and decreasing when f' is negative.

54. (a) $f(x) = 2\sin 3x + 4\cos 3x, [0, \pi]$

$f'(x) = 6\cos 3x - 12\sin 3x$

(b)



(c) $f'(x) = 0 \Rightarrow \tan 3x = \frac{1}{2}$

Critical numbers: $x \approx 0.1545, 1.2017, 2.2489$

(d) Intervals:

$(0, 0.1545)$	$(0.1545, 1.2017)$	$(1.2017, 2.2489)$	$(2.2489, \pi)$
$f' > 0$	$f' < 0$	$f' > 0$	$f' < 0$
Increasing	Decreasing	Increasing	Decreasing

f is increasing when f' is positive and decreasing when f' is negative.

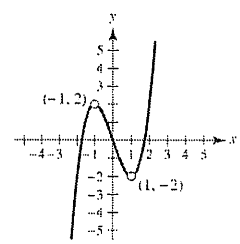
55. $f(x) = \frac{x^5 - 4x^3 + 3x}{x^2 - 1} = \frac{(x^2 - 1)(x^3 - 3x)}{x^2 - 1} = x^3 - 3x, x \neq \pm 1$

$f(x) = g(x) = x^3 - 3x$ for all $x \neq \pm 1$.

$f'(x) = 3x^2 - 3 = 3(x^2 - 1), x \neq \pm 1 \Rightarrow f'(x) \neq 0$

f symmetric about origin

zeros of f: $(0, 0), (\pm\sqrt{3}, 0)$



$g(x)$ is continuous on $(-\infty, \infty)$ and $f(x)$ has holes at $(-1, 2)$ and $(1, -2)$.

56. $f(t) = \cos^2 t - \sin^2 t = 1 - 2\sin^2 t = g(t)$

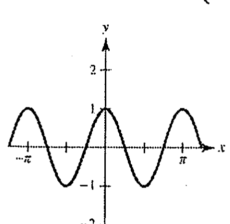
$f'(t) = -4\sin t\cos t = -2\sin 2t$

f symmetric with respect to y-axis

zeros of f: $\pm\frac{\pi}{4}$

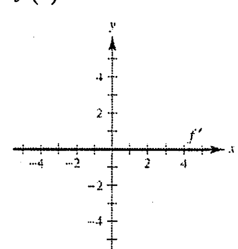
Relative maximum: $(0, 1)$

Relative minimum: $\left(-\frac{\pi}{2}, -1\right), \left(\frac{\pi}{2}, -1\right)$

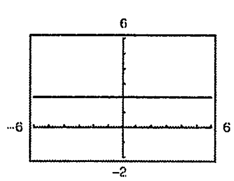


The graphs of $f(x)$ and $g(x)$ are the same.

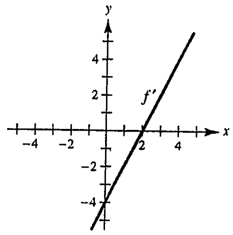
57. $f(x) = c$ is constant $\Rightarrow f'(x) = 0$.



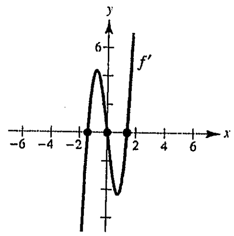
58. $f(x)$ is a line of slope $\approx 2 \Rightarrow f'(x) = 2$.



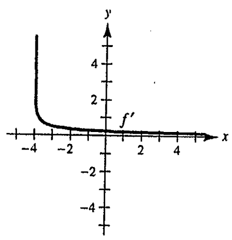
59. f is quadratic $\Rightarrow$ f' is a line.



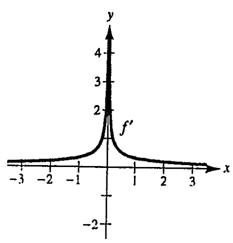
60. f is a 4^{th} degree polynomial $\Rightarrow$ f' is a cubic polynomial.



61. f has positive, but decreasing slope.



62. f has positive slope.



In Exercises 63–68, $f'(x) > 0$ on $(-\infty, -4)$, $f'(x) < 0$ on $(-4, 6)$ and $f'(x) > 0$ on $(6, \infty)$.

63. $g(x) = f(x) + 5$
$g'(x) = f'(x)$
$g'(0) = f'(0) < 0$

64. $g(x) = 3f(x) - 3$
$g'(x) = 3f'(x)$
$g'(-5) = 3f'(-5) > 0$

65. $g(x) = -f(x)$
$g'(x) = -f'(x)$
$g'(-6) = -f'(-6) < 0$

66. $g(x) = -f(x)$
$g'(x) = -f'(x)$
$g'(0) = -f'(0) > 0$

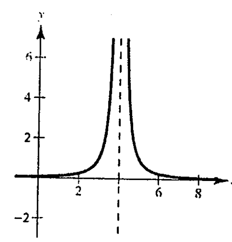
67. $g(x) = f(x - 10)$
$g'(x) = f'(x - 10)$
$g'(0) = f'(-10) > 0$

68. $g(x) = f(x - 10)$
$g'(x) = f'(x - 10)$
$g'(8) = f'(-2) < 0$

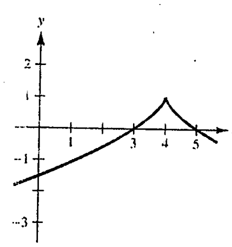
69. $$f'(x)\begin{cases} > 0, & x < 4 \Rightarrow f \text{ is increasing on } (-\infty, 4). \\ \text{undefined}, & x = 4 \\ < 0, & x > 4 \Rightarrow f \text{ is decreasing on } (4\ \infty). \end{cases}$$

Two possibilities for $f(x)$ are given below.

(a)



(b)



70. (i) (a) Critical number: $x = 2$ (Because $f'(2) = 0$)

(b) f increasing on

$(2, \infty)$ (Because $f' > 0$ on $(2, \infty)$)

f decreasing on

$(-\infty, 2)$ (Because $f' < 0$ on $(-\infty, 2)$)

(c) f has a relative minimum at $x = 2$.

(ii) (a) Critical numbers:

$x = 0, 1$ (Because $f'(1) = 0$)

(b) f increasing on $(-\infty, 0)$ and $(1, \infty)$

(Because $f' > 0$ on these intervals)

f decreasing on

$(0, 1)$ (Because $f' < 0$ on $(0, 1)$)

(c) f has a relative maximum at $x = 0$, and a relative minimum at $x = 1$.

(iii) (a) Critical numbers: $x = -1, 0, 1$

(Because $f'(-1) = f'(0) = f'(1) = 0$)

(b) f increasing on $(-\infty, -1)$ and $(0, 1)$

(Because $f' > 0$ on these intervals)

f decreasing on $(-1, 0)$ and $(1, \infty)$

(Because $f' < 0$ on these intervals)

(c) f has a relative maximum at $x = -1$ and $x = 1$. f has a relative minimum at $x = 0$.

(iv) (a) Critical numbers: $x = -3, 1, 5$

(Because $f'(-3) = f'(1) = f'(s) = 0$)

(b) f increasing on $(-3, 1)$ and $(1, 5)$

(Because $f' > 0$ on these intervals). In fact, f is increasing on $(-3, 5)$.

f decreasing on $(-\infty, -3)$ and $(5, \infty)$

(Because $f' < 0$ on these intervals)

(c) f has a relative minimum at $x = -3$, and a relative maximum at $x = 5$.

$x = 1$ is not a relative extremum.

71. Critical number: $x = 5$

$f'(4) = -2.5 \Rightarrow f$ is decreasing at $x = 4$.

$f'(6) = 3 \Rightarrow f$ is increasing at $x = 6$.

$(5, f(5))$ is a relative minimum.

72. Critical number: $x = 2$

$f'(1) = 2 \Rightarrow f$ is decreasing at $x = 1$.

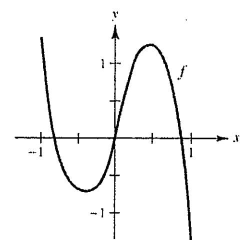
$f'(3) = 6 \Rightarrow f$ is increasing at $x = 3$.

$(2, f(2))$ is not a relative extremum.

In Exercises 73 and 74, answers will vary.

Sample answers:

73. (a)

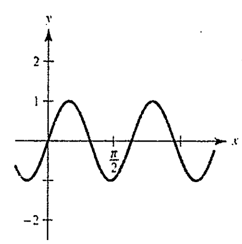


(b) The critical numbers are in intervals $(-0.50, -0.25)$ and $(0.25, 0.50)$ because the sign of f' changes in these intervals. f is decreasing on approximately $(-1, -0.40)$, $(0.48, 1)$, and increasing on $(-0.40, 0.48)$.

(c) Relative minimum when $x \approx -0.40$: $(-0.40, 0.75)$

Relative maximum when $x \approx 0.48$: $(0.48, 1.25)$

74. (a)



(b) The critical numbers are in the intervals $\left(0, \frac{\pi}{6}\right), \left(\frac{\pi}{3}, \frac{\pi}{2}\right)$, and $\left(\frac{3\pi}{4}, \frac{5\pi}{6}\right)$ because the sign of f' changes in these intervals. f is increasing on approximately $\left(0, \frac{\pi}{7}\right)$ and $\left(\frac{3\pi}{7}, \frac{6\pi}{7}\right)$ and decreasing on $\left(\frac{\pi}{7}, \frac{3\pi}{7}\right)$ and $\left(\frac{6\pi}{7}, \pi\right)$.

(c) Relative minima when $x \approx \frac{3\pi}{7}, \pi$

Relative maxima when $x \approx \frac{\pi}{7}, \frac{6\pi}{7}$

75. $s(t) = 4.9(\sin\theta)t^2$

(a) $s'(t) = 4.9(\sin\theta)(2t) = 9.8(\sin\theta)t$

speed $= |s'(t)| = |9.8(\sin\theta)t|$

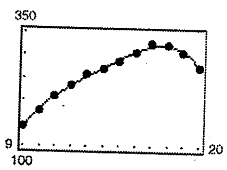
(b)

θ	0	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	π
$\lvert s'(t)\rvert$	0	$4.9\sqrt{2}t$	$4.9\sqrt{3}t$	$9.8t$	$4.9\sqrt{3}t$	$4.9\sqrt{2}t$	0

The speed is maximum for $\theta = \frac{\pi}{2}$.

76. (a) $M = -0.06803t^4 + 3.7162t^3 - 76.281t^2 + 716.56t - 2393.0$

(b)



(c) Using a graphing utility, the maximum is approximately $(17.7, 322.0)$, which compares well with the actual maximum in 2007: $(17, 326.0)$.

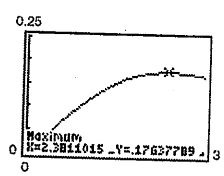
77. $C = \frac{3t}{27 + t^3}, t \geq 0$

(a)

t	0	0.5	1	1.5	2	2.5	3
$C(t)$	0	0.055	0.107	0.148	0.171	0.176	0.167

The concentration seems greatest near $t = 2.5$ hours.

(b)



The concentration is greatest when $t \approx 2.38$ hours.

(c) $$C' = \frac{(27 + t^3)(3) - (3t)(3t^2)}{(27 + t^3)^2}$$

$$= \frac{3(27 - 2t^3)}{(27 + t^3)^2}$$

$C' = 0$ when $t = 3/\sqrt[3]{2} \approx 2.38$ hours.

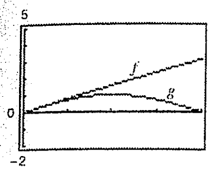
By the First Derivative Test, this is a maximum.

78. $f(x) = x, g(x) = \sin x, 0 < x < \pi$

(a)

x	0.5	1	1.5	2	2.5	3
$f(x)$	0.5	1	1.5	2	2.5	3
$g(x)$	0.479	0.841	0.997	0.909	0.598	0.141

$f(x)$ seems greater than $g(x)$ on $(0, \pi)$.

(b) 

$x > \sin x$ on $(0, \pi)$ so, $f(x) > g(x)$.

(c) Let $h(x) = f(x) - g(x) = x - \sin x$

$$h'(x) = 1 - \cos x > 0 \text{ on } (0, \pi).$$

Therefore, $h(x)$ is increasing on $(0, \pi)$. Because $h(0) = 0$ and $h'(x) > 0$ on $(0, \pi)$,

$$h(x) > 0$$
$$x - \sin x > 0$$
$$x > \sin x$$
$$f(x) > g(x) \text{ on } (0, \pi)$$

79. $v = k(R - r)r^2 = k(Rr^2 - r^3)$

$$v' = k(2Rr - 3r^2)$$
$$= kr(2R - 3r) = 0$$
$$r = 0 \text{ or } \tfrac{2}{3}R$$

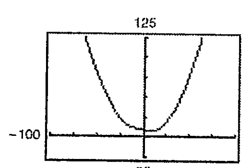
Maximum when $r = \frac{2}{3}R$.

80. $R = \sqrt{0.001T^4 - 4T + 100}$

(a) $R' = \dfrac{0.004T^3 - 4}{2\sqrt{0.001T^4 - 4T + 100}} = 0$

Critical number: $T = 10°$

Minimum resistance: $R \approx 8.3666$ ohms

(b) 

The minimum resistance is approximately $R \approx 8.37$ ohms at $T = 10°$.

81. (a) $s(t) = 6t - t^2, t \geq 0$

$$v(t) = 6 - 2t$$

(b) $v(t) = 0$ when $t = 3$.

Moving in positive direction for $0 \leq t < 3$ because $v(t) > 0$ on $0 \leq t < 3$.

(c) Moving in negative direction when $t > 3$.

(d) The particle changes direction at $t = 3$.

82. (a) $s(t) = t^2 - 7t + 10, t \geq 0$

$$v(t) = 2t - 7$$

(b) $v(t) = 0$ when $t = \frac{7}{2}$

Particle moving in positive direction for $t > \frac{7}{2}$ because $v'(t) > 0$ on $\left(\frac{7}{2}, \infty\right)$.

(c) Particle moving in negative direction on $\left[0, \frac{7}{2}\right)$.

(d) The particle changes direction at $t = \frac{7}{2}$.

83. (a) $s(t) = t^3 - 5t^2 + 4t, t \geq 0$

$v(t) = 3t^2 - 10t + 4$

(b) $v(t) = 0$ for $t = \dfrac{10 \pm \sqrt{100 - 48}}{6} = \dfrac{5 \pm \sqrt{13}}{3}$

Particle is moving in a positive direction on

$\left[0, \dfrac{5 - \sqrt{13}}{3}\right) \approx [0, 0.4648)$ and $\left(\dfrac{5 + \sqrt{13}}{3}, \infty\right) \approx (2.8685, \infty)$ because $v > 0$ on these intervals.

(c) Particle is moving in a negative direction on

$\left(\dfrac{5 - \sqrt{13}}{3}, \dfrac{5 + \sqrt{13}}{3}\right) \approx (0.4648, 2.8685)$

(d) The particle changes direction at $t = \dfrac{5 \pm \sqrt{13}}{3}$.

84. (a) $s(t) = t^3 - 20t^2 + 128t - 280$

$v(t) = 3t^2 - 40t + 128$

(b) $v(t) = (3t - 16)(t - 8)$

$v(t) = 0$ when $t = \frac{16}{3}, 8$

$v(t) > 0$ for $\left[0, \frac{16}{3}\right)$ and $(8, \infty)$

(c) $v(t) < 0$ for $\left(\frac{16}{3}, 8\right)$

(d) The particle changes direction at $t = \frac{16}{3}$ and 8.

85. Answers will vary.

86. Answers will vary.

87. (a) Use a cubic polynomial

$f(x) = a_3x^3 + a_2x^2 + a_1x + a_0$

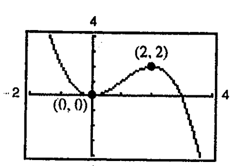
(b) $f'(x) = 3a_3x^2 + 2a_2x + a_1.$

$$\begin{aligned}
f(0) = 0:&\quad a_3(0)^3 + a_2(0)^2 + a_1(0) + a_0 = 0 \Rightarrow &\quad a_0 &= 0\\
f'(0) = 0:&\quad 3a_3(0)^2 + 2a_2(0) + a_1 = 0 \Rightarrow &\quad a_1 &= 0\\
f(2) = 2:&\quad a_3(2)^3 + a_2(2)^2 + a_1(2) + a_0 = 2 \Rightarrow &\quad 8a_3 + 4a_2 &= 2\\
f'(2) = 0:&\quad 3a_3(2)^2 + 2a_2(2) + a_1 = 0 \Rightarrow &\quad 12a_3 + 4a_2 &= 0
\end{aligned}$$

(c) The solution is $a_0 = a_1 = 0, a_2 = \frac{3}{2}, a_3 = -\frac{1}{2}$:

$f(x) = -\frac{1}{2}x^3 + \frac{3}{2}x^2.$

(d)



88. (a) Use a cubic polynomial

$$f(x) = 3a_3x^3 + a_2x^2 + a_1x + a_0$$

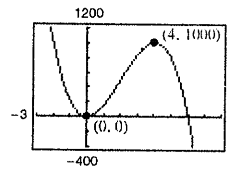
(b) $f'(x) = 3a_3x^2 + 2a_2x + a_1$

$$\begin{aligned}
f(0) &= 0: & a_3(0)^3 + a_2(0)^2 + a_1(0) + a_0 &= 0 \Rightarrow & a_0 &= 0\\
f'(0) &= 0: & 3a_3(0)^2 + 2a_2(0) + a_1 &= 0 \Rightarrow & a_1 &= 0\\
f(4) &= 1000: & a_3(4)^3 + a_2(4)^2 + a_1(4) + a_0 &= 1000 \Rightarrow & 64a_3 + 16a_2 &= 100\\
f'(4) &= 0: & 3a_3(4)^2 + 2a_2(4) + a_1 &= 0 \Rightarrow & 48a_3 + 8a_2 &= 0
\end{aligned}$$

(c) The solution is $a_0 = a_1 = 0, a_2 = \frac{375}{2}, a_3 = -\frac{125}{4}$

$$f(x) = -\frac{125}{4}x^3 + \frac{375}{2}x^2.$$

(d)



89. (a) Use a fourth degree polynomial

$$f(x) = a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0.$$

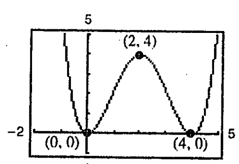
(b) $f'(x) = 4a_4x^3 + 3a_3x^2 + 2a_2x + a_1$

$$\begin{aligned}
f(0) &= 0: & a_4(0)^4 + a_3(0)^3 + a_2(0)^2 + a_1(0) + a_0 &= 0 \Rightarrow & a_0 &= 0\\
f'(0) &= 0: & 4a_4(0)^3 + 3a_3(0)^2 + 2a_2(0) + a_1 &= 0 \Rightarrow & a_1 &= 0\\
f(4) &= 0: & a_4(4)^4 + a_3(4)^3 + a_2(4)^2 + a_1(4) + a_0 &= 0 \Rightarrow & 256a_4 + 64a_3 + 16a_2 &= 0\\
f'(4) &= 0: & 4a_4(4)^3 + 3a_3(4)^2 + 2a_2(4) + a_1 &= 0 \Rightarrow & 256a_4 + 48a_3 + 8a_2 &= 0\\
f(2) &= 4: & a_4(2)^4 + a_3(2)^3 + a_2(2)^2 + a_1(2) + a_0 &= 4 \Rightarrow & 16a_4 + 8a_3 + 4a_2 &= 4\\
f'(2) &= 0: & 4a_4(2)^3 + 3a_3(2)^2 + 2a_2(2) + a_1 &= 0 \Rightarrow & 32a_4 + 12a_3 + 4a_2 &= 0
\end{aligned}$$

(c) The solution is $a_0 = a_1 = 0, a_2 = 4, a_3 = -2, \quad a_4 = \frac{1}{4}.$

$$f(x) = \frac{1}{4}x^4 - 2x^3 + 4x^2$$

(d)



90. (a) Use a fourth-degree polynomial

$$f(x) = a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0.$$

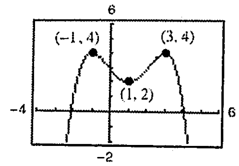
(b) $f'(x) = 4a_4x^3 + 3a_3x^2 + 2a_2x + a_1$

$$\begin{aligned}
f(1) &= 2: & a_4(1)^4 + a_3(1)^3 + a_2(1)^2 + a_1(1) + a_0 &= 2 \Rightarrow & a_4 + a_3 + a_2 + a_1 + a_0 &= 2\\
f'(1) &= 0: & 4a_4(1)^3 + 3a_3(1)^2 + 2a_2(1) + a_1 &= 0 \Rightarrow & 4a_4 + 3a_3 + 2a_2 + a_1 &= 0\\
f(-1) &= 4: & a_4(-1)^4 + a_3(-1)^3 + a_2(-1)^2 + a_1(-1) + a_0 &= 4 \Rightarrow & a_4 - a_3 + a_2 - a_1 + a_0 &= 4\\
f'(-1) &= 0: & 4a_4(-1)^3 + 3a_3(-1)^2 + 2a_2(-1) + a_1 &= 0 \Rightarrow & -4a_4 + 3a_3 - 2a_2 + a_1 &= 0\\
f(3) &= 4: & a_4(3)^4 + a_3(3)^3 + a_2(3)^2 + a_1(3) + a_0 &= 4 \Rightarrow & 81a_4 + 27a_3 + 9a_2 + a_1 + a_0 &= 4\\
f'(3) &= 0: & 4a_4(3)^3 + 3a_3(3)^2 + 2a_2(3) + a_1 &= 0 \Rightarrow & 108a_4 + 27a_3 + 6a_2 + a_1 &= 0
\end{aligned}$$

(c) The solution is $a_0 = \frac{23}{8}, a_1 = -\frac{3}{2}, a_2 = \frac{1}{4}, a_3 = \frac{1}{2}, a_4 = -\frac{1}{8}$

$f(x) = -\frac{1}{8}x^4 + \frac{1}{2}x^3 + \frac{1}{4}x^2 - \frac{3}{2}x + \frac{23}{8}.$

(d)



91. True.

Let $h(x) = f(x) + g(x)$ where f and g are increasing. Then $h'(x) = f'(x) + g'(x) > 0$ because $f'(x) > 0$ and $g'(x) > 0$.

92. False.

Let $h(x) = f(x)g(x)$ where $f(x) = g(x) = x$. Then $h(x) = x^2$ is decreasing on $(-\infty, 0)$.

93. False.

Let $f(x) = x^3$, then $f'(x) = 3x^2$ and f only has one critical number. Or, let $f(x) = x^3 + 3x + 1$, then $f'(x) = 3(x^2 + 1)$ has no critical numbers.

94. True.

If $f(x)$ is an nth-degree polynomial, then the degree of $f'(x)$ is $n - 1$.

95. False. For example, $f(x) = x^3$ does not have a relative extrema at the critical number $x = 0$.

96. False. The function might not be continuous on the interval.

97. Assume that $f'(x) < 0$ for all x in the interval (a, b) and let $x_1 < x_2$ be any two points in the interval. By the Mean Value Theorem, you know there exists a number c such that $x_1 < c < x_2$, and

$$f'(c) = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Because $f'(c) < 0$ and $x_2 - x_1 > 0$, then $f(x_2) - f(x_1) < 0$, which implies that $f(x_2) < f(x_1)$. So, f is decreasing on the interval.

98. Suppose $f'(x)$ changes from positive to negative at c. Then there exists a and b in I such that $f'(x) > 0$ for all x in (a, c) and $f'(x) < 0$ for all x in (c, b). By Theorem 3.5, f is increasing on (a, c) and decreasing on (c, b). Therefore, $f(c)$ is a maximum of f on (a, b) and so, a relative maximum of f.

99. Let x_1 and x_2 be two real numbers, $x_1 < x_2$. Then $x_1^3 < x_2^3 \Rightarrow f(x_1) < f(x_2)$. So f is increasing on $(-\infty, \infty)$.

100. Let x_1 and x_2 be two positive real numbers, $0 < x_1 < x_2$. Then

$$\frac{1}{x_1} > \frac{1}{x_2}$$

$$f(x_1) > f(x_2)$$

So, f is decreasing on $(0, \infty)$.

101. First observe that

$$\tan x + \cot x + \sec x + \csc x = \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} + \frac{1}{\cos x} + \frac{1}{\sin x}$$

$$= \frac{\sin^2 x + \cos^2 x + \sin x + \cos x}{\sin x \cos x}$$

$$= \frac{1 + \sin x + \cos x}{\sin x \cos x}\left(\frac{\sin x + \cos x - 1}{\sin x + \cos x - 1}\right)$$

$$= \frac{(\sin x + \cos x)^2 - 1}{\sin x \cos x(\sin x + \cos x - 1)}$$

$$= \frac{2 \sin x \cos x}{\sin x \cos x(\sin x + \cos x - 1)}$$

$$= \frac{2}{\sin x + \cos x - 1}$$

Let $t = \sin x + \cos x - 1$. The expression inside the absolute value sign is

$$f(t) = \sin x + \cos x + \frac{2}{\sin x + \cos x - 1}$$

$$= (\sin x + \cos x - 1) + 1 + \frac{2}{\sin x + \cos x - 1}$$

$$= t + 1 + \frac{2}{t}$$

Because $\sin\left(x + \frac{\pi}{4}\right) = \sin x \cos\frac{\pi}{4} + \cos x \sin\frac{\pi}{4}$

$$= \frac{\sqrt{2}}{2}(\sin x + \cos x),$$

$\sin x + \cos x \in \left[-\sqrt{2}, \sqrt{2}\right]$ and

$t = \sin x + \cos x - 1 \in \left[-1 - \sqrt{2}, -1 + \sqrt{2}\right].$

$$f'(t) = 1 - \frac{2}{t^2} = \frac{t^2 - 2}{t^2} = \frac{(t + \sqrt{2})(t - \sqrt{2})}{t^2}$$

$$f\left(-1 + \sqrt{2}\right) = -1 + \sqrt{2} + 1 + \frac{2}{-1 + \sqrt{2}} = \sqrt{2} + \frac{2}{\sqrt{2} - 1}$$

$$= \frac{4 - \sqrt{2}}{\sqrt{2} - 1}\left(\frac{\sqrt{2} + 1}{\sqrt{2} + 1}\right) = \frac{4\sqrt{2} - 2 + 4 - \sqrt{2}}{1} = 2 + 3\sqrt{2}$$

For $t > 0$, f is decreasing and $f(t) > f\left(-1 + \sqrt{2}\right) = 2 + 3\sqrt{2}$

For $t < 0$, f is increasing on $\left(-\sqrt{2} - 1, -\sqrt{2}\right)$, then decreasing on $\left(-\sqrt{2}, 0\right)$. So $f(t) < f\left(-\sqrt{2}\right) = 1 - 2\sqrt{2}$.

Finally, $\left|f(t)\right| \geq 2\sqrt{2} - 1$.

(You can verify this easily with a graphing utility.)

Section 3.4 Concavity and the Second Derivative Test

1. The graph of f is increasing and concave downward:
$f' > 0, f'' < 0$

2. The graph of f is decreasing and concave upward:
$f' < 0, f'' > 0$