Key

## 3.4 AP Practice Problems (p. 261) – Derivatives of Log Functions

1. If $f(x) = e^x + x^2$, find $\frac{d}{dx}[f(\ln x)]$.

(A) $\frac{1+2\ln x}{x}$ $\boxed{\text{(B)}}$ $1 + \frac{2\ln x}{x}$

(C) $1 + \frac{2}{x^2}$ (D) $x + 2x\ln x$

$f(\ln x) = e^{\ln x} + (\ln x)^2 = x + (\ln x)^2$

$f'(\ln x) = 1 + 2(\ln x)\cdot\left(\frac{1}{x}\right)$

$\boxed{f'(\ln x) = 1 + \frac{2\ln x}{x}}$

2. $\frac{d}{dx}(x^2 e^{\ln x^3}) =$

(A) $2x + 3x^2$ (B) $5x^3$

$\boxed{\text{(C)}}$ $5x^4$ (D) $6x^4$

$y = x^2 \cdot e^{\ln x^3}$

$y = x^2 \cdot x^3$

$y = x^5$

$\boxed{\frac{dy}{dx} = 5x^4}$

3. If $h(x) = \ln(x^2+4)$, then $h'(x)$ equals

(A) $\left|\frac{2x}{x^2+4}\right|$ $\boxed{\text{(B)}}$ $\frac{2x}{x^2+4}$

(C) $\frac{x^2}{x^2+4}$ (D) $\frac{1}{x^2+4}$

$* \frac{d}{dx}\ln u = \frac{u'}{u}$

$h(x) = \ln(x^2+4)$

$\boxed{h'(x) = \frac{2x}{x^2+4}}$

4. Find the rate of change of $y$ with respect to $x$ when $x = 1$ if $\ln(xy) = x$.

(A) $e$ $\boxed{\text{(B)}}$ 0 (C) 1 (D) $e - 1$

$\ln(xy) = x$

when $x = 1$, $\ln(1y) = 1 \rightarrow e^{\ln y} = e^{1} \rightarrow y = e$

$\frac{d}{dx}\ln u = \frac{u'}{u}$

$\ln(\underbrace{x}_{f}\underbrace{y}_{g}) = x$

$\frac{\underbrace{1}_{f'}\cdot\underbrace{y}_{g} + \underbrace{x}_{f}\cdot\underbrace{(dy/dx)}_{g'}}{xy} = 1$

$y + x\left(\frac{dy}{dx}\right) = xy$

$x\left(\frac{dy}{dx}\right) = xy - y$

$\frac{dy}{dx} = \frac{xy - y}{x}$

$\left.\frac{dy}{dx}\right|_{(1,e)} = \frac{1e - e}{1}$

$= \frac{0}{1} = \boxed{0}$

5. If $f(x) = \ln(e^{x^2-3x})$, then $f'(x)$ equals

(A) $\dfrac{1}{e^{x^2-3x}}$ (B) $\dfrac{2x-3}{e^{x^2-3x}}$

(C) $x^2 - 3x$ $\boxed{\text{(D)}}$ $2x - 3$

$f(x) = \ln e^{x^2-3x}$

$f(x) = (x^2-3x)\ln e$

$f(x) = x^2 - 3x$

$\boxed{f'(x) = 2x-3}$

6. Find the slope of the tangent line to the graph of $y = \ln(\sec^2 x)$ at $x = \dfrac{\pi}{4}$.

$\boxed{\text{(A)}}$ 2 (B) $\dfrac{\sqrt{2}}{2}$

(C) $\sqrt{2}$ (D) $2\sqrt{2}$

$y = \ln(\sec^2 x) = \ln\left[(\sec x)^2\right]$

$\dfrac{dy}{dx} = \dfrac{2(\sec x)(\sec x \tan x)}{\sec^2 x}$

$\dfrac{dy}{dx} = 2\tan x$

$\left.\dfrac{dy}{dx}\right|_{x=\pi/4} = 2\tan\left(\dfrac{\pi}{4}\right)$

$\boxed{\left.\dfrac{dy}{dx}\right|_{x=\pi/4} = 2(1) = 2}$

7. $\dfrac{d}{dx} \ln\left|\sin\dfrac{\pi}{x}\right| =$

(A) $\cot\dfrac{\pi}{x}$ (B) $-\dfrac{\pi}{x^2}\csc\dfrac{\pi}{x}$

$\boxed{\text{(C)}}$ $-\dfrac{\pi}{x^2}\cot\dfrac{\pi}{x}$ (D) $\dfrac{\pi}{x}\cot\dfrac{\pi}{x}$

$y = \ln\left|\sin(\pi x^{-1})\right|$

$y' = \dfrac{\cos(\pi x^{-1}) \cdot -1\pi x^{-2}}{\sin(\pi x^{-1})}$

$\boxed{y' = \dfrac{-\pi}{x^2} \cdot \cot(\pi x^{-1})}$

8. Find $\frac{d}{dx}(x^4+2)^x$.

(A) $4x^4(x^4+2)^{x-1}$

(B) $\frac{4x^4}{x^4+2} + \ln(x^4+2)$

$\boxed{\text{(C)}}$ $(x^4+2)^x\left[\frac{4x^4}{x^4+2} + \ln(x^4+2)\right]$

(D) $(x^4+2)^x \ln(x^4+2)$

$$\frac{dy}{dx} = y\left[\ln(x^4+2) + \frac{4x^4}{x^4+2}\right]$$

$$\boxed{\frac{dy}{dx} = (x^4+2)^x\left[\ln(x^4+2) + \frac{4x^4}{x^4+2}\right]}$$

*Apply Log Differentiation for form $y = (\text{variable})^{(\text{variable})}$

$y = (x^4+2)^x$

$\ln y = \ln(x^4+2)^x$

$\ln y = \underbrace{x}_{f} \cdot \underbrace{\ln(x^4+2)}_{g}$

$\frac{1}{y}\left(\frac{dy}{dx}\right) = \underbrace{(1)}_{f'} \underbrace{\ln(x^4+2)}_{g} + \underbrace{x}_{f} \cdot \underbrace{\frac{4x^3}{x^4+2}}_{g'}$

9. Find $\frac{d^2y}{dx^2}$ for $y = \ln(x\sqrt{x})$.

$\boxed{\text{(A)}}$ $-\frac{3}{2x^2}$ (B) $\frac{3}{2x^2}$ (C) $\frac{3}{2x}$ (D) $-\frac{3}{x^3}$

$y = \ln(x \cdot x^{1/2}) = \ln x^{3/2}$

$y = \frac{3}{2}\ln x$

$y' = \frac{3}{2}\left(\frac{1}{x}\right)$

$y' = \frac{3}{2}x^{-1}$

$y'' = \frac{-3}{2}x^{-2}$

$$\boxed{y'' = \frac{-3}{2x^2}}$$

10. $\frac{d}{dx}\frac{\log_2 x}{x} =$

(A) $\frac{1-\log_2 x}{x^2}$ (B) $-\frac{\ln 2 + \log_2 x}{x^2}$

(C) $\frac{\ln 2 - \log_2 x}{x^2 \ln 2}$ $\boxed{\text{(D)}}$ $\frac{1 - \ln 2 \log_2 x}{x^2 \ln 2}$

$* \frac{d}{dx}\log_a u = \frac{1}{\ln a}\cdot\frac{u'}{u}$

$y = \frac{\overbrace{\log_2 x}^{f}}{\underbrace{x}_{g}}$

$y' = \frac{\overbrace{\frac{1}{\ln 2}\cdot\left(\frac{1}{x}\right)}^{f'} \cdot \overbrace{x}^{g} - \overbrace{\log_2 x}^{f} \cdot \overbrace{(1)}^{g'}}{\underbrace{x^2}_{g^2}}$

$y' = \frac{\frac{1}{\ln 2} - \log_2 x}{x^2}$

$y' = \frac{\frac{1}{\ln 2} - \frac{(\ln 2)\log_2 x}{\ln 2}}{x^2}$

$y' = \frac{\frac{1-(\ln 2)(\log_2 x)}{\ln 2}}{x^2}$

$$\boxed{y' = \frac{1-(\ln 2)(\log_2 x)}{x^2 \ln 2}}$$

11. If $f(x) = x \ln x$, then $f'(x)$ equals

(A) $x + \ln x$ (B) $1$ (C) $1 + \ln x$ (D) $\frac{1}{x} + \ln x$

$$f'(x) = \underbrace{(1)}_{f'}\underbrace{\ln x}_{g} + \underbrace{x}_{f} \cdot \underbrace{\left(\frac{1}{x}\right)}_{g'}$$

$$\boxed{f'(x) = \ln x + 1}$$

12. Suppose $g(x) = \ln(f(x))$, where $f(x) > 0$ for all real numbers and $f$ is differentiable for all real numbers. If $f(4) = 2$ and $f'(4) = -\frac{1}{5}$, find $g'(4)$. Show the computations that lead to the answer.

$-1/10$

*chain Rule

$\frac{d}{dx} \ln u = \frac{u'}{u}$

$$g(x) = \ln[f(x)]$$

$$g'(x) = \frac{f'(x)}{f(x)}$$

$$g'(4) = \frac{f'(4)}{f(4)} = \frac{-1/5}{2} \rightarrow -\frac{1}{5} \cdot \frac{1}{2} = \boxed{\frac{-1}{10}}$$