Key

AP Calculus – 3.4 Notes – Log Derivatives and Log Differentiation

Recall:

$\ln 1 = 0$ $\ln 0 = \text{undefined}$ $e^0 = 1$ $e^{\ln a} = a$ $\ln e^a = a$

Derivatives of Exponential Functions

$$\frac{d}{dx} a^u = \ln a \cdot a^u \cdot u' \qquad \frac{d}{dx} e^u = e^u \cdot u'$$

Example: Find $f'(x)$ if $f(x) = 2^x + 3e^x$

Derivatives of Logarithmic Functions

$$\frac{d}{dx}\log_a u = \frac{1}{\ln a} \cdot \frac{u'}{u} \qquad \frac{d}{dx}\ln u = \frac{u'}{u}$$

Example: Find $f'(x)$ if $f(x) = \log_4 x - 4\ln x$

$$\frac{1}{\ln 4} \cdot \frac{1}{x} - 4 \cdot \frac{1}{x} \rightarrow \frac{1}{x \ln 4} - \frac{4}{x}$$

$$\boxed{f'(x) = \frac{1}{x\ln 4} - \frac{4}{x}}$$

Find the derivative of each function.

1. $f(x) = 2\sin x + 5e^x$

$$\boxed{f'(x) = 2\cos x + 5e^x}$$

2. $f(x) = 3^x - 4\cos x$

$$f'(x) = \ln 3 \cdot 3^x - 4(-\sin x)$$

$$\downarrow$$

$$\boxed{f'(x) = (\ln 3)3^x + 4\sin x}$$

3. $f(x) = \log_2 x - \sin x$

$$f'(x) = \frac{1}{\ln 2} \cdot \frac{1}{x} - \cos x$$

$$\boxed{f'(x) = \frac{1}{x \ln 2} - \cos x}$$

Natural Log Properties:

$\ln 1 = 0$ | $\ln e = 1$	$\ln a^n = n * \ln a$
$\ln(ab) = \ln a + \ln b$	$ln\left(\frac{a}{b}\right) = \ln a - \ln b$

Log Differentiation

Logarithmic Differentiation : Simplifying non-log functions using log properties to expand before differentiating.

Log differentiation steps:

1. Take the **ln** (natural log) of both sides
2. Simplify and expand using log properties
3. Use implicit differentiation
4. Substitute for y

Example 1: Find the derivative of $y = \frac{(x-2)^2}{\sqrt{x^2+1}}$

$$\ln y = \ln\left[\frac{(x-2)^2}{(x^2+1)^{1/2}}\right]$$

$$\ln y = \ln(x-2)^2 - \ln(x^2+1)^{1/2}$$

$$\ln y = 2\ln(x-2) - \frac{1}{2}\ln(x^2+1)$$

$$\frac{1}{y}\left(\frac{dy}{dx}\right) = 2\cdot\frac{1}{x-2} - \frac{1}{2}\cdot\frac{2x}{x^2+1}$$

$$\frac{dy}{dx} = y\left[\frac{2}{x-2} - \frac{x}{x^2+1}\right]$$

$$\boxed{\frac{dy}{dx} = \frac{(x-2)^2}{\sqrt{x^2+1}}\left[\frac{2}{x-2} - \frac{x}{x^2+1}\right]}$$

Example 2: Find the derivative of $y = x^{2x+3}$

$$\ln y = \ln x^{(2x+3)}$$

product Rule

$$\ln y = \overbrace{(2x+3)}^{f}\overbrace{(\ln x)}^{g}$$

$$\frac{1}{y}\left(\frac{dy}{dx}\right) = \overbrace{(2)}^{f'}\overbrace{(\ln x)}^{g} + \overbrace{(2x+3)}^{f}\overbrace{\left(\frac{1}{x}\right)}^{g'}$$

$$\frac{dy}{dx} = y\left[2\ln x + \frac{2x+3}{x}\right]$$

$$\frac{2x}{x} + \frac{3}{x}$$

$$\boxed{\frac{dy}{dx} = x^{2x+3}\left[2\ln x + 2 + \frac{3}{x}\right]}$$