**101.** First observe that

$$\begin{aligned}\tan x + \cot x + \sec x + \csc x &= \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} + \frac{1}{\cos x} + \frac{1}{\sin x}\\ &= \frac{\sin^2 x + \cos^2 x + \sin x + \cos x}{\sin x \cos x}\\ &= \frac{1 + \sin x + \cos x}{\sin x \cos x}\left(\frac{\sin x + \cos x - 1}{\sin x + \cos x - 1}\right)\\ &= \frac{(\sin x + \cos x)^2 - 1}{\sin x \cos x(\sin x + \cos x - 1)}\\ &= \frac{2 \sin x \cos x}{\sin x \cos x(\sin x + \cos x - 1)}\\ &= \frac{2}{\sin x + \cos x - 1}\end{aligned}$$

Let $t = \sin x + \cos x - 1$. The expression inside the absolute value sign is

$$\begin{aligned}f(t) &= \sin x + \cos x + \frac{2}{\sin x + \cos x - 1}\\ &= (\sin x + \cos x - 1) + 1 + \frac{2}{\sin x + \cos x - 1}\\ &= t + 1 + \frac{2}{t}\end{aligned}$$

Because $\sin\left(x + \frac{\pi}{4}\right) = \sin x \cos\frac{\pi}{4} + \cos x \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}(\sin x + \cos x)$,

$\sin x + \cos x \in \left[-\sqrt{2}, \sqrt{2}\right]$ and

$t = \sin x + \cos x - 1 \in \left[-1 - \sqrt{2}, -1 + \sqrt{2}\right]$.

$$f'(t) = 1 - \frac{2}{t^2} = \frac{t^2 - 2}{t^2} = \frac{\left(t + \sqrt{2}\right)\left(t - \sqrt{2}\right)}{t^2}$$

$$\begin{aligned}f\left(-1 + \sqrt{2}\right) &= -1 + \sqrt{2} + 1 + \frac{2}{-1 + \sqrt{2}} = \sqrt{2} + \frac{2}{\sqrt{2} - 1}\\ &= \frac{4 - \sqrt{2}}{\sqrt{2} - 1}\left(\frac{\sqrt{2} + 1}{\sqrt{2} + 1}\right) = \frac{4\sqrt{2} - 2 + 4 - \sqrt{2}}{1} = 2 + 3\sqrt{2}\end{aligned}$$

For $t > 0$, $f$ is decreasing and $f(t) > f\left(-1 + \sqrt{2}\right) = 2 + 3\sqrt{2}$

For $t < 0$, $f$ is increasing on $\left(-\sqrt{2} - 1, -\sqrt{2}\right)$, then decreasing on $\left(-\sqrt{2}, 0\right)$. So $f(t) < f\left(-\sqrt{2}\right) = 1 - 2\sqrt{2}$.

Finally, $|f(t)| \ge 2\sqrt{2} - 1$.

(You can verify this easily with a graphing utility.)

## Section 3.4 Concavity and the Second Derivative Test

**1.** The graph of $f$ is increasing and concave downward: $f' > 0, f'' < 0$

**2.** The graph of $f$ is decreasing and concave upward: $f' < 0, f'' > 0$

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3. $y = x^2 - x - 2$

$y' = 2x - 1$

$y'' = 2$

$y'' > 0$ for all $x$.

Concave upward: $(-\infty, \infty)$

4. $g(x) = 3x^2 - x^3$

$g'(x) = 6x - 3x^2$

$g''(x) = 6 - 6x$

$g''(x) = 0$ when $x = 1$.

Concave upward: $(-\infty, 1)$

Concave downward: $(1, \infty)$

| Intervals: | $-\infty < x < 1$ | $1 < x < \infty$ |
|---|---|---|
| Sign of $g''$: | $g'' > 0$ | $g'' < 0$ |
| Conclusion: | Concave upward | Concave downward |

5. $f(x) = -x^3 + 6x^2 - 9x - 1$

$f'(x) = -3x^2 + 12x - 9$

$f''(x) = -6x + 12 = -6(x - 2)$

$f''(x) = 0$ when $x = 2$.

Concave upward: $(-\infty, 2)$

Concave downward: $(2, \infty)$

| Intervals: | $-\infty < x < 2$ | $2 < x < \infty$ |
|---|---|---|
| Sign of $f''$: | $f'' > 0$ | $f'' < 0$ |
| Conclusion: | Concave upward | Concave downward |

6. $h(x) = x^5 - 5x + 2$

$h'(x) = 5x^4 - 5$

$h''(x) = 20x^3$

$h''(x) = 0$ when $x = 0$.

Concave upward: $(0, \infty)$

Concave downward: $(-\infty, 0)$

| Intervals: | $-\infty < x < 0$ | $0 < x < \infty$ |
|---|---|---|
| Sign of $h''$: | $h'' < 0$ | $h'' > 0$ |
| Conclusion: | Concave downward | Concave upward |

7. $f(x) = \dfrac{24}{x^2 + 12}$

$f'(x) = \dfrac{-48x}{(x^2 + 12)^2}$

$f''(x) = \dfrac{-144(4 - x^2)}{(x^2 + 12)^3}$

$f''(x) = 0$ when $x = \pm 2$.

Concave upward: $(-\infty, -2), (2, \infty)$

Concave downward: $(-2, 2)$

| Intervals: | $-\infty < x < -2$ | $-2 < x < 2$ | $2 < x < \infty$ |
|---|---|---|---|
| Sign of $f''$: | $f'' > 0$ | $f'' < 0$ | $f'' > 0$ |
| Conclusion: | Concave upward | Concave downward | Concave upward |

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8. $f(x) = \dfrac{2x^2}{3x^2 + 1}$

$f'(x) = \dfrac{4x}{\left(3x^2 + 1\right)^2}$

$f''(x) = \dfrac{-4(3x - 1)(3x + 1)}{\left(3x^2 + 1\right)^3}$

$f''(x) = 0$ when $x = \pm\dfrac{1}{3}$.

| Intervals: | $-\infty < x < -\frac{1}{3}$ | $-\frac{1}{3} < x < \frac{1}{3}$ | $\frac{1}{3} < x < \infty$ |
|---|---|---|---|
| Sign of $f''$: | $f'' < 0$ | $f'' > 0$ | $f'' < 0$ |
| Conclusion: | Concave downward | Concave upward | Concave downward |

Concave upward: $\left(-\dfrac{1}{3}, \dfrac{1}{3}\right)$

Concave downward: $\left(-\infty, -\dfrac{1}{3}\right)\left(\dfrac{1}{3}, \infty\right)$

9. $f(x) = \dfrac{x^2 + 1}{x^2 - 1}$

$f' = \dfrac{-4x}{\left(x^2 - 1\right)^2}$

$f'' = \dfrac{4\left(3x^2 + 1\right)}{\left(x^2 - 1\right)^3}$

$f$ is not continuous at $x = \pm 1$.

| Intervals: | $-\infty < x < -1$ | $-1 < x < 1$ | $1 < x < \infty$ |
|---|---|---|---|
| Sign of $f''$: | $f'' > 0$ | $f'' < 0$ | $f'' > 0$ |
| Conclusion: | Concave upward | Concave downward | Concave upward |

Concave upward: $(-\infty, -1), (1, \infty)$

Concave downward: $(-1, 1)$

10. $y = \frac{1}{270}\left(-3x^5 + 40x^3 + 135x\right)$

$y' = \frac{1}{270}\left(-15x^4 + 120x^2 + 135\right)$

$y'' = -\frac{2}{9}x(x - 2)(x + 2)$

$y'' = 0$ when $x = 0, \ \pm 2$.

| Intervals: | $-\infty < x < -2$ | $-2 < x < 0$ | $0 < x < 2$ | $2 < x < \infty$ |
|---|---|---|---|---|
| Sign of $y''$: | $y'' > 0$ | $y'' < 0$ | $y'' > 0$ | $y'' < 0$ |
| Conclusion: | Concave upward | Concave downward | Concave upward | Concave downward |

Concave upward: $(-\infty, -2), (0, 2)$

Concave downward: $(-2, 0), (2, \infty)$

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**11.** $g(x) = \dfrac{x^2 + 4}{4 - x^2}$

$g'(x) = \dfrac{16x}{(4 - x^2)^2}$

$g''(x) = \dfrac{16(3x^2 + 4)}{(4 - x^2)^3} = \dfrac{16(3x^2 + 4)}{(2 - x)^3(2 + x)^3}$

$f$ is not continuous at $x = \pm 2$.

| Intervals: | $-\infty < x < -2$ | $-2 < x < 2$ | $2 < x < \infty$ |
|---|---|---|---|
| Sign of $g''$: | $g'' < 0$ | $g'' > 0$ | $g'' < 0$ |
| Conclusion: | Concave downward | Concave upward | Concave downward |

Concave upward: $(-2, 2)$

Concave downward: $(-\infty, -2), (2, \infty)$

**12.** $h(x) = \dfrac{x^2 - 1}{2x - 1}$

$h'(x) = \dfrac{2(x^2 - x + 1)}{(2x - 1)^2}$

$h''(x) = \dfrac{-6}{(2x - 1)^3}$

| Intervals: | $-\infty < x < \frac{1}{2}$ | $\frac{1}{2} < x < \infty$ |
|---|---|---|
| Sign of $h''$: | $h'' > 0$ | $h'' < 0$ |
| Conclusion: | Concave upward | Concave downward |

$f''$ is not continuous at $x = \dfrac{1}{2}$.

Concave upward: $\left(-\infty, \dfrac{1}{2}\right)$

Concave downward: $\left(\dfrac{1}{2}, \infty\right)$

**13.** $y = 2x - \tan x, \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$

$y' = 2 - \sec^2 x$

$y'' = -2\sec^2 x \tan x$

$y'' = 0$ when $x = 0$.

| Intervals: | $-\frac{\pi}{2} < x < 0$ | $0 < x < \frac{\pi}{2}$ |
|---|---|---|
| Sign of $y''$: | $y'' > 0$ | $y'' < 0$ |
| Conclusion: | Concave upward | Concave downward |

Concave upward: $\left(-\dfrac{\pi}{2}, 0\right)$

Concave downward: $\left(0, \dfrac{\pi}{2}\right)$

**14.** $y = x + 2\csc x, \quad (-\pi, \pi)$

$y' = 1 - 2\csc x \cot x$

$y'' = -2\csc x(-\csc^2 x) - 2\cot x(-\csc x \cot x)$

$= 2(\csc^3 x + \csc x \cot^2 x)$

$y'' = 0$ when $x = 0$.

| Intervals: | $-\pi < x < 0$ | $0 < x < \pi$ |
|---|---|---|
| Sign of $y''$: | $y'' < 0$ | $y'' > 0$ |
| Conclusion: | Concave downward | Concave upward |

Concave upward: $(0, \pi)$

Concave downward: $(-\pi, 0)$

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**15.** $f(x) = x^3 - 6x^2 + 12x$

$f'(x) = 3x^2 - 12x + 12$

$f''(x) = 6(x - 2) = 0$ when $x = 2$.

| Intervals: | $-\infty < x < 2$ | $2 < x < \infty$ |
|---|---|---|
| Sign of $f''$: | $f'' < 0$ | $f'' > 0$ |
| Conclusion: | Concave downward | Concave upward |

Concave upward: $(2, \infty)$

Concave downward: $(-\infty, 2)$

Point of inflection: $(2, 8)$

**16.** $f(x) = -x^3 + 6x^2 - 5$

$f'(x) = -3x^2 + 12x$

$f''(x) = -6x + 12 = -6(x - 2) = 0$ when $x = 2$.

| Intervals: | $-\infty < x < 2$ | $2 < x < \infty$ |
|---|---|---|
| Sign of $f''$: | $f'' > 0$ | $f'' < 0$ |
| Conclusion: | Concave upward | Concave downward |

Concave upward: $(-\infty, 2)$

Concave downward: $(2, \infty)$

Point of inflection: $(2, 11)$

**17.** $f(x) = \frac{1}{2}x^4 + 2x^3$

$f'(x) = 2x^3 + 6x^2$

$f''(x) = 6x^2 + 12x = 6x(x + 2)$

$f''(x) = 0$ when $x = 0, -2$

| Intervals: | $-\infty < x < -2$ | $-2 < x < 2$ | $0 < x < \infty$ |
|---|---|---|---|
| Sign of $f''$: | $f'' > 0$ | $f'' < 0$ | $f'' > 0$ |
| Conclusion: | Concave upward | Concave downward | Concave upward |

Concave upward: $(-\infty, -2), (0, \infty)$

Concave downward: $(-2, 0)$

Points of inflection: $(-2, -8)$ and $(0, 0)$

**18.** $f(x) = 4 - x - 3x^4$

$f'(x) = -1 - 12x^3$

$f''(x) = -36x^2 = 0$ when $x = 0$.

| Intervals: | $-\infty < x < 0$ | $0 < x < \infty$ |
|---|---|---|
| Sign of $f''$: | $f'' < 0$ | $f'' < 0$ |
| Conclusion: | Concave downward | Concave downward |

Concave downward: $(-\infty, \infty)$

No points of inflection

**19.** $f(x) = x(x - 4)^3$

$f'(x) = x\left[3(x - 4)^2\right] + (x - 4)^3 = (x - 4)^2(4x - 4)$

$f''(x) = 4(x - 1)\left[2(x - 4)\right] + 4(x - 4)^2 = 4(x - 4)\left[2(x - 1) + (x - 4)\right] = 4(x - 4)(3x - 6) = 12(x - 4)(x - 2)$

$f''(x) = 12(x - 4)(x - 2) = 0$ when $x = 2, 4$.

| Intervals: | $-\infty < x < 2$ | $2 < x < 4$ | $4 < x < \infty$ |
|---|---|---|---|
| Sign of $f''(x)$: | $f''(x) > 0$ | $f''(x) < 0$ | $f''(x) > 0$ |
| Conclusion: | Concave upward | Concave downward | Concave upward |

Concave upward: $(-\infty, 2), (4, \infty)$

Concave downward: $(2, 4)$

Points of inflection: $(2, -16), (4, 0)$

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**20.** $f(x) = (x-2)^3(x-1)$

$f'(x) = (x-2)^2(4x-5)$

$f''(x) = 6(x-2)(2x-3)$

$f''(x) = 0$ when $x = \frac{3}{2}, 2.$

| Intervals: | $-\infty < x < \frac{3}{2}$ | $\frac{3}{2} < x < 2$ | $2 < x < \infty$ |
|---|---|---|---|
| Sign of $f''$: | $f'' > 0$ | $f'' < 0$ | $f'' > 0$ |
| Conclusion: | Concave upward | Concave downward | Concave upward |

Concave upward: $\left(-\infty, \frac{3}{2}\right), (2, \infty)$

Concave downward: $\left(\frac{3}{2}, 2\right)$

Points of inflection: $\left(\frac{3}{2}, -\frac{1}{16}\right), (2, 0)$

**21.** $f(x) = x\sqrt{x+3}$, Domain: $[-3, \infty)$

$f'(x) = x\left(\frac{1}{2}\right)(x+3)^{-1/2} + \sqrt{x+3} = \frac{3(x+2)}{2\sqrt{x+3}}$

$f''(x) = \frac{6\sqrt{x+3} - 3(x+2)(x+3)^{-1/2}}{4(x+3)}$

$= \frac{3(x+4)}{4(x+3)^{3/2}} = 0$ when $x = -4$.

$x = -4$ is not in the domain. $f''$ is not continuous at $x = -3$.

| Interval: | $-3 < x < \infty$ |
|---|---|
| Sign of $f''$: | $f'' > 0$ |
| Conclusion: | Concave upward |

Concave upward: $(-3, \infty)$

There are no points of inflection.

**22.** $f(x) = x\sqrt{9-x}$, Domain: $x \le 9$

$f'(x) = \frac{3(6-x)}{2\sqrt{9-x}}$

$f''(x) = \frac{3(x-12)}{4(9-x)^{3/2}} = 0$ when $x = 12$.

$x = 12$ is not in the domain. $f''$ is not continuous at $x = 9$.

| Interval: | $-\infty < x < 9$ |
|---|---|
| Sign of $f''$: | $f'' < 0$ |
| Conclusion: | Concave downward |

Concave downward: $(-\infty, 9)$

No point of inflection

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23. $f(x) = \frac{4}{x^2 + 1}$

$f'(x) = \frac{-8x}{(x^2 + 1)^2}$

$f''(x) = \frac{8(3x^2 - 1)}{(x^2 + 1)^3}$

$f''(x) = 0$ for $x = \pm\frac{\sqrt{3}}{3}$

| Intervals: | $-\infty < x < -\frac{\sqrt{3}}{3}$ | $-\frac{\sqrt{3}}{3} < x < \frac{\sqrt{3}}{3}$ | $\frac{\sqrt{3}}{3} < x < \infty$ |
|---|---|---|---|
| Sign of $f''$: | $f'' > 0$ | $f'' < 0$ | $f'' > 0$ |
| Conclusion: | Concave upward | Concave downward | Concave upward |

Concave upward: $\left(-\infty, -\frac{\sqrt{3}}{3}\right), \left(\frac{\sqrt{3}}{3}, \infty\right)$

Concave downward: $\left(-\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right)$

Points of inflection: $\left(-\frac{\sqrt{3}}{3}, 3\right)$ and $\left(\frac{\sqrt{3}}{3}, 3\right)$

24. $f(x) = \frac{x + 3}{\sqrt{x}}$, Domain: $x > 0$

$f'(x) = \frac{x - 3}{2x^{3/2}}$

$f''(x) = \frac{9 - x}{4x^{5/2}} = 0$ when $x = 9$

| Intervals: | $0 < x < 9$ | $9 < x < \infty$ |
|---|---|---|
| Sign of $f''$: | $f'' > 0$ | $f'' < 0$ |
| Conclusion: | Concave upward | Concave downward |

Concave upward: $(0, 9)$

Concave downward: $(9, \infty)$

Points of inflection: $(9, 4)$

25. $f(x) = \sin\frac{x}{2}, 0 \le x \le 4\pi$

$f'(x) = \frac{1}{2}\cos\left(\frac{x}{2}\right)$

$f''(x) = -\frac{1}{4}\sin\left(\frac{x}{2}\right)$

$f''(x) = 0$ when $x = 0, 2\pi, 4\pi$.

| Intervals: | $0 < x < 2\pi$ | $2\pi < x < 4\pi$ |
|---|---|---|
| Sign of $f''$: | $f'' < 0$ | $f'' > 0$ |
| Conclusion: | Concave downward | Concave upward |

Concave upward: $(2\pi, 4\pi)$

Concave downward: $(0, 2\pi)$

Point of inflection: $(2\pi, 0)$

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**26.** $f(x) = 2\csc\frac{3x}{2}, 0 < x < 2\pi$

$f'(x) = -3\csc\frac{3x}{2}\cot\frac{3x}{2}$

$f''(x) = \frac{9}{2}\left(\csc^3\frac{3x}{2} + \csc\frac{3x}{2}\cot^2\frac{3x}{2}\right) \neq 0$ for any $x$ in the domain of $f$.

$f''$ is not continuous at $x = \frac{2\pi}{3}$ and $x = \frac{4\pi}{3}$.

| Intervals: | $0 < x < \frac{2\pi}{3}$ | $\frac{2\pi}{3} < x < \frac{4\pi}{3}$ | $\frac{4\pi}{3} < x < 2\pi$ |
|---|---|---|---|
| Sign of $f''(x)$: | $f'' > 0$ | $f'' < 0$ | $f'' > 0$ |
| Conclusion: | Concave upward | Concave downward | Concave upward |

Concave upward: $\left(0, \frac{2\pi}{3}\right), \left(\frac{4\pi}{3}, 2\pi\right)$

Concave downward: $\left(\frac{2\pi}{3}, \frac{4\pi}{3}\right)$

No point of inflection

**27.** $f(x) = \sec\left(x - \frac{\pi}{2}\right), 0 < x < 4\pi$

$f'(x) = \sec\left(x - \frac{\pi}{2}\right)\tan\left(x - \frac{\pi}{2}\right)$

$f''(x) = \sec^3\left(x - \frac{\pi}{2}\right) + \sec\left(x - \frac{\pi}{2}\right)\tan^2\left(x - \frac{\pi}{2}\right) \neq 0$ for any $x$ in the domain of $f$.

$f''$ is not continuous at $x = \pi, x = 2\pi$, and $x = 3\pi$.

| Intervals: | $0 < x < \pi$ | $\pi < x < 2\pi$ | $2\pi < x < 3\pi$ | $3\pi < x < 4\pi$ |
|---|---|---|---|---|
| Sign of $f''$: | $f'' > 0$ | $f'' < 0$ | $f'' > 0$ | $f'' < 0$ |
| Conclusion: | Concave upward | Concave downward | Concave upward | Concave upward |

Concave upward: $(0, \pi), (2\pi, 3\pi)$

Concave downward: $(\pi, 2\pi), (3\pi, 4\pi)$

No point of inflection

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**28.** $f(x) = \sin x + \cos x, 0 \le x \le 2\pi$

$f'(x) = \cos x - \sin x$

$f''(x) = \sin x - \cos x$

$f''(x) = 0$ when $x = \frac{3\pi}{4}, \frac{7\pi}{4}$.

| Intervals: | $0 < x < \frac{3\pi}{4}$ | $\frac{3\pi}{4} < x < \frac{7\pi}{4}$ | $\frac{7\pi}{4} < x < 2\pi$ |
|---|---|---|---|
| Sign of $f''$: | $f''(x) < 0$ | $f''(x) > 0$ | $f''(x) < 0$ |
| Conclusion: | Concave downward | Concave upward | Concave downward |

Concave upward: $\left(\frac{3\pi}{4}, \frac{7\pi}{4}\right)$

Concave downward: $\left(0, \frac{3\pi}{4}\right), \left(\frac{7\pi}{4}, 2\pi\right)$

Points of inflection: $\left(\frac{3\pi}{4}, 0\right), \left(\frac{7\pi}{4}, 0\right)$

**29.** $f(x) = 2 \sin x + \sin 2x, 0 \le x \le 2\pi$

$f'(x) = 2 \cos x + 2 \cos 2x$

$f''(x) = -2 \sin x - 4 \sin 2x = -2 \sin x(1 + 4 \cos x)$

$f''(x) = 0$ when $x = 0, 1.823, \pi, 4.460$.

| Intervals: | $0 < x < 1.823$ | $1.823 < x < \pi$ | $\pi < x < 4.460$ | $4.460 < x < 2\pi$ |
|---|---|---|---|---|
| Sign of $f''$: | $f'' < 0$ | $f'' > 0$ | $f'' < 0$ | $f'' > 0$ |
| Conclusion: | Concave downward | Concave upward | Concave downward | Concave upward |

Concave upward: $(1.823, \pi), (4.460, 2\pi)$

Concave downward: $(0, 1.823), (\pi, 4.460)$

Points of inflection: $(1.823, 1.452), (\pi, 0), (4.46, -1.452)$

**30.** $f(x) = x + 2 \cos x, [0, 2\pi]$

$f'(x) = 1 - 2 \sin x$

$f''(x) = -2 \cos x$

$f''(x) = 0$ when $x = \frac{\pi}{2}, \frac{3\pi}{2}$.

| Intervals: | $0 < x < \frac{\pi}{2}$ | $\frac{\pi}{2} < x < \frac{3\pi}{2}$ | $\frac{3\pi}{2} < x < 2\pi$ |
|---|---|---|---|
| Sign of $f''$: | $f'' < 0$ | $f'' > 0$ | $f'' < 0$ |
| Conclusion: | Concave downward | Concave upward | Concave downward |

Concave upward: $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

Concave downward: $\left(0, \frac{\pi}{2}\right), \left(\frac{3\pi}{2}, 2\pi\right)$

Points of inflection: $\left(\frac{\pi}{2}, \frac{\pi}{2}\right), \left(\frac{3\pi}{2}, \frac{3\pi}{2}\right)$

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**31.** $f(x) = 6x - x^2$

$f'(x) = 6 - 2x$

$f''(x) = -2$

Critical number: $x = 3$

$f''(3) = -2 < 0$

Therefore, $(3, 9)$ is a relative maximum.

**32.** $f(x) = x^2 + 3x - 8$

$f'(x) = 2x + 3$

$f''(x) = 2$

Critical number: $x = -\frac{3}{2}$

$f''\left(-\frac{3}{2}\right) = 2 > 0$

Therefore, $\left(-\frac{3}{2}, -\frac{41}{4}\right)$ is a relative minimum.

**33.** $f(x) = x^3 - 3x^2 + 3$

$f'(x) = 3x^2 - 6x = 3x(x - 2)$

$f''(x) = 6x - 6 = 6(x - 1)$

Critical numbers: $x = 0, x = 2$

$f''(0) = -6 < 0$

Therefore, $(0, 3)$ is a relative maximum.

$f''(2) = 6 > 0$

Therefore, $(2, -1)$ is a relative minimum.

**34.** $f(x) = -x^3 + 7x^2 - 15x$

$f'(x) = -3x^2 + 14x - 15 = -(x - 3)(3x - 5)$

$f''(x) = -6x + 14 = -2(3x - 7)$

Critical numbers: $x = 3, \frac{5}{3}$

$f''(3) = -4 < 0$

Therefore, $(3, 9)$ is a relative maximum.

$f''\left(\frac{5}{3}\right) = 4 > 0$

Therefore, $\left(\frac{5}{3}, -\frac{275}{27}\right)$ is a relative minimum.

**35.** $f(x) = x^4 - 4x^3 + 2$

$f'(x) = 4x^3 - 12x^2 = 4x^2(x - 3)$

$f''(x) = 12x^2 - 24x = 12x(x - 2)$

Critical numbers: $x = 0, x = 3$

However, $f''(0) = 0$, so you must use the First Derivative Test. $f'(x) < 0$ on the intervals $(-\infty, 0)$ and $(0, 3)$; so, $(0, 2)$ is not an extremum. $f''(3) > 0$ so $(3, -25)$ is a relative minimum.

**36.** $f(x) = -x^4 + 4x^3 + 8x^2$

$f'(x) = -4x^3 + 12x^2 + 16x = -4x(x - 4)(x + 1)$

$f''(x) = -12x^2 + 24x + 16 = -4(3x^2 - 6x - 4)$

Critical numbers: $x = -1, 0, 4$

$f''(-1) = -20 < 0$

Therefore $(-1, 3)$ is a relative maximum.

$f''(0) = 16 > 0$

Therefore, $(0, 0)$ is a relative minimum.

$f''(4) = -80 < 0$

Therefore, $(4, 128)$ is a relative maximum.

**37.** $f(x) = x^{2/3} - 3$

$f'(x) = \dfrac{2}{3x^{1/3}}$

$f''(x) = -\dfrac{2}{9x^{4/3}}$

Critical number: $x = 0$

However, $f''(0)$ is undefined, so you must use the First Derivative Test. Because $f'(x) < 0$ on $(-\infty, 0)$ and $f'(x) > 0$ on $(0, \infty)$, $(0, -3)$ is a relative minimum.

**38.** $f(x) = \sqrt{x^2 + 1}$

$f'(x) = \dfrac{x}{\sqrt{x^2 + 1}}$

$f''(x) = \dfrac{1}{(x^2 + 1)^{3/2}}$

Critical number: $x = 0$

$f''(0) = 1 > 0$

Therefore, $(0, 1)$ is a relative minimum.

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