Key

Calculus **Ch. 3.5 Notes Limits at Infinity (End behavior)**

A. Checking for Horizontal Asymptotes (H.A.) $\left(\lim_{x\to\infty} f(x) \quad or \quad \lim_{x\to-\infty} f(x)\right)$

If $f(x) = \frac{p(x)}{q(x)}$, then **compare the degrees between numerator and denominator**

i) If Numerator degree < Denominator degree, then the H.A. is y = 0

Example 1: $\lim_{x\to\infty} \frac{3x^2-7}{2x^3+1} = \boxed{0}$

ii) If Denominator degree = Numerator degree, then H.A. is $y = \frac{numerator\ coefficient}{denominator\ coefficient}$

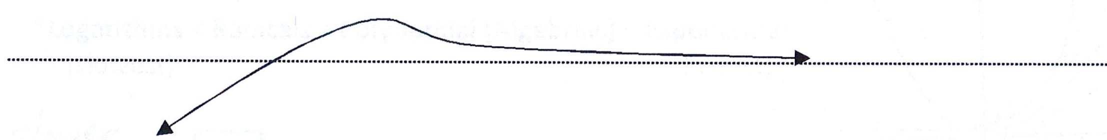
Example 2: $\lim_{x\to\infty} \frac{5x^2+3}{2x^2+4x-9} = \boxed{\frac{5}{2}}$ $\quad \lim_{x\to-\infty} \frac{5x^2+3}{2x^2+4x-9} = \boxed{\frac{5}{2}}$

iii) If Numerator degree > Denominator degree, then H.A. does not exist (limit is therefore $+\infty$ or $-\infty$)

test $x=100$

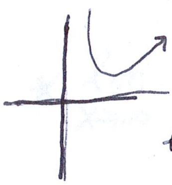
Example 3: $\lim_{x\to\infty} \frac{2x^3+1}{7x^2+5x+10} =$ $+\infty$ / $-\infty$ $\quad \frac{2(100)^3+1}{7(100)^2+5(100)+10} \to \frac{+}{+}$ $\boxed{+\infty}$

Note: a H.A. is a description of end behavior, not a boundary that the graph can't cross. A function can NEVER cross a vertical asymptote, but it might cross a horizontal asymptote.



Use Horizontal Asymptote Rules for the following:

same degree, take ratio of coefficients



4) $\lim_{x\to\infty} \frac{3x^2+1}{2x-5}$ $\to +\infty$ / $-\infty$

test $x=100$

$\frac{3(100)^2+1}{2(100)-5} \to \frac{+}{+} \to \boxed{+\infty}$

5) $\lim_{x\to-\infty} \frac{3x^2+1}{2x-5}$ $\to +\infty$ / $-\infty$

test $x=-100$

$\frac{3(-100)^2+1}{2(-100)-5} \to \frac{+}{-} \to \boxed{-\infty}$

6) $\lim_{x\to-\infty} \frac{3x+1}{5-2x} \to \frac{3}{-2} = \boxed{\frac{-3}{2}}$

7) $\lim_{x\to\infty} \frac{3x+1}{5-2x} \to \boxed{\frac{3}{-2}}$

8) $\lim_{x\to\infty} \frac{3x+1}{2x^2-5} = \boxed{0}$

9) $\lim_{x\to-\infty} \frac{3x^3+1}{2x^2-5}$ $\to +\infty$ / $-\infty$

test $x=-100$

$\frac{3(-100)^3+1}{2(-100)^2-5} \to \frac{-}{+} \to \boxed{-\infty}$

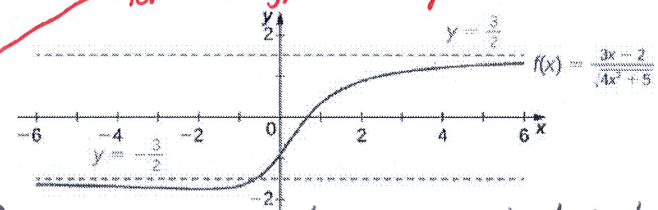
Think of this as a special case. Split horizontal asymptotes only apply for this type of setup.

B. Finding Horizontal Asymptotes with **Radicals in denominator**

Ex. 10: Find the Horizontal asymptotes for:

$$y = \frac{3x-2}{\sqrt{4x^2+5}}$$

*compare degrees



* Evaluate $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$

$$\lim_{x\to\infty} \frac{3x-2}{\sqrt{4x^2+5}} \to \frac{3}{\sqrt{4}} \to \frac{3}{2}$$

$$\lim_{x\to-\infty} \frac{3x-2}{\sqrt{4x^2+5}} \to -\frac{3}{\sqrt{4}} = -\frac{3}{2}$$

Horizontal Asymptotes at $y = \frac{3}{2}$ and $y = -3/2$

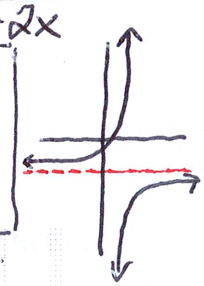
* Need to change sign of ratio when $\lim_{x\to-\infty} f(x)$

* Important Note! We do not change signs for $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$ for rational functions

Ex: $f(x) = \frac{3x-1}{1-2x}$

$\lim_{x\to\infty} f(x) = \frac{3}{-2}$

$\lim_{x\to-\infty} f(x) = -\frac{3}{2}$

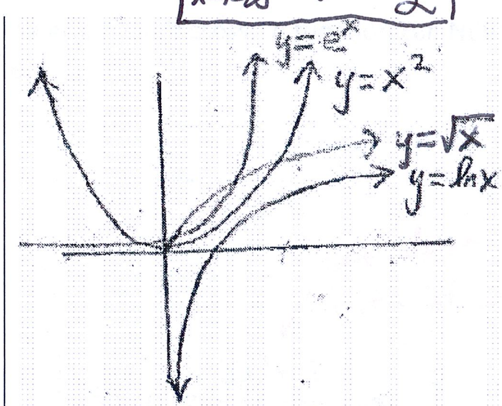


C. Comparative Growth Rates

*Families of Functions grow at predictable rates in relations to each other as x **approaches** $+\infty$

***Logarithms < Radicals < Polynomial (Algebraic) < Exponential**

(slowest) (fastest)



* $\lim_{x\to\infty} \frac{\text{slower}}{\text{faster}} = \boxed{0}$

* $\lim_{x\to\infty} \frac{\text{faster}}{\text{slower}} \to \boxed{+\infty \text{ or } -\infty}$

test x=100 to determine between +∞ and −∞

*Note: Comparative Growth Rates relationship <u>**only apply**</u> when limit approaches infinity. (NOT $-\infty$)

Ex. 11 $\lim_{x\to\infty} \frac{\sqrt{5000x+1000}}{x^2}$ Radical / Polynomial

$= \boxed{0}$

Ex. 13 $\lim_{x\to\infty} \frac{\ln(40000000x)}{2x}$ → logarithm / algebraic

$= \boxed{0}$

Ex. 12 $\lim_{x\to\infty} \frac{-e^{2x}}{1000x^4+x^5}$ exponential / polynomial

test x=100

$$\frac{-e^{2(100)}}{1000(100)^4+100^5} \to \frac{-}{+} \to \boxed{-\infty}$$

Ex. 14 $\lim_{x\to\infty} \frac{-\sqrt{3000x-4}}{\ln(5x+1)}$ → Radical / logarithm ↗ +∞ ↘ −∞

test x=100

$$\frac{-\sqrt{3000(100)-4}}{\ln(5(100)+1)} \to \frac{-}{+} \to \boxed{-\infty}$$