

**Calculus Ch. 3.5 Notes Limits at Infinity ( End behavior)**

A. Checking for Horizontal Asymptotes (H.A.)  $\left(\lim_{x \rightarrow \infty} f(x) \text{ or } \lim_{x \rightarrow -\infty} f(x)\right)$

If  $f(x) = \frac{p(x)}{q(x)}$ , then **compare the degrees between numerator and denominator**

i) If Numerator degree < Denominator degree, then the H.A. is  $y = 0$

Example 1:  $\lim_{x \rightarrow \infty} \frac{3x^2 - 7}{2x^3 + 1} =$

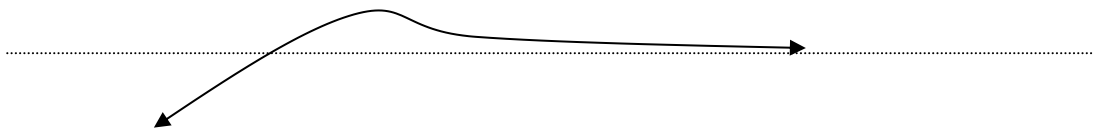
ii) If Denominator degree = Numerator degree, then H.A. is  $y = \frac{\text{numerator coefficient}}{\text{denominator coefficient}}$

Example 2:  $\lim_{x \rightarrow \infty} \frac{5x^2 + 3}{2x^2 + 4x - 9} =$

iii) If Numerator degree > Denominator degree, then H.A. does not exist (limit is therefore  $+\infty$  or  $-\infty$ )

Example 3:  $\lim_{x \rightarrow \infty} \frac{2x^3 + 1}{7x^2 + 5x + 10} =$

Note: a H.A. is a description of end behavior, not a boundary that the graph can't cross. A function can NEVER cross a vertical asymptote, but it might cross a horizontal asymptote.



**Use Horizontal Asymptote Rules for the following:**

4)  $\lim_{x \rightarrow \infty} \frac{3x^2 + 1}{2x - 5}$

5)  $\lim_{x \rightarrow -\infty} \frac{3x^2 + 1}{2x - 5}$

6)  $\lim_{x \rightarrow -\infty} \frac{3x + 1}{5 - 2x}$

7)  $\lim_{x \rightarrow \infty} \frac{3x + 1}{5 - 2x}$

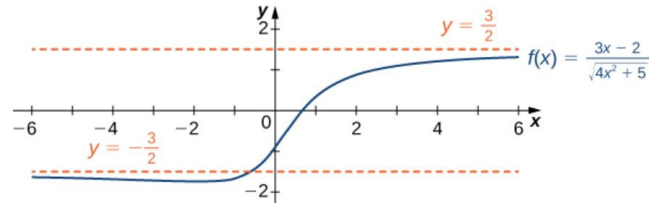
8)  $\lim_{x \rightarrow \infty} \frac{3x + 1}{2x^2 - 5}$

9)  $\lim_{x \rightarrow -\infty} \frac{3x^3 + 1}{2x^2 - 5}$

B. Finding Horizontal Asymptotes with **Radicals in denominator**

Ex. 10: Find the Horizontal asymptotes for:

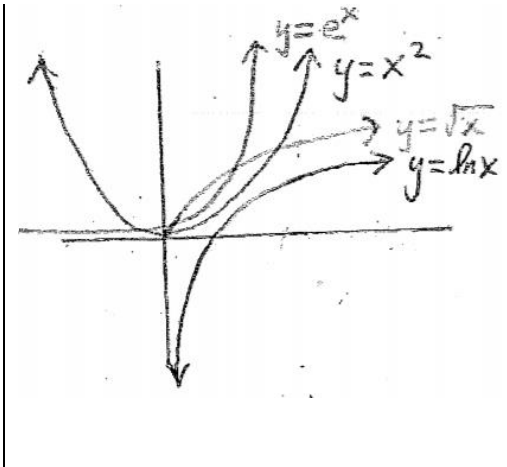
$$y = \frac{3x - 2}{\sqrt{4x^2 + 5}}$$



C. Comparative Growth Rates

\*Families of Functions grow at predictable rates in relations to each other as  $x$  **approaches**  $+\infty$

\***Logarithms < Radicals < Polynomial (Algebraic) < Exponential**  
(slowest) (fastest)



\*Note: Comparative Growth Rates relationship **only apply** when limit approaches infinity. (NOT  $-\infty$ )

Ex. 11  $\lim_{x \rightarrow \infty} \frac{\sqrt{5000x+1000}}{x^2}$

Ex. 13  $\lim_{x \rightarrow \infty} \frac{\ln(4000000x)}{2x}$

Ex. 12  $\lim_{x \rightarrow \infty} \frac{-e^{2x}}{1000x^4 + x^5}$

Ex. 14  $\lim_{x \rightarrow \infty} \frac{-\sqrt{3000x-4}}{\ln(5x+1)}$