73. $f(x) = x \sin\left(\frac{1}{x}\right)$

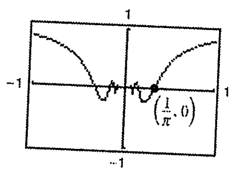
$$f'(x) = x\left[-\frac{1}{x^2}\cos\left(\frac{1}{x}\right)\right] + \sin\left(\frac{1}{x}\right) = -\frac{1}{x}\cos\left(\frac{1}{x}\right) + \sin\left(\frac{1}{x}\right)$$

$$f''(x) = -\frac{1}{x}\left[\frac{1}{x^2}\sin\left(\frac{1}{x}\right)\right] + \frac{1}{x^2}\cos\left(\frac{1}{x}\right) - \frac{1}{x^2}\cos\left(\frac{1}{x}\right) = -\frac{1}{x^3}\sin\left(\frac{1}{x}\right) = 0$$

$$x = \frac{1}{\pi}$$

Point of inflection: $\left(\frac{1}{\pi}, 0\right)$

When $x > 1/\pi$, $f'' < 0$, so the graph is concave downward.



74. $f(x) = x(x - 6)^2 = x^3 - 12x^2 + 36x$

$f'(x) = 3x^2 - 24x + 36 = 3(x - 2)(x - 6) = 0$

$f''(x) = 6x - 24 = 6(x - 4) = 0$

Relative extrema: $(2, 32)$ and $(6, 0)$

Point of inflection $(4, 16)$ is midway between the relative extrema of f.

75. True. Let $y = ax^3 + bx^2 + cx + d$, $a \neq 0$. Then $y'' = 6ax + 2b = 0$ when $x = -(b/3a)$, and the concavity changes at this point.

76. False. $f(x) = 1/x$ has a discontinuity at $x = 0$.

77. False. Concavity is determined by f''. For example, let $f(x) = x$ and $c = 2$. $f'(c) = f'(2) > 0$, but f is not concave upward at $c = 2$.

78. False. For example, let $f(x) = (x - 2)^4$.

79. f and g are concave upward on (a, b) implies that f' and g' are increasing on (a, b), and $f'' > 0$ and $g'' > 0$. So, $(f + g)'' > 0 \Rightarrow f + g$ is concave upward on (a, b) by Theorem 3.7.

80. f, g are positive, increasing, and concave upward on $(a, b) \Rightarrow f(x) > 0$, $f'(x) \geq 0$ and $f''(x) > 0$, and $g(x) > 0$, $g'(x) \geq 0$ and $g''(x) > 0$ on (a, b). For $x \in (a, b)$,

$$(fg)'(x) = f'(x)g(x) + f(x)g'(x)$$

$$(fg)''(x) = f''(x)g(x) + 2f'(x)g'(x) + f(x)g''(x) > 0$$

So, fg is concave upward on (a, b).

Section 3.5 Limits at Infinity

1. $f(x) = \dfrac{2x^2}{x^2 + 2}$

No vertical asymptotes

Horizontal asymptote: $y = 2$

Matches (f).

2. $f(x) = \dfrac{2x}{\sqrt{x^2 + 2}}$

No vertical asymptotes

Horizontal asymptotes: $y = \pm 2$

Matches (c).

3. $f(x) = \dfrac{x}{x^2 + 2}$

No vertical asymptotes

Horizontal asymptote: $y = 0$

$f(1) < 1$

Matches (d).

4. $f(x) = 2 + \dfrac{x^2}{x^4 + 1}$

No vertical asymptotes

Horizontal asymptote: $y = 2$

Matches (a).

5. $f(x) = \dfrac{4 \sin x}{x^2 + 1}$

No vertical asymptotes

Horizontal asymptote: $y = 0$

$f(1) > 1$

Matches (b).

6. $f(x) = \dfrac{2x^2 - 3x + 5}{x^2 + 1}$

No vertical asymptotes

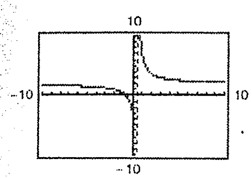
Horizontal asymptote: $y = 2$

Matches (e).

7. $f(x) = \dfrac{4x + 3}{2x - 1}$

x	10^0	10^1	10^2	10^3	10^4	10^5	10^6
$f(x)$	7	2.26	2.025	2.0025	2.0003	2	2

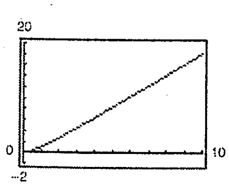
$\lim_{x\to\infty} f(x) = 2$



8. $f(x) = \dfrac{2x^2}{x + 1}$

x	10^0	10^1	10^2	10^3	10^4	10^5	10^6
$f(x)$	1	18.18	198.02	1998.02	19,998	199,998	1,999,998

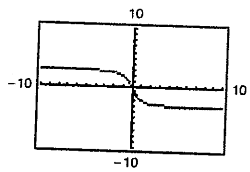
$\lim_{x\to\infty} f(x) = \infty$ (Limit does not exist)



9. $f(x) = \dfrac{-6x}{\sqrt{4x^2 + 5}}$

x	10^0	10^1	10^2	10^3	10^4	10^5	10^6
$f(x)$	−2	−2.98	−2.9998	−3	−3	−3	−3

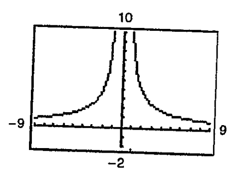
$\lim_{x\to\infty} f(x) = -3$



10. $f(x) = \dfrac{10}{\sqrt{2x^2 - 1}}$

x	10^0	10^1	10^2	10^3	10^4	10^5	10^6
$f(x)$	10.0	0.7089	0.0707	0.0071	0.0007	0.00007	0.000007

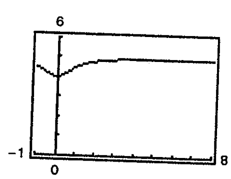
$\lim_{x\to\infty} f(x) = 0$



11. $f(x) = 5 - \dfrac{1}{x^2 + 1}$

x	10^0	10^1	10^2	10^3	10^4	10^5	10^6
$f(x)$	4.5	4.99	4.9999	4.999999	5	5	5

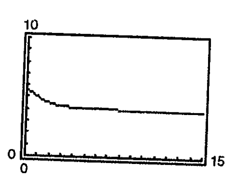
$\lim_{x\to\infty} f(x) = 5$



12. $f(x) = 4 + \dfrac{3}{x^2 + 2}$

x	10^0	10^1	10^2	10^3	10^4	10^5	10^6
$f(x)$	5	4.03	4.0003	4.0	4.0	4	4

$\lim_{x\to\infty} f(x) = 4$



13. (a) $h(x) = \frac{f(x)}{x^2} = \frac{5x^3 - 3x^2 + 10x}{x^2} = 5x - 3 + \frac{10}{x}$

$\lim_{x\to\infty} h(x) = \infty$ (Limit does not exist)

(b) $h(x) = \frac{f(x)}{x^3} = \frac{5x^3 - 3x^2 + 10x}{x^3} = 5 - \frac{3}{x} + \frac{10}{x^2}$

$\lim_{x\to\infty} h(x) = 5$

(c) $h(x) = \frac{f(x)}{x^4} = \frac{5x^3 - 3x^2 + 10x}{x^4} = \frac{5}{x} - \frac{3}{x^2} + \frac{10}{x^3}$

$\lim_{x\to\infty} h(x) = 0$

14. (a) $h(x) = \frac{f(x)}{x} = \frac{-4x^2 + 2x - 5}{x} = -4x + 2 - \frac{5}{x}$

$\lim_{x\to\infty} h(x) = -\infty$ (Limit does not exist)

(b) $h(x) = \frac{f(x)}{x^2} = \frac{-4x^2 + 2x - 5}{x^2} = -4 + \frac{2}{x} - \frac{5}{x^2}$

$\lim_{x\to\infty} h(x) = -4$

(c) $h(x) = \frac{f(x)}{x^3} = \frac{-4x^2 + 2x - 5}{x^3} = -\frac{4}{x} + \frac{2}{x^2} - \frac{5}{x^3}$

$\lim_{x\to\infty} h(x) = 0$

15. (a) $\lim_{x\to\infty} \frac{x^2 + 2}{x^3 - 1} = 0$

(b) $\lim_{x\to\infty} \frac{x^2 + 2}{x^2 - 1} = 1$

(c) $\lim_{x\to\infty} \frac{x^2 + 2}{x - 1} = \infty$ (Limit does not exist)

16. (a) $\lim_{x\to\infty} \frac{3 - 2x}{3x^3 - 1} = 0$

(b) $\lim_{x\to\infty} \frac{3 - 2x}{3x - 1} = -\frac{2}{3}$

(c) $\lim_{x\to\infty} \frac{3 - 2x^2}{3x - 1} = -\infty$ (Limit does not exist)

17. (a) $\lim_{x\to\infty} \frac{5 - 2x^{3/2}}{3x^2 - 4} = 0$

(b) $\lim_{x\to\infty} \frac{5 - 2x^{3/2}}{3x^{3/2} - 4} = -\frac{2}{3}$

(c) $\lim_{x\to\infty} \frac{5 - 2x^{3/2}}{3x - 4} = -\infty$ (Limit does not exist)

18. (a) $\lim_{x\to\infty} \frac{5x^{3/2}}{4x^2 + 1} = 0$

(b) $\lim_{x\to\infty} \frac{5x^{3/2}}{4x^{3/2} + 1} = \frac{5}{4}$

(c) $\lim_{x\to\infty} \frac{5x^{3/2}}{4\sqrt{x} + 1} = \infty$ (Limit does not exist)

19. $\lim_{x\to\infty}\left(4 + \frac{3}{x}\right) = 4 + 0 = 4$

20. $\lim_{x\to-\infty}\left(\frac{5}{x} - \frac{x}{3}\right) = \infty$ (Limit does not exist)

21. $\lim_{x\to\infty} \frac{2x - 1}{3x + 2} = \lim_{x\to\infty} \frac{2 - (1/x)}{3 + (2/x)} = \frac{2 - 0}{3 + 0} = \frac{2}{3}$

22. $\lim_{x\to-\infty} \frac{4x^2 + 5}{x^2 + 3} = \lim_{x\to-\infty} \frac{4 + (5/x^2)}{1 + (3/x^2)} = 4$

23. $\lim_{x\to\infty} \frac{x}{x^2 - 1} = \lim_{x\to\infty} \frac{1/x}{1 - (1/x^2)} = \frac{0}{1} = 0$

24. $\lim_{x\to\infty} \frac{5x^3 + 1}{10x^3 - 3x^2 + 7} = \lim_{x\to\infty} \frac{5 + 1/x^3}{10 - 3/x + 7/x^3}$

$= \frac{5 + 0}{10 - 0} = \frac{1}{2}$

25. $\lim_{x\to-\infty} \frac{5x^2}{x + 3} = \lim_{x\to-\infty} \frac{5x}{1 + (3/x)} = -\infty$

Limit does not exist.

26. $\lim_{x\to-\infty} \frac{x^3 - 4}{x^2 + 1} = \lim_{x\to-\infty} \frac{x - (4/x^2)}{1 + (1/x^2)} = -\infty$

Limit does not exist.

27. $\lim_{x\to-\infty} \frac{x}{\sqrt{x^2 - x}}$

$= \lim_{x\to-\infty} \frac{1}{\left(\frac{\sqrt{x^2 - x}}{-\sqrt{x^2}}\right)}$

$= \lim_{x\to-\infty} \frac{-1}{\sqrt{1 - (1/x)}}$

$= -1$, (for $x < 0$ we have $x = -\sqrt{x^2}$)

28. $\lim_{x\to-\infty} \frac{x}{\sqrt{x^2+1}}$

$= \lim_{x\to-\infty} \frac{1}{\left(\frac{\sqrt{x^2+1}}{-\sqrt{x^2}}\right)}$

$= \lim_{x\to-\infty} \frac{-1}{\sqrt{1+(1/x^2)}}$

$= -1$, (for $x < 0$ we have $x = -\sqrt{x^2}$)

29. $\lim_{x\to-\infty} \frac{2x+1}{\sqrt{x^2-x}}$

$= \lim_{x\to-\infty} \frac{2+\frac{1}{x}}{\left(\frac{\sqrt{x^2-x}}{-\sqrt{x^2}}\right)}$

$= \lim_{x\to-\infty} \frac{-2-\left(\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x}}}$

$= -2$, (for $x < 0$, $x = -\sqrt{x^2}$)

30. $\lim_{x\to\infty} \frac{5x^2+2}{\sqrt{x^2+3}}$

$= \lim_{x\to\infty} \frac{5x^2+2}{x\sqrt{1+(3/x^2)}}$

$= \lim_{x\to\infty} \frac{5x^2+(2/x)}{\sqrt{1+3/x^2}}$

$= \infty$

Limit does not exist.

31. $\lim_{x\to\infty} \frac{\sqrt{x^2-1}}{2x-1} = \lim_{x\to\infty} \frac{\sqrt{x^2-1}/\sqrt{x^2}}{2-1/x}$

$= \lim_{x\to\infty} \frac{\sqrt{1-1/x^2}}{2-1/x} = \frac{1}{2}$

32. $\lim_{x\to-\infty} \frac{\sqrt{x^4-1}}{x^3-1} = \lim_{x\to-\infty} \frac{\sqrt{x^4-1}}{x^3-1}\left(\frac{1/\left(-\sqrt{x^6}\right)}{1/x^3}\right)$

$= \lim_{x\to-\infty} \frac{\sqrt{1/x^2-1/x^6}}{-1+1/x^3} = 0,$

(for $x < 0$, we have $-\sqrt{x^6} = x^3$)

33. $\lim_{x\to\infty} \frac{x+1}{\left(x^2+1\right)^{1/3}} = \lim_{x\to\infty} \frac{x+1}{\left(x^2+1\right)^{1/3}}\left(\frac{1/x^{2/3}}{1/\left(x^2\right)^{1/3}}\right)$

$= \lim_{x\to\infty} \frac{x^{1/3}+1/x^{2/3}}{\left(1+1/x^2\right)^{1/3}} = \infty$

Limit does not exist.

34. $\lim_{x\to-\infty} \frac{2x}{\left(x^6-1\right)^{1/3}} = \lim_{x\to-\infty} \frac{2x}{\left(x^6-1\right)^{1/3}} \frac{\left(1/x^2\right)}{\left(1/\left(x^6\right)^{1/3}\right)}$

$= \lim_{x\to-\infty} \frac{2/x}{\left(1-1/x^6\right)^{1/3}} = 0$

35. $\lim_{x\to\infty} \frac{1}{2x+\sin x} = 0$

36. $\lim_{x\to\infty} \cos\left(\frac{1}{x}\right) = \cos 0 = 1$

37. Because $(-1/x) \le (\sin 2x)/x \le (1/x)$ for all $x \ne 0$, you have by the Squeeze Theorem,

$\lim_{x\to\infty} -\frac{1}{x} \le \lim_{x\to\infty} \frac{\sin 2x}{x} \le \lim_{x\to\infty} \frac{1}{x}$

$0 \le \lim_{x\to\infty} \frac{\sin 2x}{x} \le 0.$

Therefore, $\lim_{x\to\infty} \frac{\sin 2x}{x} = 0.$

38. $\lim_{x\to\infty} \frac{x-\cos x}{x} = \lim_{x\to\infty}\left(1-\frac{\cos x}{x}\right)$

$= 1 - 0 = 1$

Note:

$\lim_{x\to\infty} \frac{\cos x}{x} = 0$ by the Squeeze Theorem because

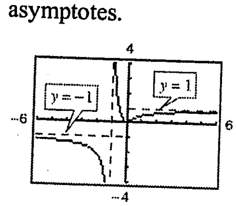
$-\frac{1}{x} \le \frac{\cos x}{x} \le \frac{1}{x}.$

39. $f(x) = \frac{|x|}{x+1}$

$\lim_{x\to\infty} \frac{|x|}{x+1} = 1$

$\lim_{x\to-\infty} \frac{|x|}{x+1} = -1$

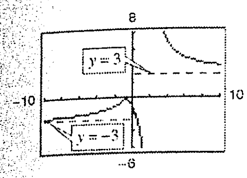
Therefore, $y = 1$ and $y = -1$ are both horizontal asymptotes.



40. $f(x) = \dfrac{|3x + 2|}{x - 2}$

$y = 3$ is a horizontal asymptote (to the right).

$y = -3$ is a horizontal asymptote (to the left).

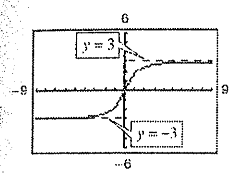


41. $f(x) = \dfrac{3x}{\sqrt{x^2 + 2}}$

$\lim_{x \to \infty} f(x) = 3$

$\lim_{x \to -\infty} f(x) = -3$

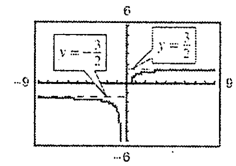
Therefore, $y = 3$ and $y = -3$ are both horizontal asymptotes.



42. $f(x) = \dfrac{\sqrt{9x^2 - 2}}{2x + 1}$

$y = \dfrac{3}{2}$ is a horizontal asymptote (to the right).

$y = -\dfrac{3}{2}$ is a horizontal asymptote (to the left).



43. $\lim_{x \to \infty} x \sin \dfrac{1}{x} = \lim_{t \to 0^+} \dfrac{\sin t}{t} = 1$

(Let $x = 1/t$.)

44. $\lim_{x \to \infty} x \tan \dfrac{1}{x} = \lim_{x \to 0^+} \dfrac{\tan t}{t} = \lim_{x \to 0^+} \left[\dfrac{\sin t}{t} \cdot \dfrac{1}{\cos t}\right]$

$= (1)(1) = 1$

(Let $x = 1/t$.)

45. $$\lim_{x \to -\infty}\left(x + \sqrt{x^2 + 3}\right) = \lim_{x \to -\infty}\left[\left(x + \sqrt{x^2 + 3}\right) \cdot \frac{x - \sqrt{x^2 + 3}}{x - \sqrt{x^2 + 3}}\right] = \lim_{x \to -\infty} \frac{-3}{x - \sqrt{x^2 + 3}} = 0$$

46. $$\lim_{x \to \infty}\left(x - \sqrt{x^2 + x}\right) = \lim_{x \to \infty}\left[\left(x - \sqrt{x^2 + x}\right) \cdot \frac{x + \sqrt{x^2 + x}}{x + \sqrt{x^2 + x}}\right] = \lim_{x \to \infty} \frac{-x}{x + \sqrt{x^2 + x}} = \lim_{x \to \infty} \frac{-1}{1 + \sqrt{1 + (1/x)}} = -\frac{1}{2}$$

47. $$\lim_{x \to -\infty}\left(3x + \sqrt{9x^2 - x}\right) = \lim_{x \to -\infty}\left[\left(3x + \sqrt{9x^2 - x}\right) \cdot \frac{3x - \sqrt{9x^2 - x}}{3x - \sqrt{9x^2 - x}}\right]$$

$$= \lim_{x \to -\infty} \frac{x}{3x - \sqrt{9x^2 - x}}$$

$$= \lim_{x \to -\infty} \frac{1}{3 - \dfrac{\sqrt{9x^2 - x}}{-\sqrt{x^2}}} \quad \left(\text{for } x < 0 \text{ you have } x = -\sqrt{x^2}\right)$$

$$= \lim_{x \to -\infty} \frac{1}{3 + \sqrt{9 - (1/x)}} = \frac{1}{6}$$

48. $$\lim_{x \to \infty}\left(4x - \sqrt{16x^2 - x}\right)\frac{4x + \sqrt{16x^2 - x}}{4x + \sqrt{16x^2 - x}} = \lim_{x \to \infty} \frac{16x^2 - \left(16x^2 - x\right)}{4x + \sqrt{\left(16x^2 - x\right)}}$$

$$= \lim_{x \to \infty} \frac{x}{4x + \sqrt{16x^2 - x}}$$

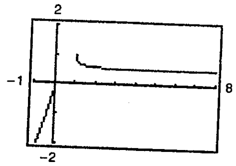
$$= \lim_{x \to \infty} \frac{1}{4 + \sqrt{16 - 1/x}}$$

$$= \frac{1}{4 + 4} = \frac{1}{8}$$

49.

x	10^0	10^1	10^2	10^3	10^4	10^5	10^6
$f(x)$	1	0.513	0.501	0.500	0.500	0.500	0.500

$$\lim_{x\to\infty}\left(x - \sqrt{x(x-1)}\right) = \lim_{x\to\infty}\frac{x - \sqrt{x^2 - x}}{1}\cdot\frac{x + \sqrt{x^2 - x}}{x + \sqrt{x^2 - x}} = \lim_{x\to\infty}\frac{x}{x + \sqrt{x^2 - x}} = \lim_{x\to\infty}\frac{1}{1 + \sqrt{1 - (1/x)}} = \frac{1}{2}$$

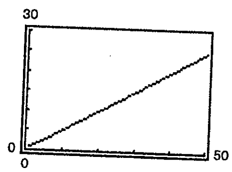


50.

x	10^0	10^1	10^2	10^3	10^4	10^5	10^6
$f(x)$	1.0	5.1	50.1	500.1	5000.1	50,000.1	500,000.1

$$\lim_{x\to\infty}\frac{x^2 - x\sqrt{x^2 - x}}{1}\cdot\frac{x^2 + x\sqrt{x^2 - x}}{x^2 + x\sqrt{x^2 - x}} = \lim_{x\to\infty}\frac{x^3}{x^2 + x\sqrt{x^2 - x}} = \infty$$

Limit does not exist.

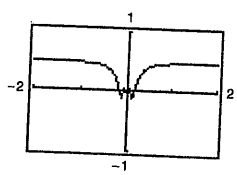


51.

x	10^0	10^1	10^2	10^3	10^4	10^5	10^6
$f(x)$	0.479	0.500	0.500	0.500	0.500	0.500	0.500

Let $x = 1/t$.

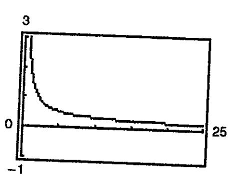
$$\lim_{x\to\infty} x\sin\left(\frac{1}{2x}\right) = \lim_{t\to 0^+}\frac{\sin(t/2)}{t} = \lim_{t\to 0^+}\frac{1}{2}\frac{\sin(t/2)}{t/2} = \frac{1}{2}$$



52.

x	10^0	10^1	10^2	10^3	10^4	10^5	10^6
$f(x)$	2.000	0.348	0.101	0.032	0.010	0.003	0.001

$$\lim_{x\to\infty}\frac{x + 1}{x\sqrt{x}} = 0$$



53. $\lim_{x\to\infty} f(x) = 4$ means that $f(x)$ approaches 4 as x becomes large.

54. $\lim_{x\to-\infty} f(x) = 2$ means that $f(x)$ approaches 2 as x becomes very large (in absolute value) and negative.

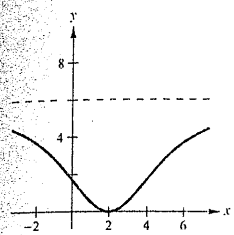
55. $x = 2$ is a critical number.

$f'(x) < 0$ for $x < 2$.

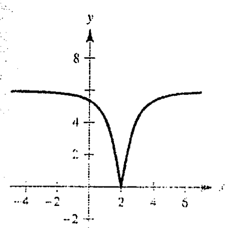
$f'(x) > 0$ for $x > 2$.

$\lim_{x\to-\infty} f(x) = \lim_{x\to\infty} f(x) = 6$

For example, let $f(x) = \dfrac{-6}{0.1(x-2)^2 + 1} + 6$.



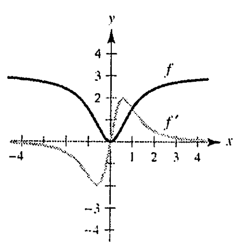
56. Yes. For example, let $f(x) = \dfrac{6|x-2|}{\sqrt{(x-2)^2 + 1}}$.



57. (a) The function is even: $\lim_{x\to-\infty} f(x) = 5$

(b) The function is odd: $\lim_{x\to-\infty} f(x) = -5$

58. (a)



(b) $\lim_{x\to\infty} f(x) = 3 \qquad \lim_{x\to\infty} f'(x) = 0$

(c) Because $\lim_{x\to\infty} f(x) = 3$, the graph approaches that of a horizontal line, $\lim_{x\to\infty} f'(x) = 0$.

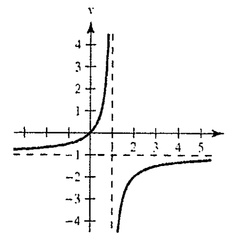
59. $y = \dfrac{x}{1-x}$

Intercept: $(0, 0)$

Symmetry: none

Horizontal asymptote: $y = -1$

Vertical asymptote: $x = 1$



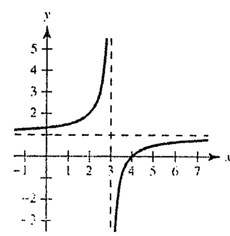
60. $y = \dfrac{x-4}{x-3}$

Intercepts: $(0, 4/3), (4, 0)$

Symmetry: none

Horizontal asymptote: $y = 1$

Vertical asymptote: $x = 3$



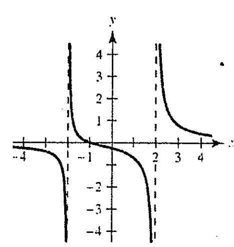
61. $y = \dfrac{x+1}{x^2-4}$

Intercepts: $(0, -1/4), (-1, 0)$

Symmetry: none

Horizontal asymptote: $y = 0$

Vertical asymptotes: $x = \pm 2$



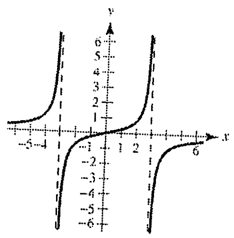
62. $y = \dfrac{2x}{9 - x^2}$

Intercept: $(0, 0)$

Symmetry: origin

Horizontal asymptote: $y = 0$

Vertical asymptotes: $x = \pm 3$



63. $y = \dfrac{x^2}{x^2 + 16}$

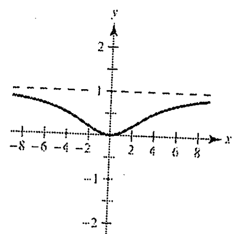
Intercept: $(0, 0)$

Symmetry: y-axis

Horizontal asymptote: $y = 1$

$y' = \dfrac{32x}{\left(x^2 + 16\right)^2}$

Relative minimum: $(0, 0)$



64. $y = \dfrac{2x^2}{x^2 - 4}$

Intercept: $(0, 0)$

Symmetry: y-axis

Horizontal asymptote: $y = 2$

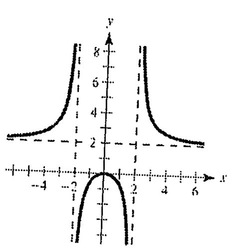
Vertical asymptotes: $x = \pm 2$

$y' = -\dfrac{4x}{\left(x^2 - 4\right)^2}$

$y'' = \dfrac{16\left(x^2 + 4\right)}{\left(x^2 - 4\right)^3}$

$y'' = (0) < 0$

Relative maximum: $(0, 0)$



65. $xy^2 = 9$

Domain: $x > 0$

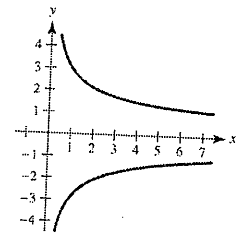
Intercepts: none

Symmetry: x-axis

$y = \pm\dfrac{3}{\sqrt{x}}$

Horizontal asymptote: $y = 0$

Vertical asymptote: $x = 0$



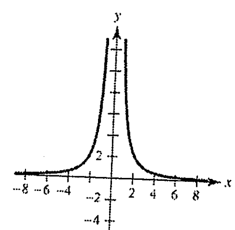
66. $x^2y = 9 \Rightarrow y = \dfrac{9}{x^2}$

Intercepts: none

Symmetry: y-axis

Horizontal asymptote: $y = 0$

Vertical asymptote: $x = 0$



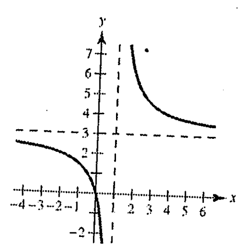
67. $y = \dfrac{3x}{x - 1}$

Intercept: $(0, 0)$

Symmetry: none

Horizontal asymptote: $y = 3$

Vertical asymptote: $x = 1$



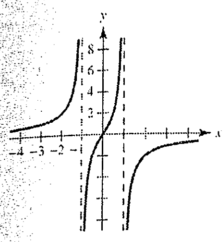
68. $y = \dfrac{3x}{1 - x^2}$

Intercept: $(0, 0)$

Symmetry: origin

Horizontal asymptote: $y = 0$

Vertical asymptotes: $x = \pm 1$



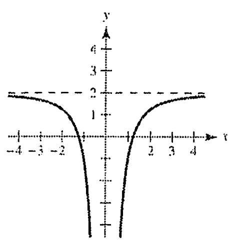
69. $y = 2 - \dfrac{3}{x^2} = \dfrac{2x^2 - 3}{x^2}$

Intercepts: $\left(\pm\sqrt{\dfrac{3}{2}}, 0\right)$

Symmetry: y-axis

Horizontal asymptote: $y = 2$

Vertical asymptote: $x = 0$



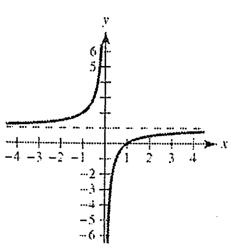
70. $y = 1 - \dfrac{1}{x} = \dfrac{x - 1}{x}$

Intercept: $(1, 0)$

Symmetry: none

Horizontal asymptote: $y = 1$

Vertical asymptote: $x = 0$



71. $y = 3 + \dfrac{2}{x}$

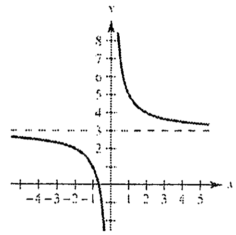
Intercept:

$y = 0 = 3 + \dfrac{2}{x} \Rightarrow \dfrac{2}{x} = -3 \Rightarrow x = -\dfrac{2}{3}; \left(-\dfrac{2}{3}, 0\right)$

Symmetry: none

Horizontal asymptote: $y = 3$

Vertical asymptote: $x = 0$



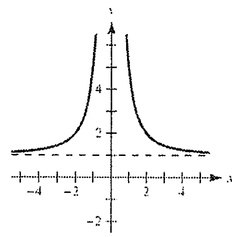
72. $y = \dfrac{4}{x^2} + 1 = \dfrac{4 + x^2}{x^2}$

Intercept: none

Symmetry: none

Horizontal asymptote: $y = 1$

Vertical asymptote: $x = 0$



73. $y = \dfrac{x^3}{\sqrt{x^2 - 4}}$

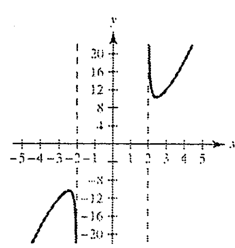
Domain: $(-\infty, -2), (2, \infty)$

Intercepts: none

Symmetry: origin

Horizontal asymptote: none

Vertical asymptotes: $x = \pm 2$ (discontinuities)



74. $y = \dfrac{x}{\sqrt{x^2 - 4}}$

Domain: $(-\infty, -2), (2, \infty)$

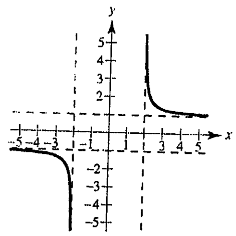
Intercepts: none

Symmetry: origin

Horizontal asymptotes: $y = \pm 1$ because

$$\lim_{x\to\infty} \frac{x}{\sqrt{x^2 - 4}} = 1, \quad \lim_{x\to-\infty} \frac{x}{\sqrt{x^2 - 4}} = -1.$$

Vertical asymptotes: $x = \pm 2$ (discontinuities)



75. $f(x) = 9 - \dfrac{5}{x^2}$

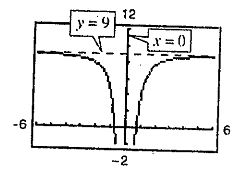
Domain: all $x \neq 0$

$f'(x) = \dfrac{10}{x^3} \Rightarrow$ No relative extrema

$f''(x) = -\dfrac{30}{x^4} \Rightarrow$ No points of inflection

Vertical asymptote: $x = 0$

Horizontal asymptote: $y = 9$



76. $f(x) = \dfrac{1}{x^2 - x - 2} = \dfrac{1}{(x+1)(x-2)}$

$$f'(x) = \frac{-(2x-1)}{(x^2 - x - 2)^2} = 0 \text{ when } x = \frac{1}{2}.$$

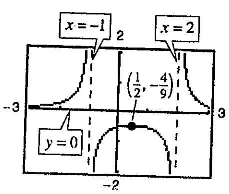
$$f''(x) = \frac{(x^2 - x - 2)^2(-2) + (2x-1)(2)(x^2 - x - 2)(2x-1)}{(x^2 - x - 2)^4} = \frac{6(x^2 - x + 1)}{(x^2 - x - 2)^3}$$

Because $f''\left(\frac{1}{2}\right) < 0$, $\left(\frac{1}{2}, -\frac{4}{9}\right)$ is a relative maximum.

Because $f''(x) \neq 0$, and it is undefined in the domain of f, there are no points of inflection.

Vertical asymptotes: $x = -1, x = 2$

Horizontal asymptote: $y = 0$



77. $f(x) = \dfrac{x-2}{x^2-4x+3} = \dfrac{x-2}{(x-1)(x-3)}$

$$f'(x) = \frac{(x^2-4x+3)-(x-2)(2x-4)}{(x^2-4x+3)^2} = \frac{-x^2+4x-5}{(x^2-4x+3)^2} \neq 0$$

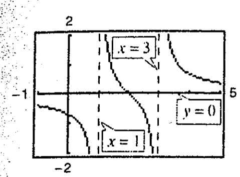
$$f''(x) = \frac{(x^2-4x+3)^2(-2x+4)-(-x^2+4x-5)(2)(x^2-4x+3)(2x-4)}{(x^2-4x+3)^4}$$

$$= \frac{2(x^3-6x^2+15x-14)}{(x^2-4x+3)^3} = \frac{2(x-2)(x^2-4x+7)}{(x^2-4x+3)^3} = 0 \text{ when } x = 2.$$

Because $f''(x) > 0$ on $(1, 2)$ and $f''(x) < 0$ on $(2, 3)$, then $(2, 0)$ is a point of inflection.

Vertical asymptotes: $x = 1, x = 3$

Horizontal asymptote: $y = 0$



78. $f(x) = \dfrac{x+1}{x^2+x+1}$

$f'(x) = \dfrac{-x(x+2)}{(x^2+x+1)^2} = 0$ when $x = 0, -2$.

$f''(x) = \dfrac{2(x^3+3x^2-1)}{(x^2+x+1)^3} = 0$

when $x \approx 0.5321, -0.6527, -2.8794$.

$f''(0) < 0$

Therefore, $(0, 1)$ is a relative maximum.

$f''(-2) > 0$

Therefore,

$\left(-2, -\dfrac{1}{3}\right)$

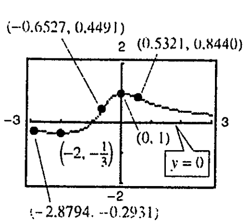
is a relative minimum.

Points of inflection:

$(0.5321, 0.8440)$, $(-0.6527, 0.4491)$ and

$(-2.8794, -0.2931)$

Horizontal asymptote: $y = 0$



79. $f(x) = \dfrac{3x}{\sqrt{4x^2+1}}$

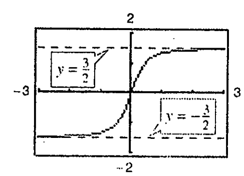
$f'(x) = \dfrac{3}{(4x^2+1)^{3/2}} \Rightarrow$ No relative extrema

$f''(x) = \dfrac{-36x}{(4x^2+1)^{5/2}} = 0$ when $x = 0$.

Point of inflection: $(0, 0)$

Horizontal asymptotes: $y = \pm\dfrac{3}{2}$

No vertical asymptotes



80. $g(x) = \dfrac{2x}{\sqrt{3x^2 + 1}}$

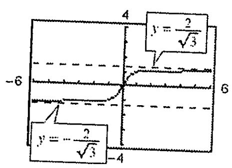
$g'(x) = \dfrac{2}{\left(3x^2 + 1\right)^{3/2}}$

$g''(x) = \dfrac{-18x}{\left(3x^2 + 1\right)^{5/2}}$

No relative extrema. Point of inflection: $(0, 0)$.

Horizontal asymptotes: $y = \pm\dfrac{2}{\sqrt{3}}$

No vertical asymptotes



81. $g(x) = \sin\left(\dfrac{x}{x - 2}\right), 3 < x < \infty$

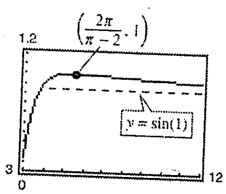
$g'(x) = \dfrac{-2\cos\left(\dfrac{x}{x - 2}\right)}{(x - 2)^2}$

Horizontal asymptote: $y = \sin(1)$

Relative maximum:

$\dfrac{x}{x - 2} = \dfrac{\pi}{2} \Rightarrow x = \dfrac{2\pi}{\pi - 2} \approx 5.5039$

No vertical asymptotes



82. $f(x) = \dfrac{2 \sin 2x}{x}$; Hole at $(0, 4)$

$f'(x) = \dfrac{4x \cos 2x - 2 \sin 2x}{x^2}$

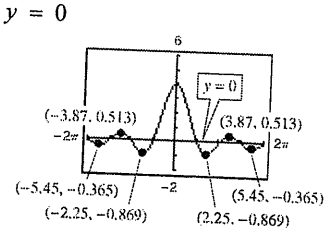
There are an infinite number of relative extrema. In the interval $(-2\pi, 2\pi)$, you obtain the following.

Relative minima: $(\pm 2.25, -0.869), (\pm 5.45, -0.365)$

Relative maxima: $(\pm 3.87, 0.513)$

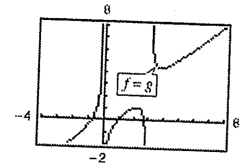
Horizontal asymptote: $y = 0$

No vertical asymptotes



83. $f(x) = \dfrac{x^3 - 3x^2 + 2}{x(x - 3)}, g(x) = x + \dfrac{2}{x(x - 3)}$

(a)

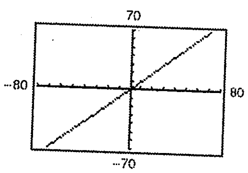


(b) $f(x) = \dfrac{x^3 - 3x^2 + 2}{x(x - 3)}$

$= \dfrac{x^2(x - 3)}{x(x - 3)} + \dfrac{2}{x(x - 3)}$

$= x + \dfrac{2}{x(x - 3)} = g(x)$

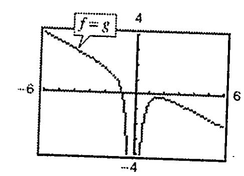
(c)



The graph appears as the slant asymptote $y = x$.

84. $f(x) = -\dfrac{x^3 - 2x^2 + 2}{2x^2}, g(x) = -\dfrac{1}{2}x + 1 - \dfrac{1}{x^2}$

(a)

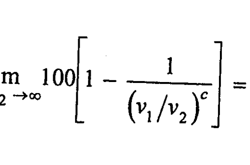


(b) $f(x) = -\dfrac{x^3 - 2x^2 + 2}{2x^2}$

$= -\left[\dfrac{x^3}{2x^2} - \dfrac{2x^2}{2x^2} + \dfrac{2}{2x^2}\right]$

$= -\dfrac{1}{2}x + 1 - \dfrac{1}{x^2} = g(x).$

(c)



The graph appears as the slant asymptote $y = -\dfrac{1}{2}x + 1$.

85. $\displaystyle\lim_{v_1/v_2 \to \infty} 100\left[1 - \frac{1}{\left(v_1/v_2\right)^c}\right] = 100[1 - 0] = 100\%$

86. $C = 0.5x + 500$

$\overline{C} = \dfrac{C}{x}$

$\overline{C} = 0.5 + \dfrac{500}{x}$

$\lim_{x\to\infty}\left(0.5 + \dfrac{500}{x}\right) = 0.5$

87. $\lim_{t\to\infty} N(t) = \infty$

$\lim_{t\to\infty} E(t) = c$

88. (a) $\lim_{t\to 0^+} T = 1700°$

This is the temperature of the kiln.

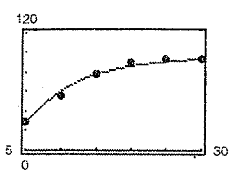
(b) $\lim_{t\to\infty} T = 72°$

This is the temperature of the room.

(c) No. $y = 72$ is the horizontal asymptote.

89. $S = \dfrac{100t^2}{65 + t^2}, t > 0$

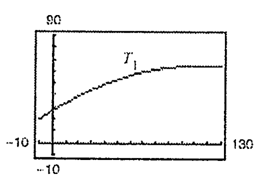
(a)



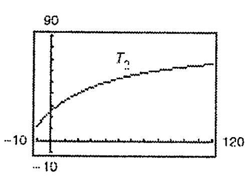
(b) Yes. $\lim_{t\to\infty} S = \dfrac{100}{1} = 100$

90. (a) $T_1(t) = -0.003t^2 + 0.68t + 26.6$

(b)



(c)



$T_2 = \dfrac{1451 + 86t}{58 + t}$

(d) $T_1(0) \approx 26.6°$

$T_2(0) \approx 25.0°$

(e) $\lim_{t\to\infty} T_2 = \dfrac{86}{1} = 86$

(f) No. The limiting temperature is 86°.

T_1 has no horizontal asymptote.

91. $f(x) = \dfrac{2x^2}{x^2 + 2}$

(a) $\lim_{x\to\infty} f(x) = 2 = L$

(b) $f(x_1) + \varepsilon = \dfrac{2x_1^2}{x_1^2 + 2} + \varepsilon = 2$

$$2x_1^2 + \varepsilon x_1^2 + 2\varepsilon = 2x_1^2 + 4$$

$$x_1^2\varepsilon = 4 - 2\varepsilon$$

$$x_1 = \sqrt{\frac{4 - 2\varepsilon}{\varepsilon}}$$

$$x_2 = -x_1 \text{ by symmetry}$$

(c) Let $M = \sqrt{\dfrac{4 - 2\varepsilon}{\varepsilon}} > 0$. For $x > M$:

$$x > \sqrt{\frac{4 - 2\varepsilon}{\varepsilon}}$$

$$x^2\varepsilon > 4 - 2\varepsilon$$

$$2x^2 + x^2\varepsilon + 2\varepsilon > 2x^2 + 4$$

$$\frac{2x^2}{x^2 + 2} + \varepsilon > 2$$

$$\left|\frac{2x^2}{x^2 + 2} - 2\right| > |-\varepsilon| = \varepsilon$$

$$|f(x) - L| > \varepsilon$$

(d) Similarly, $N = -\sqrt{\dfrac{4 - 2\varepsilon}{\varepsilon}}$.

92. $f(x) = \dfrac{6x}{\sqrt{x^2 + 2}}$

(a) $\lim_{x\to\infty} f(x) = 6 = L$

$\lim_{x\to-\infty} f(x) = -6 = K$

(b) $f(x_1) + \varepsilon = \dfrac{6x_1}{\sqrt{x_1^2 + 2}} + \varepsilon = 6$

$$6x_1 = (6 - \varepsilon)\sqrt{x_1^2 + 2}$$

$$36x_1^2 = (x_1^2 + 2)(6 - \varepsilon)^2$$

$$36x_1^2 - (6 - \varepsilon)^2 x_1^2 = 2(6 - \varepsilon)^2$$

$$x_1^2[36 - 36 + 12\varepsilon - \varepsilon^2] = 2(6 - \varepsilon)^2$$

$$x_1^2 = \frac{2(6 - \varepsilon)^2}{12\varepsilon - \varepsilon^2}$$

$$x_1 = (6 - \varepsilon)\sqrt{\frac{2}{12\varepsilon - \varepsilon^2}}$$

$$x_2 = -x_1 \text{ by symmetry}$$

(c) $M = x_1 = (6 - \varepsilon)\sqrt{\dfrac{2}{12\varepsilon - \varepsilon^2}}$

(d) $N = x_2 = (\varepsilon - 6)\sqrt{\dfrac{2}{12\varepsilon - \varepsilon^2}}$