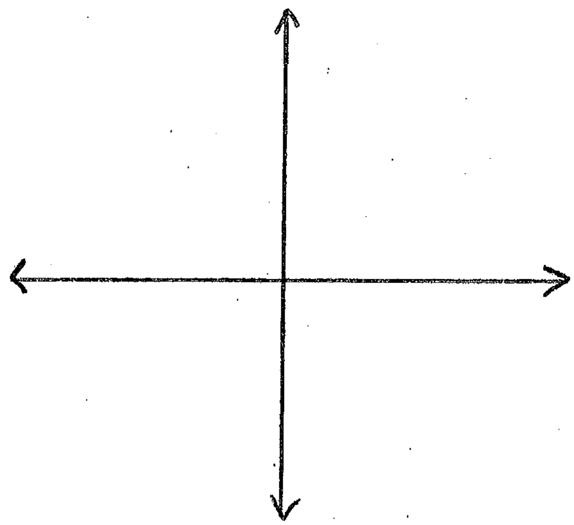
Ch. 3.6 Curve Sketching

1. Sketch the graph of the function and find the below information: $f(x) = -3x^5 + 5x^3$



x-ints: ______________ y-ints: ______________

V.A. ______________ H.A. ______________

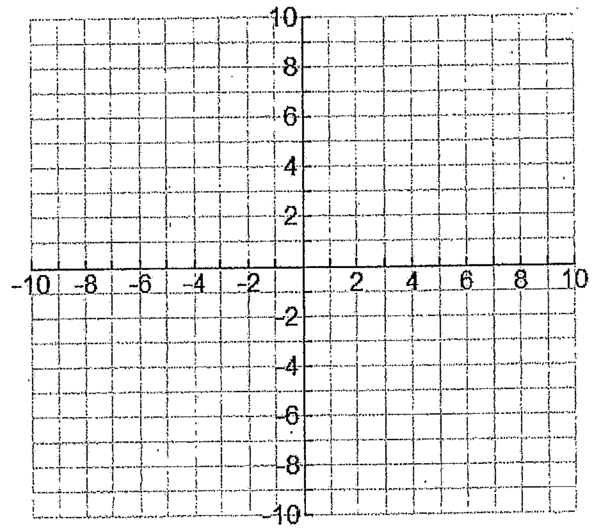
Domain: Interval Increasing

Interval Decreasing Relative Maximum

Relative Minimum: Points of Inflection:

Interval Concave Up: Interval Concave Down:

2. Sketch the graph of the function and find the below information: $f(x) = \frac{2x^2}{9-x^2}$



x-ints: ______________

y-ints: ______________

V.A. ______________

H.A. ______________

Domain:

Interval Increasing

Interval Decreasing

Relative Maximum

Relative Minimum:

Points of Inflection:

Interval Concave Up:

Interval Concave Down:

AP Calculus AB **Chapter 3 Curve Sketching** **Sketching and interpreting Derivative Graphs**

1. Sketching 1st Derivative and 2nd Derivative Graphs (Given the f (x) graph)

1. Given the f(x) graph
2. Make a sign line for f '(x) graph
   a. Label Critical points (relative max, relative min, or where slope = 0) on sign line
   b. Find intervals where graph is increasing (rising) and decreasing (falling)
   c. Use + and ↗ arrow on the sign line to indicate increasing slope
   d. Use – and ↘ arrow on the sign line to indicate decreasing slope
3. Sketch f ' (x) graph
   a. Plot critical points on the graph as x – intercepts (where slope = 0)
   b. Sketch portions of graph above the x-axis (positive slope) or below x-axis (negative slope) using the information on your sign line.
4. Make a sign line for f ''(x) graph
   a. Locate Points of Inflection on your f (x) graph.
      i. This is where graph transitions from concave up to down or from concave down to up.
   b. Label critical point on your sign line
      i. Where graph resembles parabola opening up , use + and ∪ to indicate concave up
      ii. Where graph resembles parabola opening up , use – and ∩ to indicate concave down
5. Sketch f ''(x) graph
   a. Plot critical points on the graph as x – intercepts (POI and where f '' (x) = 0)
   b. Sketch portions of graph above the x-axis (concave up) or below x-axis (concave down) using the information on your sign line.

2. Sketching f(x) graph and 2nd Derivative Graph (Given the f '(x) graph)

1. Given the f '(x) graph
2. Make a sign line for f '(x) graph
   a. Label Critical points **(x-intercepts)** on sign line
   b. Find intervals where graph is increasing(above x-axis) and decreasing(below x-axis)
   c. Use + and ↗ arrow on the sign line to indicate increasing slope
   d. Use – and ↘ arrow on the sign line to indicate decreasing slope
3. Sketch f (x) graph
   a. Follow the directional arrows on your sign line to draw the f(x) graph, along with the relative max (hills) and relative min (valleys) of your graph
4. Make a sign line for f ''(x) graph
   a. Locate critical points (Points of Inflection) on your f '(x) graph
      i. Points of Inflections are the relative max (hills) and relative mins (valleys) of your f '(x) graph

   b. Label critical point on your sign line

      i. Where f '(x) graph is increasing(rising), use + and ∪ to indicate concave up
      ii. Where f '(x) graph is decreasing(falling) , use – and ∩ to indicate concave down

5. Sketch f ''(x) graph

   a. Plot critical points on the graph as x – intercepts (POI and where f '' (x) = 0)
   b. Sketch portions of graph above the x-axis (concave up) or below x-axis (concave down) using the information on your sign line.

3. "Morgan's Method"

X – x-ints
M – max & mins
P – POI

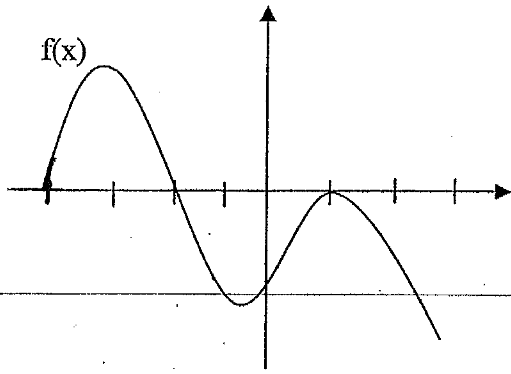
f(x)	f '(x)	f ''(x)
X		
M	X	
P	M	X
	P	M
		P

4

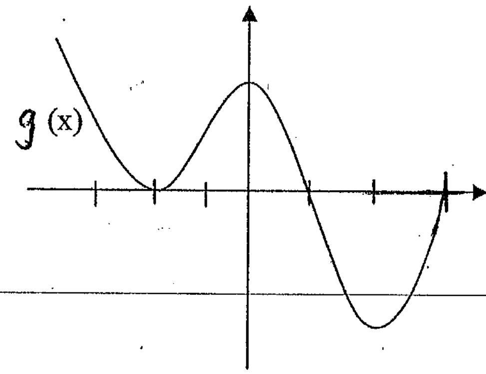
3.6b Interpreting Derivative Graphs

Make a sign line for slope and concavity for each of the following graphs

1)

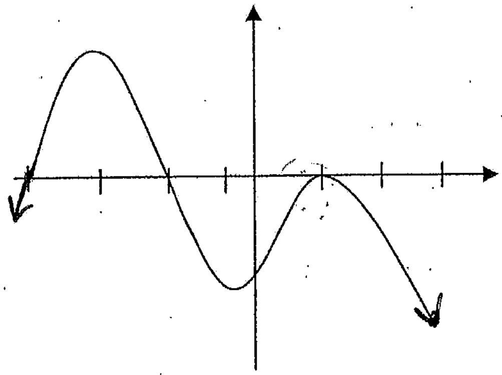


2)



(6)

3. f '(x) graph shown



Sketch f(x) graph :

Sketch f''(x) graph:

Characteristics of f(x)

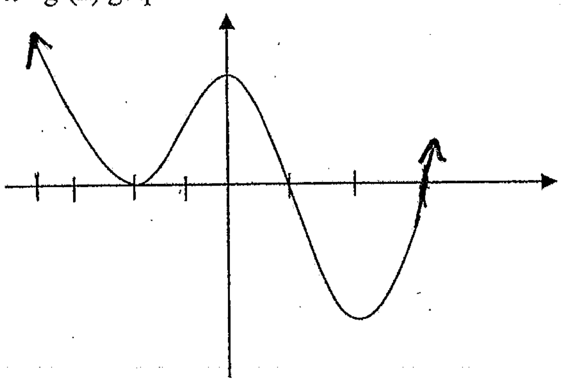
increasing: ______________ decreasing______________

rel. max______________ rel. min______________

Concave up______________ Concave Down______________

POI______________

4. g'(x) graph shown:



Sketch f(x) graph :

Sketch f''(x) graph:

Characteristics of g(x)

increasing: ______________ decreasing______________

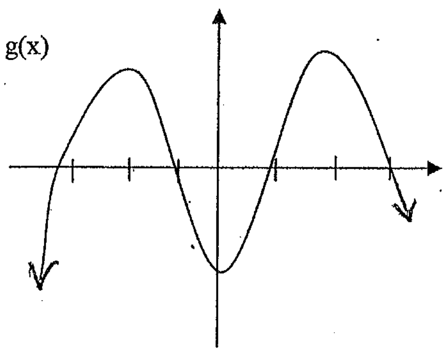
rel. max______________ rel. min______________

Concave up______________ Concave Down______________

POI______________

3.6b Interpreting Derivative Graphs – More Practice
Sketch the derivative graphs for the below f(x)

1)

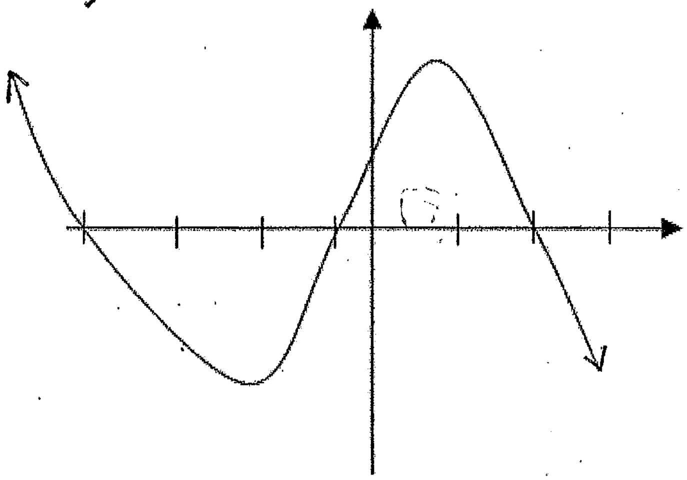


Sketch g '(x) graph:

Sketch g ''(x) graph: (POI at x = -1 and x = 1)

(8)

2) f'(x) graph show



Sketch the f (x) graph:

Sketch the f ''(x) graph:

Characteristics of f(x)

increasing:______________ decreasing________________

rel. max________________ rel. min___________________

Concave up____________ Concave Down____________

POI______________

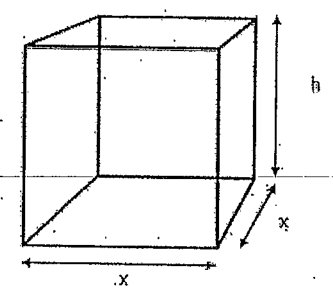
Calculus Optimization Notes

Optimization: Optimization is the process of finding the greatest (maximum optimal solution) or least value of a function (the minimum optimal solution) for some constraint, which must be true regardless of the solution. Optimization finds the most suitable value for a function within a given domain.

Optimization steps:

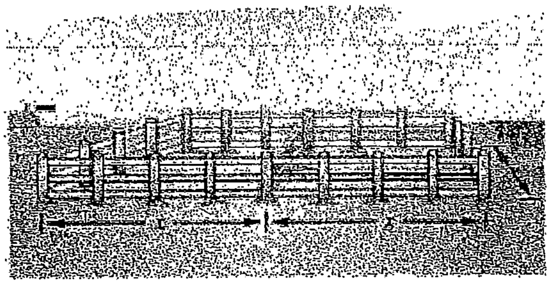
1. Write equation for variable you want to optimize
2. Substitute to get equation in terms of one variable on one side
3. Find derivative, set derivative = 0 and solve.

Example 1: A manufacturer wants to design an open box having a square base and a surface area of 108 in^2. What dimensions will produce a box with maximum volume?



(10)

2) ***Maximum Area*** A rancher has 200 feet of fencing with which to enclose two adjacent rectangular corrals (see figure). What dimensions should be used so that the enclosed area will be a maximum?



Calculus AB **Optimization Practice Problems**

1. A rectangular field is to be fenced off along the bank of a river; no fence is required along the river. If the material for the fence costs \$8 per running foot for the two ends, and \$12 per foot for the side parallel to the river, find the dimensions of the field of largest possible area that can be enclosed with \$3600 worth of fence.

2. A rectangular storage container with an open top is to have a Volume of 10 m^3. The length of its base is twice its length. Material for the base costs \$10/ m^2. Material for the sides cost \$6/m^2. Find the cost of material for the cheapest container. (Hint: Minimize surface area)

3. A piėce of cardboard measures 10 by 15 in. For equal squares are removed from corners of all sides. Find the maximum volume.

4. 1988 multiple choice problem #45
The volume of a cylindrical tin can with a top and a bottom is to be 16π cubic inches. If a minimum amount of tin is to be used to construct the can, what must be the height, in inches, of the can? (A cylinder with radius r and height h has a volume of $V = \pi r^2 h$ and a surface area of $S = 2\pi r^2 + 2\pi rh$.)

(12)

5. Highway 400 averaged 54,000 cars per day for its first 5 years charging \$0.50 per car. A scientific research study concludes that for every \$0.05 increase in the toll, the number of cars will be reduced by 500. In order to maximize revenue, what toll should Highway 400 charge?

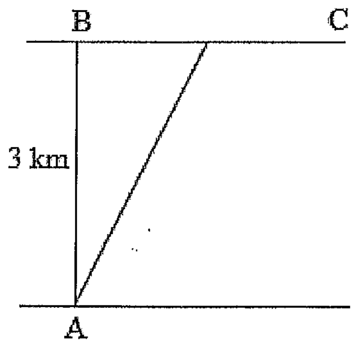
6. The manager of a department store wants to build a 600 square foot rectangular enclosure on the store's parking lot in order to display some equipment. Three sides of the enclosure will be built of redwood fencing, at a cost of \$14 per running foot. The forth side will be built of cement blocks, at a cost of \$28 per running foot. What dimensions will minimize the total cost of the building materials? What will this minimum cost be?

7. A 150-room resort hotel is filled at a room rate of \$125 per day. For each \$5 increase in the room rate, three fewer rooms are rented. What room rate will result in maximum daily revenue? How many rooms will be rented at that rate?

8. A farmer has 400 feet of fencing to make three rectangular pens. What dimensions x and y will maximize the total area?

Optimization Problems WS #2

1. Points A and B are opposite each other on shores of a straight river 3 km wide. Point C is on the same shore as B, but 5 km down the river from B. A telephone company wishes to lay cable from A to C. If the cost per kilometer of the cable is 25% more under the water than it is on land, what line of cable would be least expensive for the company?



2. You are standing at the edge of a slow-moving river which is one mile wide and wish to return to your campground on the opposite side of the river. You can swim at 2 mph and walk at 3 mph. You must first swim across the river to any point on the opposite bank. From there walk to the campground, which is one mile from the point directly across the river from where you start your swim. What route will take the least amount of time? Recall that (time = distance/velocity)

(14)

3. A cylindrical can is to hold 20π m^3 .The material for the top and bottom costs \$10/ m^2 and material for the side costs \$8/m^2 . Find the radius r and height h of the most economical can.

Surface Area = $2\pi r^2 + 2\pi rh$ Volume: $\pi r^2 h$

4. There are 50 apple trees in an orchard. Each tree produces 800 apples. For each additional tree planted in the orchard, the output per tree drops by 10 apples. How many trees should be added to the existing orchard in order to maximize the total output of trees?

Calculus AB FRQ day **Curve Sketching**

1.

1994 AB 1

Let f be the function given by $f(x) = 3x^4 + x^3 - 21x^2$.

a. Write an equation of the line tangent to the graph of f at the point (2, -28)

b. Find the absolute minimum value of f. Show the analysis that leads to your conclusion.

c. Find the x-coordinate of each point of inflection on the graph of f. Show the analysis that leads to your conclusion.

16

2.

1981 AB 3 BC 1

Let f be the function defined by $f(x) = 12x^{(2/3)} - 4x$.

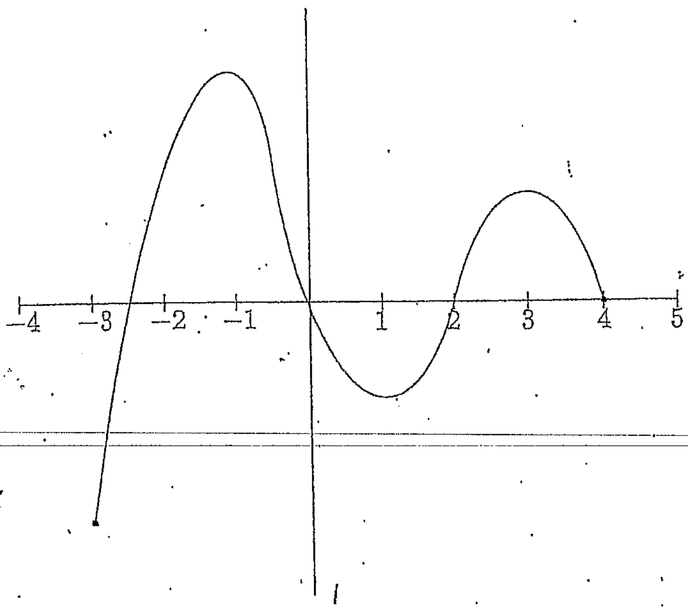
a. Find the intervals on which f is increasing.
b. Find the x- and y-coordinates of all relative maximum points.
c. Find the x- and y-coordinates of all relative minimum points.
d. Find the intervals on which f is concave downward.
e. Using the information found in parts a, b, c, and d, sketch the graph of f on the axes provided.

733. The figure below shows the graph of $g'(x)$, the derivative of a function g, with domain $[-3, 4]$.

a) Determine the values of x for which g has a relative minimum and a relative maximum. Justify your answer.

b) Determine the values of x for which g is concave down and concave up. Justify your answer.

c) Based on the information given and the fact that $g(-3) = 3$ and $g(4) = 6$, sketch a possible graph of g.



(18)

Optimization Review Problem (Involving Cost)

1)

A rectangular storage container with an open top is to have a Volume of 10 m^3. The length of its base is twice its length. Material for the base costs \$10/ m^2. Material for the sides cost \$6/m^2. Find the cost of material for the cheapest container. (Hint: Minimize surface area)

2)

The manager of a department store wants to build a 600 square foot rectangular enclosure on the store's parking lot in order to display some equipment. Three sides of the enclosure will be built of redwood fencing, at a cost of \$14 per running foot. The forth side will be built of cement blocks, at a cost of \$28 per running foot. What dimensions will minimize the total cost of the building materials? What will this minimum cost be?

Derivative Graph Practice Problem #2:

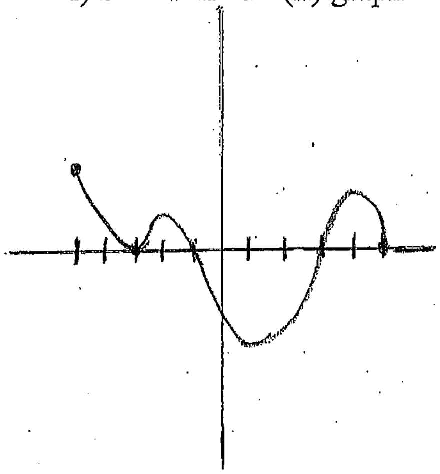
Given the f'(x) graph, find the characteristics of f(x) graph:

a) Relative minimum(s) b) Relative maximum(s) c) interval increasing

d) interval decreasing e) POI f) interval concave up g) interval concave down

h) Sketch f(x) graph given points (-5, -4) and (5, 3). The range is [-7, 5]

i) Sketch the f ''(x) graph



(20)

First Derivative Test, Concavity Test Practice Problem #3:

Given that $f(x) = x^3 - 3x^2 + 3$, find the characteristics of f(x) graph:

a) Relative minimum(s) b) Relative maximum(s) c) interval increasing

d) interval decreasing e) POI f) interval concave up g) interval concave down

h) Sketch f(x) graph