Ch. 3.6 Curve Sketching $0 = x^3(-3x^2+5)$ $x = 0, \pm\sqrt{5/3}$

1. Sketch the graph of the function and find the below information: $f(x) = -3x^5 + 5x^3$

$f'(x) = -15x^4 + 15x^2 = -15x^2(x^2-1)$

$0 = -15x^2(x+1)(x-1)$

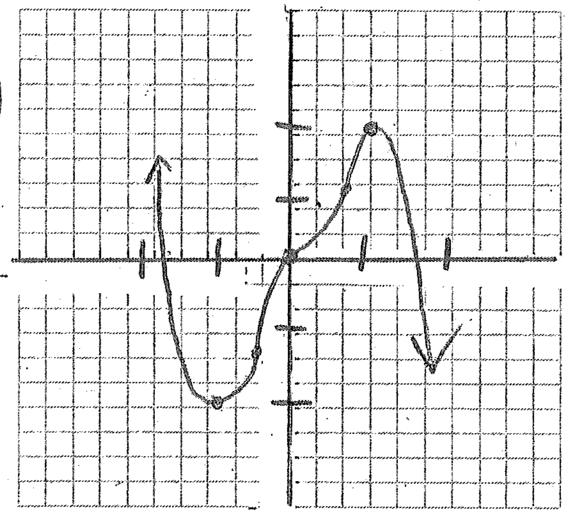
critical values: $x = 0, -1, 1$

$f'(x)$: ↘ − | −1 | ↗ + | 0 | ↗ + | 1 | ↘ −

$f''(x) = -60x^3 + 30x = -30x(2x^2-1)$

critical values: $x = 0, \pm\sqrt{1/2}$

$f''(x)$: ∪ + | $-\frac{\sqrt{2}}{2}$ | ∩ − | 0 | ∪ + | $\frac{\sqrt{2}}{2}$ | ∩ −



$f'<0$, $f''>0$ | $f'>0$, $f''>0$ | $f'>0$, $f''<0$ | $f'<0$, $f''<0$

Rel. min at $(-1, -2)$ b/c $f'(x)$ changes from − to +

Rel. max at $(1, 2)$ b/c $f'(x)$ changes from + to −

$f(x)$ increasing $(-1, 0) \cup (0, 1)$ b/c $f'(x) > 0$

$f(x)$ decreasing $(-\infty, -1) \cup (1, \infty)$ b/c $f'(x) < 0$

$f(x)$ concave up $(-\infty, -\sqrt{2}/2) \cup (0, \sqrt{2}/2)$ b/c $f''(x) > 0$

$f(x)$ concave down $(-\sqrt{2}/2, 0) \cup (\sqrt{2}/2, \infty)$ b/c $f''(x) < 0$

POI at $x = -\sqrt{2}/2, 0, \sqrt{2}/2$ b/c $f''(x)$ change signs.

↘ ∪ | −1 | ↗ ∪ | $-\sqrt{2}/2$ | ↗ ∩ | 0 | ↗ ∪ | $\sqrt{2}/2$ | ↗ ∩ | 1 | ↘ ∩

x-ints: $(0,0)$ $(\sqrt{5/3}, 0)$ $(-\sqrt{5/3}, 0)$ y-ints: $(0,0)$

V.A. none H.A. none

Domain: $(-\infty, \infty)$

Interval Increasing $(-1, 0) \cup (0, 1)$

Interval Decreasing $(-\infty, -1) \cup (1, \infty)$

Relative Maximum $(1, 2)$

Relative Minimum: $(-1, -2)$

Points of Inflection: $(-\sqrt{2}/2, -1.24)$, $(\sqrt{2}/2, 1.24)$, $(0,0)$

Interval Concave Up: $(-\infty, -\sqrt{2}/2) \cup (0, \sqrt{2}/2)$

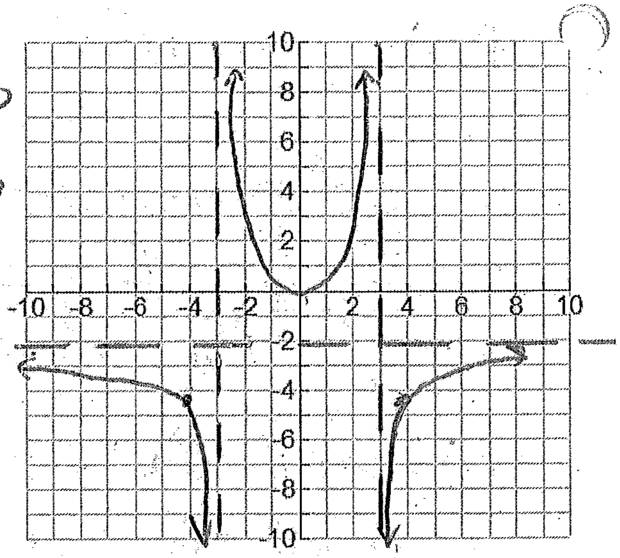
Interval Concave Down: $(-\sqrt{2}/2, 0) \cup (\sqrt{2}/2, \infty)$

2

2. Sketch the graph of the function and find the below information: $f(x) = \dfrac{2x^2}{9-x^2}$

VA: $x = 3, -3$ HA: $y = \frac{2}{-1} = -2$

x-int: $0 = 2x^2$ $\boxed{x=0}$ y-int: $y = \frac{0}{9-0} = 0$



$f'(x) = \dfrac{4x(9-x^2) - (2x^2)(-2x)}{(9-x^2)^2} = \dfrac{36x - 4x^3 + 4x^3}{(9-x^2)^2}$

$\boxed{f'(x) = \dfrac{36x}{(9-x^2)^2}}$

$-$ $-$ $+$ $+$

-3 0 3

critical pts: $x = 0, 3, -3$

$f''(x) = \dfrac{36(9-x^2)^2 - 36x[2(9-x^2)(-2x)]}{(9-x^2)^4}$

$= \dfrac{36\cancel{(9-x^2)}[9-x^2+4x^2]}{(9-x^2)^{\cancel{4}\,3}} = \boxed{\dfrac{36(9+3x^2)}{(9-x^2)^3}}$

critical values: $x = 3, -3$

∩ ∪ ∩

$-$ $+$ $-$

-3 3

∩ ∪ ∪ ∩

-3 0 3

x-ints: $(0,0)$ y-ints: $(0,0)$

V.A. $x=3$, $x=-3$ H.A. $y=-2$

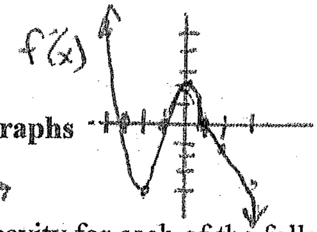
Domain: $(-\infty,-3)\cup(-3,3)\cup(3,\infty)$ Interval Increasing $(0,3)\cup(3,\infty)$

Interval Decreasing $(-\infty,-3)\cup(-3,0)$ Relative Maximum none

Relative Minimum: $(0,0)$ Points of Inflection: none

Interval Concave Up: $(-3,3)$ Interval Concave Down: $(-\infty,-3)\cup(3,\infty)$

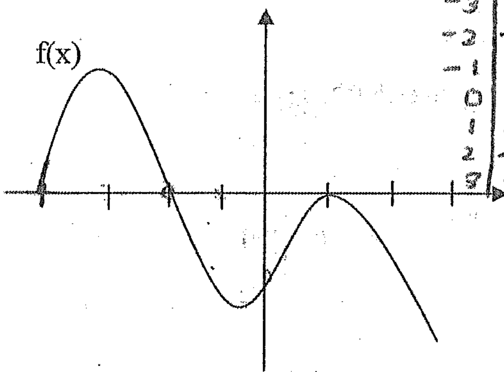
3.6b Interpreting Derivative Graphs

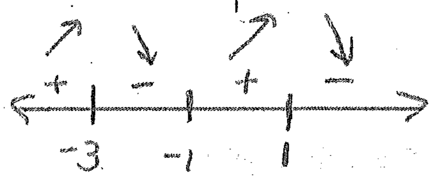


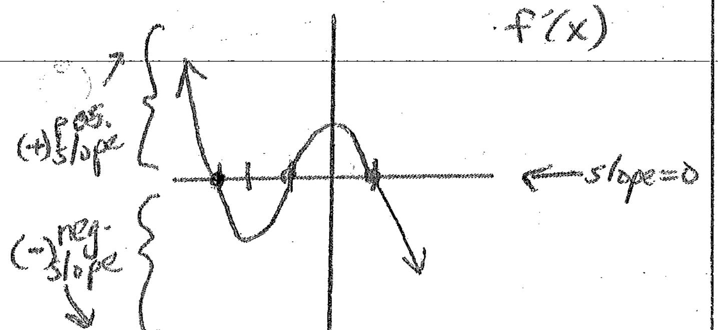
Make a sign line for slope and concavity for each of the following graphs

1)

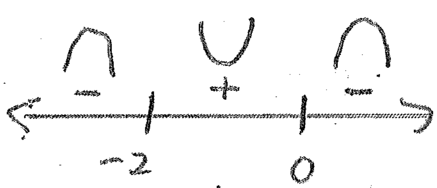
x	f'(x)
-4	5
-3	0
-2	-4
-1	0
0	2
1	0
2	-1
3	-4

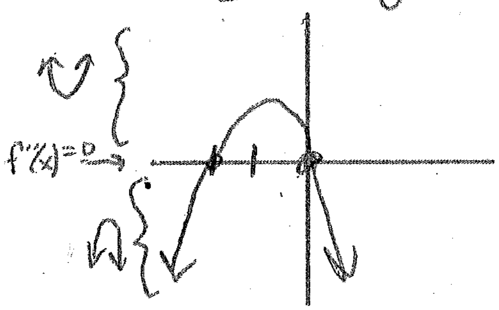




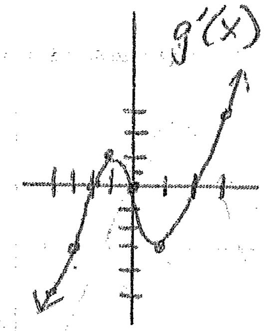


*POI at x=-2, x=0

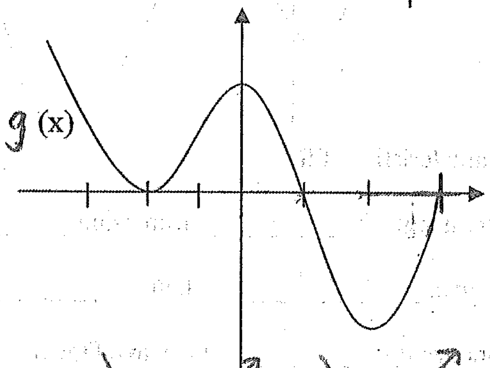


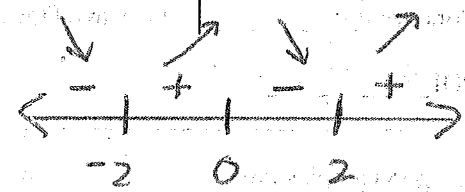


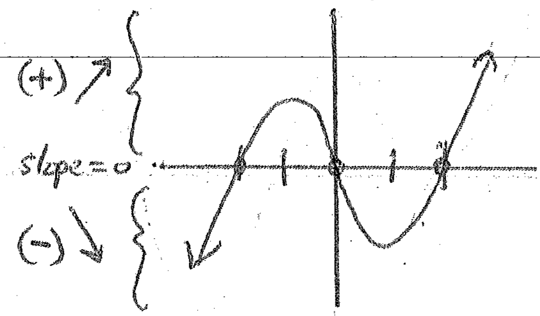
2)



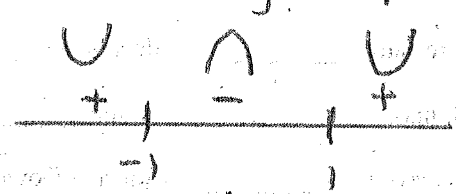
x	f'(x)
-4	-5
-3	-3
-2	0
-1	2
0	0
1	-3
2	0
3	4

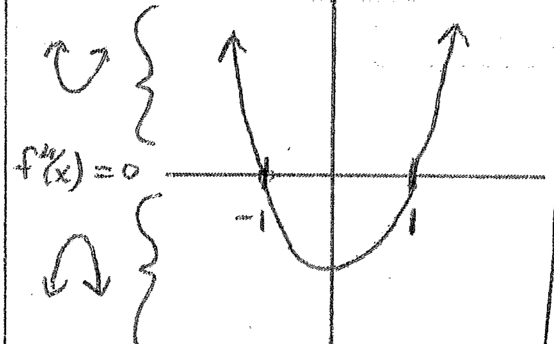






POI: x=-1, x=1





f(x)	f'(x)	f''(x)
X		
M	X	
P	M	X
	P	M
		P

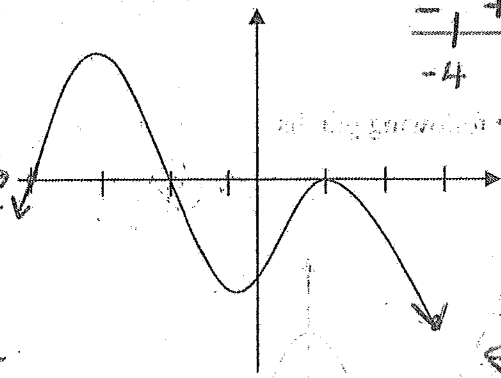
6

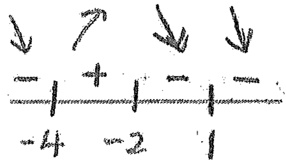
3. f '(x) graph shown

(+) pos. slope

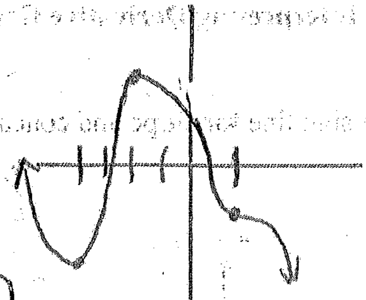
slope = 0

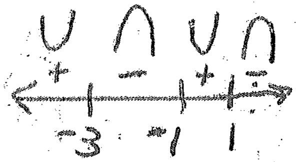
(−) neg. slope



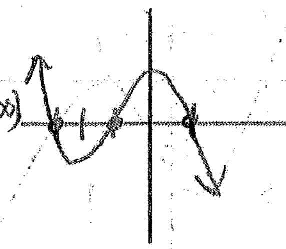


Sketch f(x) graph :





Sketch f''(x) graph:



$f''(x) = 0$

f	f'	f''
X		
M	X	
P	M	X
	P	M
		P

Characteristics of f(x)

increasing: $(-4,-2)$ decreasing $(-\infty,-4) \cup (-2,-1)(1,\infty)$

rel. max $x=-2$ rel. min $x=-4$

Concave up $(-\infty,-3) \cup (-1,1)$ Concave Down $(-3,-1) \cup (1,\infty)$

POI $x = -3, -1, 1$

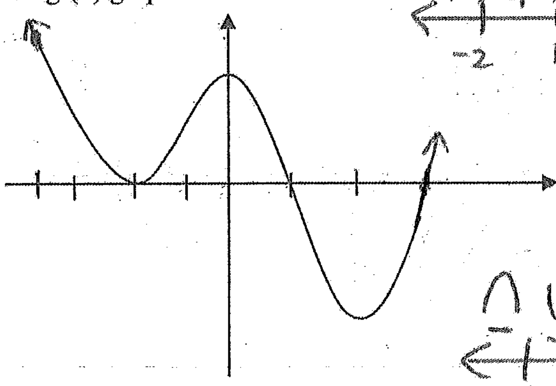
---

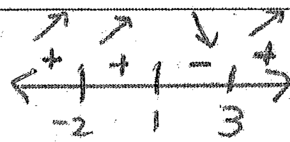
4. g'(x) graph shown:

(+) pos. slope

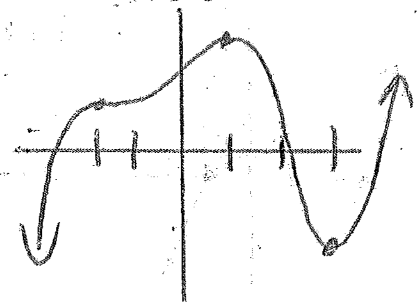
slope = 0

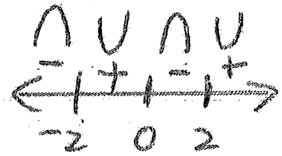
(−) neg. slope



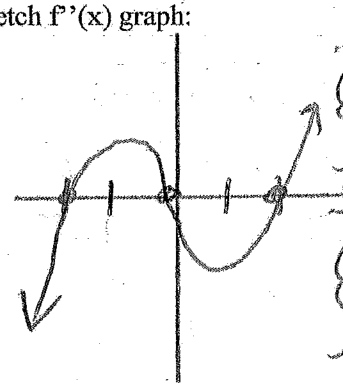


Sketch f(x) graph :





Sketch f''(x) graph:



Characteristics of g(x)

increasing: $(-\infty,-2) \cup (-2,1)$ $(3,\infty)$ decreasing $(1,3)$

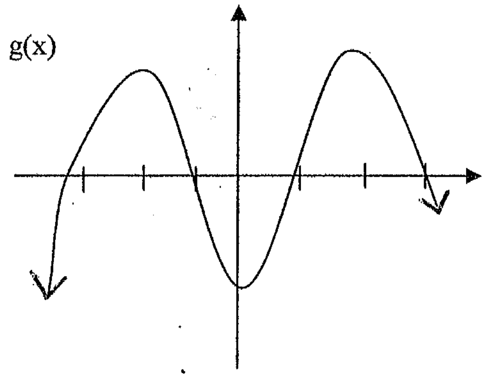
rel. max $x=1$ rel. min $x=3$

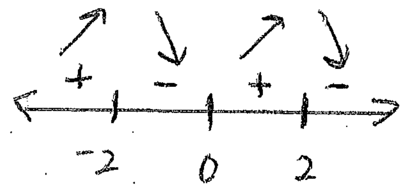
Concave up $(-2,0) \cup (2,\infty)$ Concave Down $(-\infty,-2) \cup (0,2)$

POI $x = -2, 0, 2$

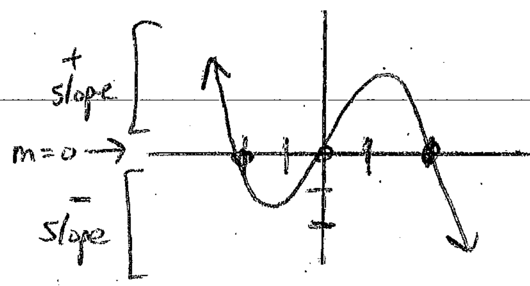
3.6b Interpreting Derivative Graphs – More Practice

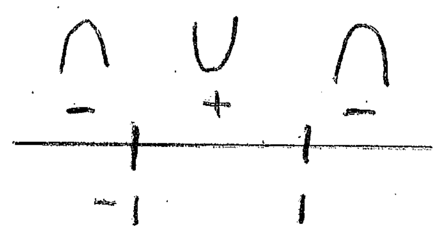
Sketch the derivative graphs for the below f(x)



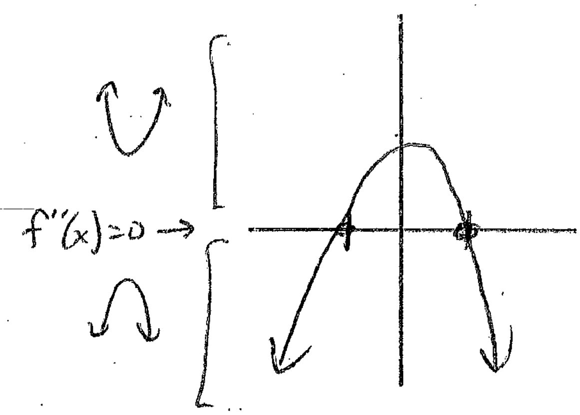


Sketch g '(x) graph:



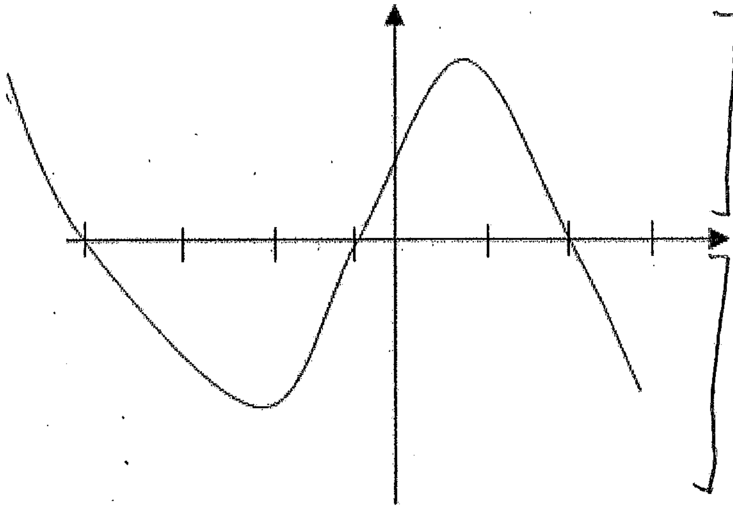


Sketch g ''(x) graph: (POI at x = -1 and x = 1)



8

2. f'(x) graph show



+ slope (↗)

slope = 0

− slope (↘)

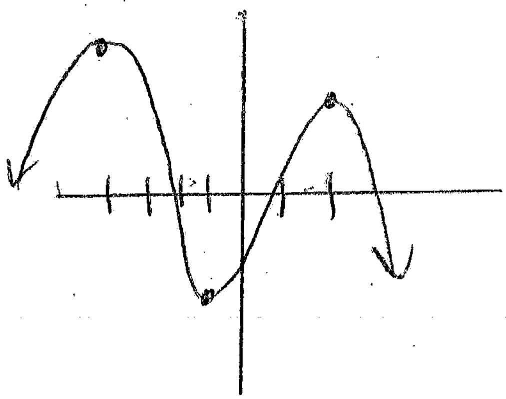
f(x)	f'(x)	f''(x)
X		
M	X	
P	M	X
	P	M
		P

Sketch the f (x) graph:

↗ ↘ ↗ ↘

+ − + −

−4 −1 2



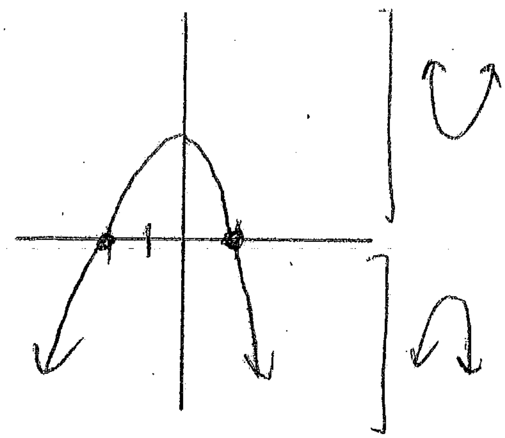
Sketch the f ''(x) graph:

*POI exists at max/min of f'(x) graph

∩ ∪ ∩

− + −

−2 1



Characteristics of f(x)

increasing: $(-\infty,-4)\cup(-1,2)$ decreasing $(-4,-1)\cup(2,\infty)$

rel. max $x=-4, x=2$ rel. min $x=-1$

Concave up $(-2,1)$ Concave Down $(-\infty,-2)\cup(1,\infty)$

POI $x=-2, 1$

Calculus Optimization Notes

Optimization: Optimization is the process of finding the greatest (maximum optimal solution) or least value of a function (the minimum optimal solution) for some constraint, which must be true regardless of the solution. Optimization finds the most suitable value for a function within a given domain.

Optimization steps:

1. Write equation for variable you want to optimize

2. Substitute to get equation in terms of one variable on one side

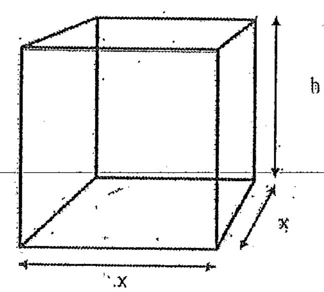
3. Find derivative, set derivative = 0 and solve.

Example 1: A manufacturer wants to design an open box having a square base and a surface area of 108 in^2. What dimensions will produce a box with maximum volume?

optimize volume

$S = 108 \text{ in}^2$ $\quad * V = x^2h$

$S = x^2 + 4xh$



$108 = x^2 + 4xh$

$108 - x^2 = 4xh$

$\frac{108 - x^2}{4x} = h$

$V = x^2\left(\frac{108 - x^2}{4x}\right)$

$V = \frac{108}{4}x - \frac{1}{4}x^3$

$V = 27x - \frac{1}{4}x^3$

$V'(x) = 27 - \frac{3}{4}x^2$

$0 = 27 - \frac{3}{4}x^2$

$\frac{3}{4}x^2 = 27$

$x^2 = 27\left(\frac{4}{3}\right)$

$x^2 = 36$

$x = 6$

$S = x^2 + 4xh$

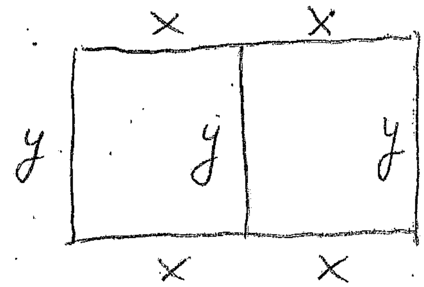
$108 = 6^2 + 4(6)h$

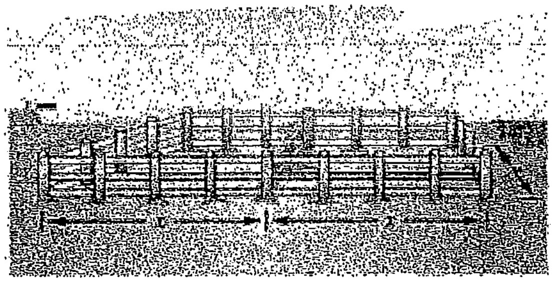
$3 = h$

$\boxed{\text{Dimensions: } 6 \text{ in.} \times 6 \text{ in.} \times 3 \text{ in.}}$

(10)

2) ***Maximum Area*** A rancher has 200 feet of fencing with which to enclose two adjacent rectangular corrals (see figure). What dimensions should be used so that the enclosed area will be a maximum?





(Optimize Area)

$P = 200\text{ft}$

$P = 4x + 3y$

$200 = 4x + 3y$

$\frac{200 - 4x}{3} = y$

$* A = 2xy$

$A = 2x\left(\frac{200-4x}{3}\right)$

$A = \frac{400}{3}x - \frac{8}{3}x^2$

$A'(x) = \frac{400}{3} - \frac{16}{3}x$

$0 = \frac{400}{3} - \frac{16}{3}x$

$\frac{16}{3}x = \frac{400}{3}$

$x = 25\text{ft}$

$P = 4x + 3y$

$200 = 4(25) + 3y$

$200 - 100 = 3y$

$\frac{100}{3} = y$

$\boxed{\text{Dimensions: } 25\text{ ft by } \frac{100}{3}\text{ ft.}}$

$50\text{ ft by } \frac{100}{3}\text{ ft.}$

3.7 Optimization HW p. 223-226 #3, 7, 9, 11, 21c, 23, 27

3) $S = x + y$ $\quad x = S - y$

$* P = xy$ $\quad P = (S-y)y$

(optimize P)

$P = Sy - y^2$

$P'(y) = S - 2y$

$0 = S - 2y$

$\underline{y = \frac{S}{2}}$

$\underline{x = \frac{S}{2}}$

5) $P = 192$ $\quad 192 = xy$

$P = xy$ $\quad \frac{192}{y} = x$

(optimize S) $S = x + y$

$S = \frac{192}{y} + y$

$S = 192y^{-1} + y$

$S'(y) = -192y^{-2} + 1$

$0 = \frac{-192}{y^2} + 1$

$y^2 = 192$

$y = \sqrt{192}$

7) $S = x + 2y$

$S = 100$

$100 = x + 2y$

$\underline{100 - 2y = x}$

(*optimize P)

$P = xy$

$P = (100 - 2y)y$

$P = 100y - 2y^2$

$P'(y) = 100 - 4y$

$0 = 100 - 4y$

$4y = 100$

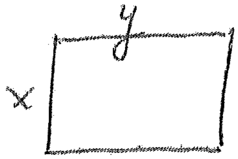
$\underline{y = 25}$

$100 = x + 2(25)$

$\underline{x = 50}$

$P_{max} = (25)(50)$

$= \underline{1250}$

9) 

$P = 100$

$P = 2x + 2y$

$100 = 2x + 2y$

$100 - 2y = 2x$

$\frac{100-2y}{2} = x$

Optimize Area

$A = xy$

$A = \left[\frac{100-2y}{2}\right] y$

$A = 50y - y^2$

$A'(y) = 50 - 2y$

$0 = 50 - 2y$

$2y = 50$

$y = 25$

$100 = 2x + 2y$

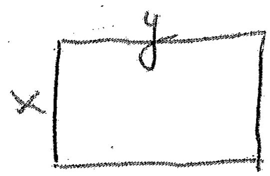
$100 = 2x + 2(25)$

$100 = 2x + 50$

$25 = x$

$A_{max} = (25)(25)$

$= \boxed{625 m^2}$

11) 

(*Minimize P)

$A = xy$

$A = 64$

$64 = xy$

$\frac{64}{x} = y$

$P = 2x + 2y$

$P = 2x + 2\left(\frac{64}{x}\right)$

$P = 2x + 128x^{-1}$

$P'(x) = 2 - 128x^{-2}$

$0 = 2 - \frac{128}{x^2}$

$x^2 = 64$

$x = 8$

$64 = xy$

$64 = 8y$

$8 = y$

$P = 2(8) + 2(8)$

$= \boxed{32 ft.}$

21c) Determine dimension of rectangular solid:

Optimize volume given surface area $= 150\text{ in}^2$.

$* \; V = x^2 h$

$S = 2x^2 + 4xh$

$150 = 2x^2 + 4xh$

$\frac{150 - 2x^2}{4x} = h$

$V = x^2 \cdot \left[\frac{150 - 2x^2}{4x}\right]$

$V = \frac{75}{2}x - \frac{1}{2}x^3$

$V'(x) = \frac{75}{2} - \frac{3}{2}x^2$

$0 = \frac{75}{2} - \frac{3}{2}x^2$

$\frac{75}{2} = \frac{3}{2}x^2$

$\underline{\underline{x = 5}}$

$150 = 2(5)^2 + 4(5)h$

$150 = 50 + 20h$

$\frac{100}{20} = h$

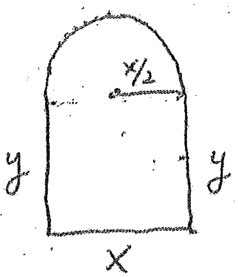
$\underline{\underline{h = 5}}$

$\boxed{V_{max} = 5\text{ in.} \times 5\text{ in.} \times 5\text{ in.}}$

3.7 Optimization

Circumference = distance around a circle

23) $P = 16$ ft. $\quad C = 2\pi r$



$P = x + 2y + \frac{1}{2}(2\pi r)$

$P = x + 2y + \frac{1}{2}(2\pi(\frac{x}{2}))$

$P = x + 2y + \frac{\pi}{2}x$

$(16 = x + 2y + \frac{\pi}{2}x)\,2$

$32 = 2x + 4y + \pi x$

$32 - 2x - \pi x = 4y$

$\frac{32 - 2x - \pi x}{4} = y$

$* \ A = xy + \frac{1}{2}(\pi r^2)$

$A = xy + \frac{1}{2}(\pi(\frac{x}{2})^2)$

$A = xy + \frac{1}{2}\pi(\frac{x^2}{4})$

$A = xy + \frac{\pi}{8}x^2$

$A = x\left(\frac{32 - 2x - \pi x}{4}\right) + \frac{\pi}{8}x^2$

$A = 8x - \frac{x^2}{2} - \frac{\pi}{4}x^2 + \frac{\pi}{8}x^2$

$A = 8x - \frac{x^2}{2} - \frac{\pi}{8}x^2$

$\frac{dA}{dx}$ or $A' = 8 - \frac{2x}{2} - \frac{\pi}{8} \cdot 2x$

$A' = 8 - x - \frac{\pi}{4}x$

$0 = 8 - x - \frac{\pi}{4}x$

$0 = 8 - x(1 + \frac{\pi}{4})$

$-8 = -x(1 + \frac{\pi}{4})$

$\frac{8}{1 + \pi/4} = x$

$x = \frac{8}{1 - \pi/4} = \frac{32}{4 + \pi}$

$+ \quad - \quad \frac{32}{4+\pi}$

$y = \frac{32 - 2x - \pi x}{4}$

$y = \frac{32 - 2\left(\frac{32}{4+\pi}\right) - \pi\left(\frac{32}{4+\pi}\right)}{4} = \boxed{\frac{16}{4+\pi} \text{ ft.}}$

7) $S = x + 2y$

$S = 100$

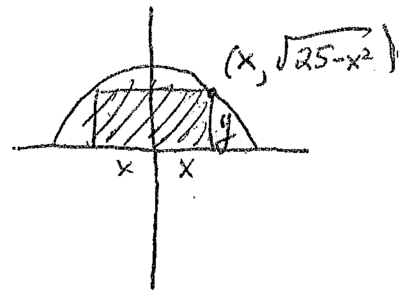
$100 = x + 2y$

$100 - 2y = x$

$* \ P = xy$

$P = (100 - 2y)(y)$

27) $A = 2xy = 2x\sqrt{25-x^2}$



$A = 2x(25-x^2)^{1/2}$

$A'(x) = 2(25-x^2)^{1/2} + 2x \cdot \frac{1}{2}(25-x^2)^{-1/2}(-2x)$

$= 2\sqrt{25-x^2} - \frac{2x^2}{\sqrt{25-x^2}}$

$A'(x) = \frac{2(25-x^2)-2x^2}{\sqrt{25-x^2}} = \frac{50-2x^2-2x^2}{\sqrt{25-x^2}} = \frac{50-4x^2}{\sqrt{25-x^2}} = 0$

$50-4x^2=0$

$4x^2 = 50$

$x^2 = \frac{50}{4}$

$x^2 = \frac{25}{2}$

$x = \pm\sqrt{25/2} = \pm\frac{5}{\sqrt{2}} = \pm\frac{5\sqrt{2}}{2}$

$\frac{5}{\sqrt{2}}$

$y = \sqrt{25-(\frac{25}{2})}$

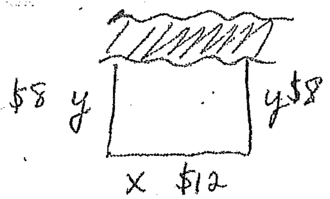
$= \sqrt{\frac{25}{2}}, = \frac{5\sqrt{2}}{2}$

length $= 5\sqrt{2}$
width $= \frac{5\sqrt{2}}{2}$

$\frac{}{}$ (11)

Calculus AB **Optimization Practice Problems** Solution Key

1. A rectangular field is to be fenced off along the bank of a river; no fence is required along the river. If the material for the fence costs \$8 per running foot for the two ends, and \$12 per foot for the side parallel to the river, find the dimensions of the field of largest possible area that can be enclosed with \$3600 worth of fence.



$P = 12x + 16y$

$3600 = 12x + 16y$

$\frac{3600-12x}{16} = y$

(* Optimize Area)

$A = xy$

$A = x\left[\frac{3600-12x}{16}\right]$

$A = 225x - \frac{3}{4}x^2$

$A'(x) = 225 - \frac{6}{4}x$

$0 = 225 - \frac{3}{2}x$

$\frac{3}{2}x = 225$

$x = 150 \text{ ft}$

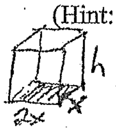
$3600 = 12(150) + 16y$

$1800 = 16y$

$y = 112.5 \text{ ft}$

$\boxed{\text{Dimensions: } 150 \text{ ft by } 112.5 \text{ ft.}}$

2. A rectangular storage container with an open top is to have a Volume of 10 m^3. The length of its base is twice its length. Material for the base costs \$10/ m^2. Material for the sides cost \$6/m^2. Find the cost of material for the cheapest container. (Hint: Minimize surface area)



$S = 2x^2 + 2xh + 2xh + xh + xh$

$S = 2x^2 + 2xh + 4xh$

$S = 2x^2 + 6xh$

Cost $\rightarrow C(x) = (\$10)2x^2 + (\$6)6xh$

$C(x) = 20x^2 + 36xh$

(Minimize Cost)

$V = 2x^2h$

$10 = 2x^2h$

$\frac{10}{2x^2} = h$

$C(x) = 20x^2 + 36x\left(\frac{10}{2x^2}\right)$

$C(x) = 20x^2 + 180x^{-1}$

$C'(x) = 40x - 180x^{-2}$

$0 = 40x - \frac{180}{x^2}$

$\frac{180}{x^2} = 40x$

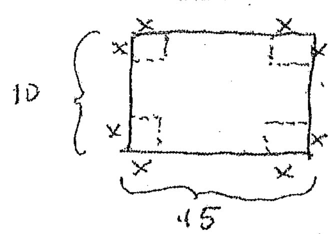
$40x^3 = 180$

$x = \sqrt[3]{9/2}$

$x \approx 1.65 \text{ m}$

$\boxed{C(1.65) = \$163.54}$

3. A piece of cardboard measures 10 by 15 in. For equal squares are removed from corners of all sides. Find the maximum volume.



$V = (x)(10-2x)(15-2x)$

$V = x(150 - 50x + 4x^2)$

$V = 150x - 50x^2 + 4x^3$

$V'(x) = 150 - 100x + 12x^2$

$0 = 2(6x^2 - 50x + 75)$

$\frac{50 \pm \sqrt{50^2 - 4(6)(75)}}{2(6)}$

$x = \cancel{6.3715}$

$x = 1.962$

$\boxed{V(1.962) = 132.038 \text{ in}^3}$

4. 1988 multiple choice problem #45
The volume of a cylindrical tin can with a top and a bottom is to be 16π cubic inches. If a minimum amount of tin is to be used to construct the can, what must be the height, in inches, of the can? (A cylinder with radius r and height h has a volume of $V = \pi r^2 h$ and a surface area of $S = 2\pi r^2 + 2\pi rh$.)

(optimize surface Area)

$V = \pi r^2 h$

$16\pi = \pi r^2 h$

$\frac{16\pi}{\pi r^2} = h$

$\frac{16}{r^2} = h$

$S = 2\pi r^2 + 2\pi r\left(\frac{16}{r^2}\right)$

$S = 2\pi r^2 + \frac{32\pi}{r}$

$S = 2\pi r^2 + 32\pi r^{-1}$

$S' = 4\pi r - 32\pi r^{-2}$

$0 = 4\pi r - \frac{32\pi}{r^2}$

$0 = 4\pi r - \frac{32\pi}{r^2}$

$\frac{32\pi}{r^2} = 4\pi r$

$32\pi = 4\pi r^3$

$2 = r$

$h = \frac{16}{r^2}$

$h = \frac{16}{4} = 4 \text{ in.}$

$h = 4 \text{ in.}$

12

5. Highway 400 averaged 54,000 cars per day for its first 5 years charging \$0.50 per car. A scientific research study concludes that for every \$0.05 increase in the toll, the number of cars will be reduced by 500. In order to maximize revenue, what toll should Highway 400 charge?

*Rate = (Change in revenue)(Change in # of cars

$R(x) = (0.50 + 0.05x)(54000 - 500x)$

$R(x) = 27000 - 250x + 2700x - 25x^2$

$R(x) = -25x^2 + 2450x + 27000$

$R'(x) = -50x + 2450$

$0 = -50x + 2450$

$50x = 2450$

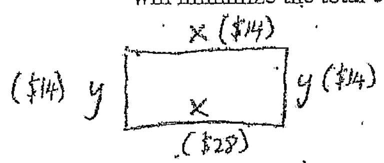
$x = 49$

Toll $= 0.50 + 0.05x$

$= 0.50 + 0.05(49)$

$= \boxed{\$2.95}$

6) The manager of a department store wants to build a 600 square foot rectangular enclosure on the store's parking lot in order to display some equipment. Three sides of the enclosure will be built of redwood fencing, at a cost of \$14 per running foot. The forth side will be built of cement blocks, at a cost of \$28 per running foot. What dimensions will minimize the total cost of the building materials? What will this minimum cost be?



$C(x) = 28x + 14x + 14y + 14y$

$C(x) = 42x + 28y$

$A = xy$

$600 = xy$

$\frac{600}{x} = y$

$C(x) = 42x + 28\left(\frac{600}{x}\right)$

$C(x) = 42x + 16800x^{-1}$

$C'(x) = 42 - 16800x^{-2}$

$0 = 42 - \frac{16800}{x^2}$

$42 = \frac{16800}{x^2}$

$x^2 = \frac{16800}{42}$ $\quad x = 20$

20

$x = 20 \quad y = 30$

$C(x) = 42(20) + 28(30)$

$\boxed{C(x) = \$1680}$

7) A 150-room resort hotel is filled at a room rate of \$125 per day. For each \$5 increase in the room rate, three fewer rooms are rented. What room rate will result in maximum daily revenue? How many rooms will be rented at that rate? Revenue = (room rate)(price rate)

(change in # rooms)

(change in price)

$R(x) = (125 + 5x)(150 - 3x)$

$= -15x^2 + 375x + 18750$

$R'(x) = -30x + 375$

$0 = -30x + 375$

$x = 12.5$

12.5

$R(12) = \$21,090$

$R(13) = \$21,090$

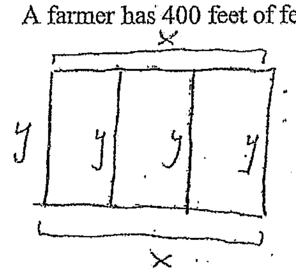
$\boxed{12 \text{ or } 13 \text{ rate increases}}$

Room rate $= 125 + 5x$

$= 125 + 5(12)$

$= \$185$

8. A farmer has 400 feet of fencing to make three rectangular pens. What dimensions x and y will maximize the total area?



$P = 2x + 4y$

$400 = 2x + 4y$

$\frac{400 - 2x}{4} = y$

$A = xy$

$A = \left[\frac{400 - 2x}{4}\right]x$

$A = 100x - \frac{1}{2}x^2$

$A'(x) = 100 - x$

$0 = 100 - x$

$x = 100$

$400 = 2(100) + 4y$

$200 = 4y$

$50 = y$

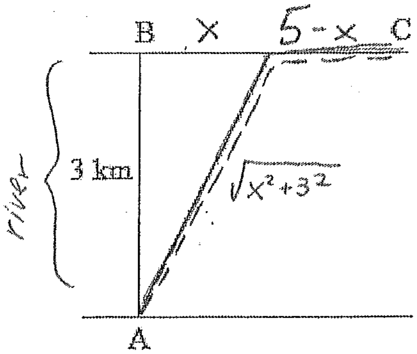
Dimensions: 100 ft by 50 ft

Key

13

Optimization Problems WS #2

1. Points A and B are opposite each other on shores of a straight river 3 km wide. Point C is on the same shore as B, but 5 km down the river from B. A telephone company wishes to lay cable from A to C. If the cost per kilometer of the cable is 25% more under the water than it is on land, what line of cable would be least expensive for the company?



$$C(x) = (1)(5-x) + (1.25)(x^2+9)^{1/2}$$

$$C'(x) = -1 + (1.25)\left(\tfrac{1}{2}\right)(x^2+9)^{-1/2}(2x)$$

$$C'(x) = -1 + \frac{1.25x}{\sqrt{x^2+9}}$$

$$0 = -1 + \frac{1.25x}{\sqrt{x^2+9}}$$

$$1 = \frac{1.25x}{\sqrt{x^2+9}}$$

$$\left(\sqrt{x^2+9}\right)^2 = (1.25x)^2$$

$$x^2+9 = 1.5625x^2$$

$$0.5625x^2 = 9$$

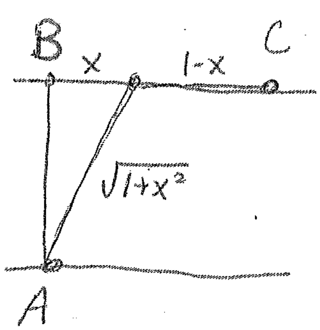
$$x^2 = 16$$

$$x = \pm 4$$

$$\boxed{x = 4}$$

Optimal line of cable would be underwater line from Point A to a point 4 miles down from point B

2. You are standing at the edge of a slow-moving river which is one mile wide and wish to return to your campground on the opposite side of the river. You can swim at 2 mph and walk at 3 mph. You must first swim across the river to any point on the opposite bank. From there walk to the campground, which is one mile from the point directly across the river from where you start your swim. What route will take the least amount of time? Recall that (time = distance/velocity)



$$T(x) = \frac{\sqrt{1+x^2}}{2} + \frac{1-x}{3}$$

$$T(x) = \tfrac{1}{2}(1+x^2)^{1/2} + \tfrac{1}{3} - \tfrac{1}{3}x$$

$$T'(x) = \tfrac{1}{2}\left(\tfrac{1}{2}\right)(1+x^2)^{-1/2}(2x) - \tfrac{1}{3}$$

$$0 = \frac{x}{2\sqrt{1+x^2}} - \frac{1}{3}$$

$$\frac{1}{3} = \frac{x}{2\sqrt{1+x^2}}$$

$$3x = 2\sqrt{1+x^2}$$

$$(3x)^2 = \left(2\sqrt{1+x^2}\right)^2$$

$$9x^2 = 4(1+x^2)$$

$$9x^2 = 4 + 4x^2$$

$$5x^2 = 4$$

$$x^2 = 4/5 \qquad x = \pm\frac{2}{\sqrt{5}}$$

$$\boxed{x = \frac{2}{\sqrt{5}}}$$

(14)

3. A cylindrical can is to hold 20π m^3. The material for the top and bottom costs \$10/ m^2 and material for the side costs \$8/m^2. Find the radius r and height h of the most economical can.

Surface Area = $2\pi r^2 + 2\pi rh$ Volume: $\pi r^2 h$ $\longrightarrow$ $20\pi = \pi r^2 h$ $\frac{20}{r^2} = h$

$10(\pi r^2) + 10(\pi r^2) + 8(2\pi r h)$

$C = 20\pi r^2 + 16\pi r h$

$C = 20\pi r^2 + 16\pi r\left(\frac{20}{r^2}\right)$

$C = 20\pi r^2 + 320\pi r^{-1}$

$C'(r) = 40\pi r - 320\pi r^{-2}$

$0 = 40\pi r - \frac{320\pi}{r^2}$

$40\pi r = \frac{320\pi}{r^2}$

$40\pi r^3 = 320\pi$

$40\pi r^3 - 320\pi = 0$

$40\pi(r^3 - 8) = 0$

$r^3 = 8$

$\boxed{r = 2m}$

$20\pi = \pi r^2 h$

$20\pi = \pi(2)^2 h$

$h = \frac{20\pi}{\pi \cdot 4} = 5$

$\boxed{h = 5m}$

4. There are 50 apple trees in an orchard. Each tree produces 800 apples. For each additional tree planted in the orchard, the output per tree drops by 10 apples. How many trees should be added to the existing orchard in order to maximize the total output of trees?

P = (# of trees)(apple output per tree)

$P = (50 + x)(800 - 10x)$

$P = 40{,}000 + 300x - 10x^2$

$P'(x) = 300 - 2x$

$0 = 20(15 - x)$

15 trees added

$x = 15$, 65 total trees

1.

1994 AB 1

Let f be the function given by $f(x) = 3x^4 + x^3 - 21x^2$.

a. Write an equation of the line tangent to the graph of f at the point (2, -28)

b. Find the absolute minimum value of f. Show the analysis that leads to your conclusion.

c. Find the x-coordinate of each point of inflection on the graph of f. Show the analysis that leads to your conclusion.

a) $f'(x) = 12x^3 + 3x^2 - 42x$

$f'(2) = 12(2)^3 + 3(2)^2 - 42(2) = 24$

point: $(2, -28)$

slope: $m = 24$

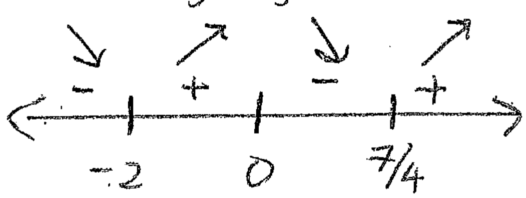
$\boxed{y + 28 = 24(x-2)}$

b) $f'(x) = 12x^3 + 3x^2 - 42x$

$0 = 3x(4x^2 + x - 14)$

$0 = 3x(4x-7)(x+2)$

$x = 0, 7/4, -2$



Abs. min must be either at $x = -2$ or $x = 7/4$ since $f'(x) < 0$ for all $x < -2$ and $f'(x) > 0$ for all $x > 7/4$

$f(-2) = -44$

$f(7/4) = -30.816$

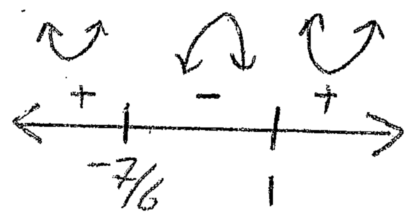
$\boxed{\text{Abs. min is } \underline{-44} \text{ at } x = -2}$

c) $f''(x) = 36x^2 + 6x - 42$

$= 6(6x^2 + x - 7)$

$0 = 6(6x+7)(x-1)$

$x = -7/6, x = 1$



$\boxed{\text{POI at } x = -7/6,\ x = 1 \text{ b/c } f''(x) \text{ change signs}}$

(16)

2.

1981 AB 3 BC 1

Let f be the function defined by f(x) = 12x $^{(2/3)}$ -4x.

a. Find the intervals on which f is increasing.
b. Find the x- and y-coordinates of all relative maximum points.
c. Find the x- and y-coordinates of all relative minimum points.
d. Find the intervals on which f is concave downward.
e. Using the information found in parts a, b, c, and d, sketch the graph of f on the axes provided.

a) $f'(x) = 12 \cdot \frac{2}{3} x^{-1/3} - 4$

$f'(x) = \frac{8}{x^{1/3}} - 4 = \frac{8 - 4x^{1/3}}{x^{1/3}}$

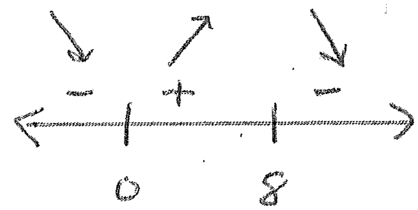
$8 - 4x^{1/3} = 0$ | $x^{1/3} = 0$

$\boxed{x = 0}$

$4x^{1/3} = 8$

$x^{1/3} = 2$

$\boxed{x = 8}$



a) f(x) increasing (0, 8] b/c $f'(x) > 0$

b) Rel. max at (8, 16) b/c $f'(x)$ changes from + to −

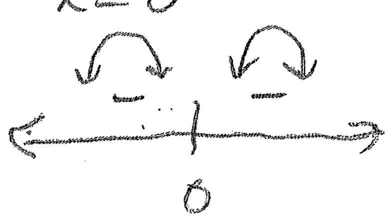
c) Rel. min at (0, 0) b/c $f'(x)$ changes from − to +

$f'(x) = 8x^{-1/3} - 4$

d) $f''(x) = 8 \cdot -\frac{1}{3} x^{-4/3} + 0$

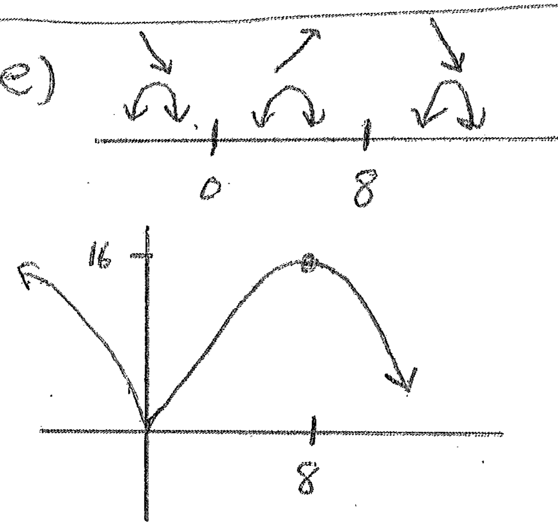
$0 = \frac{-8}{3x^{4/3}}$

$x = 0$



f(x) concave down $(-\infty, 0) \cup (0, \infty)$ b/c $f''(x) < 0$

e)



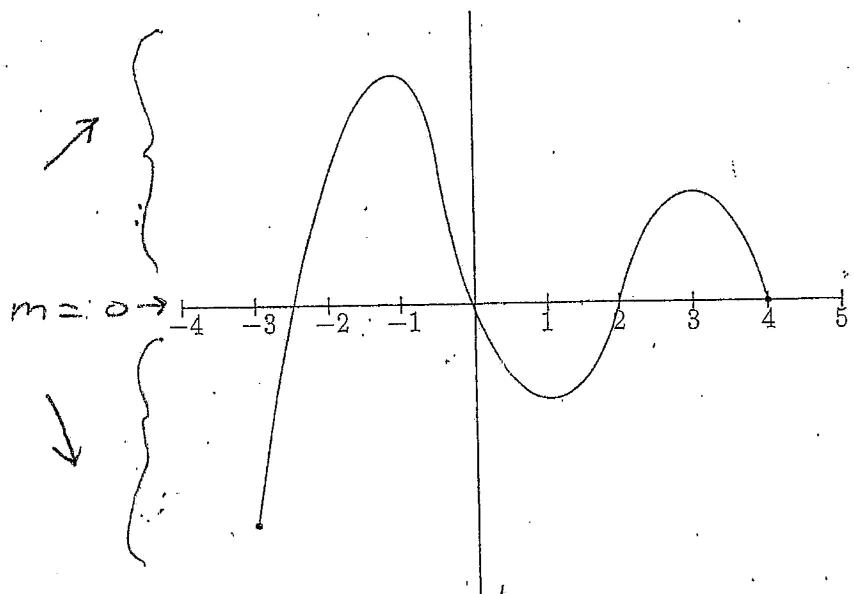
17

733. The figure below shows the graph of $g'(x)$, the derivative of a function g, with domain $[-3, 4]$.

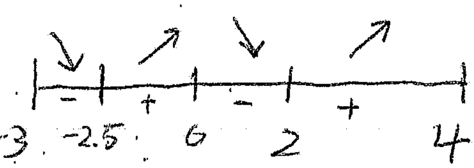
a). Determine the values of x for which g has a relative minimum and a relative maximum. Justify your answer.

b) Determine the values of x for which g is concave down and concave up. Justify your answer.

c). Based on the information given and the fact that $g(-3) = 3$ and $g(4) = 6$, sketch a possible graph of g.



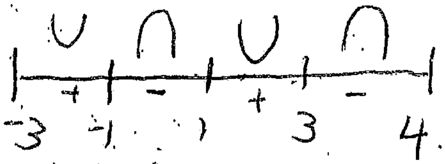
a)



Rel. min at $x = -2.5$, $x = 2$ b/c $f'(x)$ changes sign from − to +

Rel. max at $x = 0$ b/c $f'(x)$ changes sign from + to −

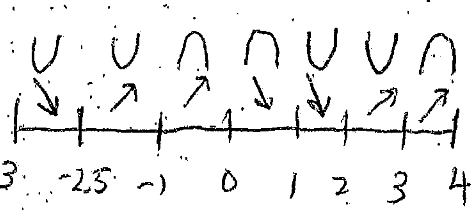
b)

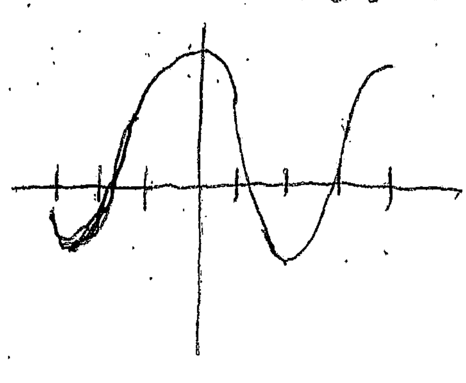


$f(x)$ is concave up $(-3, -1) \cup (1, 3)$ b/c $f''(x) > 0$

$f(x)$ is concave down $(-1, 1) \cup (3, 4)$ b/c $f''(x) < 0$

c)





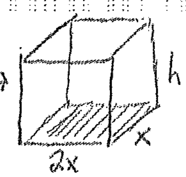
Key (18)

Optimization Review Problem (Involving Cost)

1)

A rectangular storage container with an open top is to have a Volume of 10 m^3. The length of its base is twice its length. Material for the base costs \$10/ m^2. Material for the sides cost \$6/m^2. Find the cost of material for the cheapest container. (Hint: Minimize surface area)

sides \$6



base \$10

$S = 2x^2 + xh + xh + 2xh + 2xh$

$S = 2x^2 + 6xh$

$Cost = 10(2x^2) + 6(6xh)$

$* Cost = 20x^2 + 36xh$

$Volume = (2x)(x)(h)$

$10 = 2x^2h$

$\frac{10}{2x^2} = h$

$\boxed{\frac{5}{x^2} = h}$

$Cost = 20x^2 + 36x\left(\frac{5}{x^2}\right)$

$C(x) = 20x^2 + \frac{180}{x}$

$C(x) = 20x^2 + 180x^{-1}$

$C'(x) = 40x - 180x^{-2}$

$0 = 40x - \frac{180}{x^2}$

$\frac{180}{x^2} = \frac{40x}{1}$

$40x^3 = 180$

$x^3 = \frac{180}{40} = \frac{9}{2}$

$x^3 = 9/2$

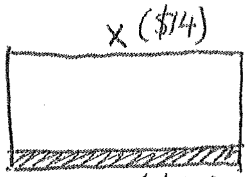
$x = \sqrt[3]{9/2} \approx 1.651$ meters

$C(1.651) = 20(1.651)^2 + \frac{180}{1.651}$

$= \boxed{\$163.54}$

2)

The manager of a department store wants to build a 600 square foot rectangular enclosure on the store's parking lot in order to display some equipment. Three sides of the enclosure will be built of redwood fencing, at a cost of \$14 per running foot. The forth side will be built of cement blocks, at a cost of \$28 per running foot. What dimensions will minimize the total cost of the building materials? What will this minimum cost be? (perimeter)



Add up the costs associated with each side

$* Cost = 28x + 14x + 14y + 14y$

$C = 42x + 28y$

$C(x) = 42x + 28\left(\frac{600}{x}\right)$

$C(x) = 42x + 16800x^{-1}$

$Area = xy$

$600 = xy$

$\boxed{\frac{600}{x} = y}$

$C'(x) = 42 - 16800x^{-2}$

$0 = 42 - \frac{16800}{x^2}$

$\frac{16800}{x^2} = \frac{42}{1}$

$42x^2 = 16800$

$x^2 = \frac{16800}{42}$

$x^2 = 400$

$x = 20$ ft

$y = \frac{600}{x} \rightarrow \frac{600}{20} = 30$ ft

$Cost = 42(20) + 28(30)$

$\boxed{Cost = \$1680}$

/ Key (19)

Derivative Graph Practice Problem #2:

Given the f'(x) graph, find the characteristics of f(x) graph:

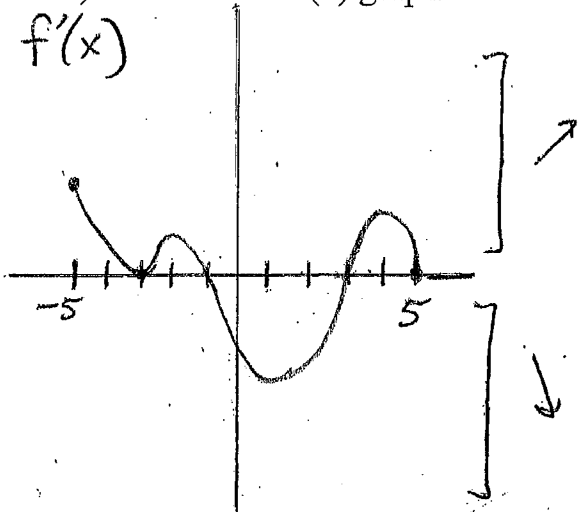
a) Relative minimum(s) b) Relative maximum(s) c) interval increasing

d) interval decreasing e) POI f) interval concave up g) interval concave down

h) Sketch f(x) graph given points (-5, -4) and (5, 3). The range is [-7, 5]

i) Sketch the f ''(x) graph

f'(x)



f'(x) ↗ ↗ ↘ ↗

\+ + − +

-5 -3 -1 3 5

a) Rel. min at x = 3 b/c f'(x) changes from − to +

b) Rel. max at x = −1 b/c f'(x) changes from + to −

c) f(x) increasing (-5, -3), (-3, -1), (3, 5) b/c f'(x) > 0

d) f(x) decreasing (-1, 3) b/c f'(x) < 0

f''(x) ∩ ∪ ∩ ∪ ∩

− + − + −

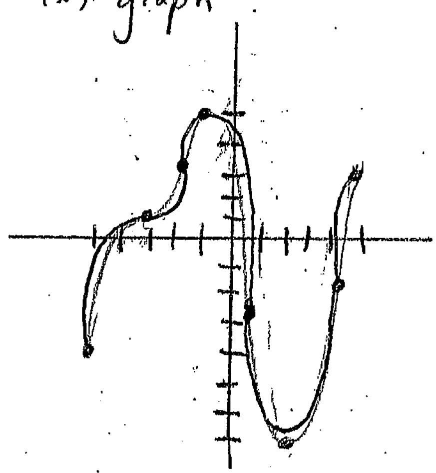
-5 -3 -2 1 4 5

e) POI at x = -3, -2, 1, 4 b/c f''(x) change signs

f) Concave up (-3, -2), (1, 4) b/c f''(x) > 0

g) concave down (-5, -3), (-2, 1), (4, 5) b/c f''(x) < 0

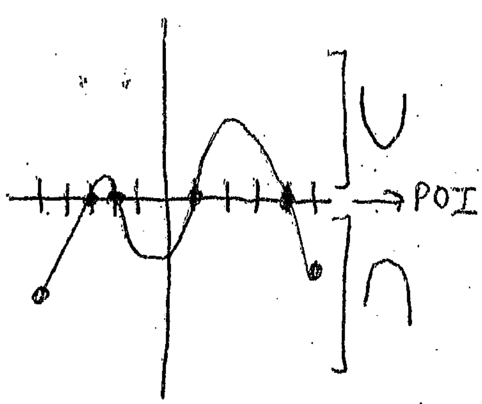
h) f(x) graph



i) f''(x) graph

∩ ∪ ∩ ∪ ∩

-5 -3 -2 1 4 5



20/

First Derivative Test, Concavity Test Practice Problem #3:

Given that $f(x) = x^3 - 3x^2 + 3$, find the characteristics of f(x) graph:

a) Relative minimum(s) b) Relative maximum(s) c) interval increasing

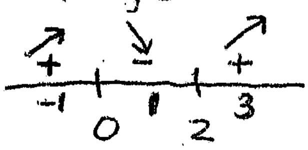
d) interval decreasing e) POI f) interval concave up g) interval concave down

h) Sketch f(x) graph

$f'(x) = 3x^2 - 6x$

$0 = 3x(x-2)$

$x = 0, 2$



a) Rel. min $(2, -1)$ b/c $f'(x)$ changes from − to +

b) Rel. max $(0, 3)$ b/c $f'(x)$ changes from + to −

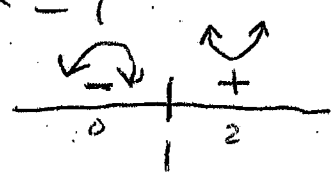
c) $f(x)$ increasing $(-\infty, 0), (2, \infty)$ b/c $f'(x) > 0$

d) $f(x)$ decreasing $(0, 2)$ b/c $f'(x) < 0$

$f''(x) = 6x - 6$

$0 = 6(x-1)$

$x = 1$



e) POI at $(1, 1)$ b/c $f''(x)$ change signs

f) concave up $(1, \infty)$ b/c $f''(x) > 0$

g) concave down $(-\infty, 1)$ b/c $f''(x) < 0$

h) $f(x)$