

AB Calculus Ch. 3.6 Select HW Problems Key

Analyzing the Graph of a Function In Exercises 5-24, analyze and sketch a graph of the function. Label any intercepts, relative extrema, points of inflection, and asymptotes. Use a graphing utility to verify your results.

15. $y = x\sqrt{4-x}$ Domain: $(-\infty, 4]$

$x\text{-int:}$
 $(0, 0)$
 $(4, 0)$

$$y = x(4-x)^{1/2}$$

$$y' = 1 \cdot (4-x)^{1/2} + x \cdot \frac{1}{2}(4-x)^{-1/2}(-1)$$

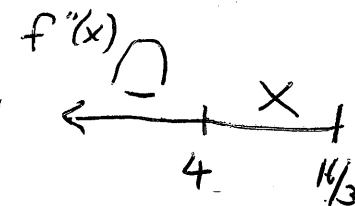
$$y' = \sqrt{4-x} - \frac{x}{2\sqrt{4-x}} = \frac{2(4-x)-x}{2\sqrt{4-x}} = \frac{8-3x}{2\sqrt{4-x}}, x = \frac{8}{3}$$

$$\begin{array}{c} \nearrow \\ + \\ \swarrow \\ - \\ \leftarrow \quad \rightarrow \\ \frac{8}{3} \quad 4 \end{array}$$

$$y'' = \frac{-3 \cdot 2\sqrt{4-x} - (8-3x) \cdot 2 \cdot \frac{1}{2}(4-x)^{-1/2}(-1)}{(2\sqrt{4-x})^2}$$

$$y'' = \frac{-6\sqrt{4-x} - \frac{8-3x}{\sqrt{4-x}}}{4(4-x)} \cdot \frac{\sqrt{4-x}}{\sqrt{4-x}} = \frac{-24+6x+8-3x}{4(4-x)^{3/2}} = \frac{-6(4-x)+8-3x}{4(4-x)^{3/2}} = \frac{3x-16}{4(4-x)^{3/2}}, x = \frac{16}{3}$$

not in domain

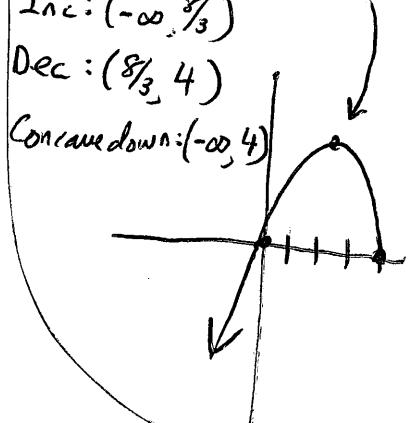


Rel. max at $(\frac{8}{3}, \frac{16\sqrt{3}}{9})$

Inc: $(-\infty, \frac{8}{3})$

Dec: $(\frac{8}{3}, 4)$

Concave down: $(-\infty, 4)$



23. $y = x^5 - 5x$ $x\text{-ints: } (-\sqrt[4]{5}, 0), (\sqrt[4]{5}, 0)$

$$y' = 5x^4 - 5$$

$$y' = 5(x^4 - 1)$$

$$0 = 5(x^2+1)(x-1)(x+1)$$

$$x = 1, -1$$

$$\begin{array}{c} \nearrow \\ + \\ \swarrow \\ - \\ \leftarrow \quad \rightarrow \\ -1 \quad 1 \end{array}$$

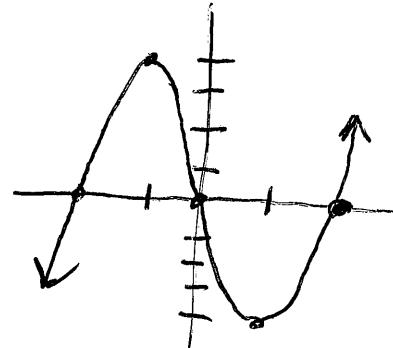
$$y'' = 20x^3 - 0$$

$$0 = 20x^3$$

$$x = 0$$

$$\begin{array}{c} \cap \\ + \\ \cup \\ 0 \end{array}$$

$$\text{POI: } (0, 0)$$



Concave up $(0, \infty)$

Concave down $(-\infty, 0)$

Rel. max $(-1, \frac{4}{5})$ b/c $f'(x)$ changes from + to -
 Rel. min $(1, -\frac{4}{5})$ b/c $f'(x)$ changes from - to +

Inc: $(-\infty, -1) \cup (1, \infty)$ Dec: $(-1, 1)$

AB Calculus Ch. 3.6 Select HW Problems

Analyzing the Graph of a Function In Exercises 5–24, analyze and sketch a graph of the function. Label any intercepts, relative extrema, points of inflection, and asymptotes. Use a graphing utility to verify your results.

15. $y = x\sqrt{4 - x}$

23. $y = x^5 - 5x$