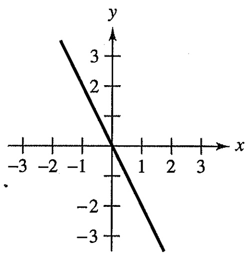
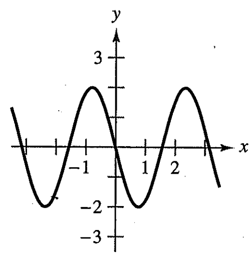
3.6 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Matching **In Exercises 1–4, match the graph of f in the left column with that of its derivative in the right column.**

Graph of f | **Graph of f'**

1.


(a)


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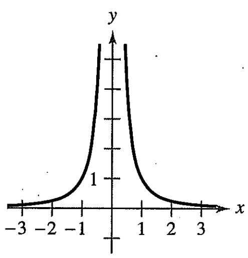

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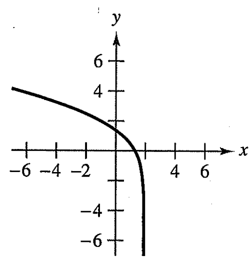
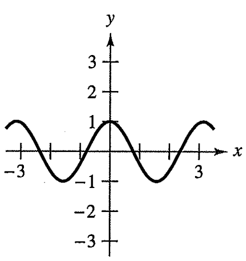
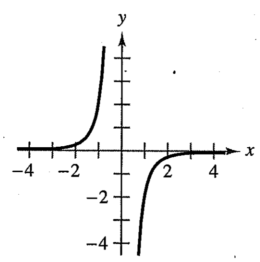
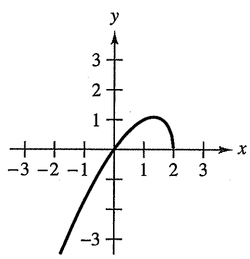
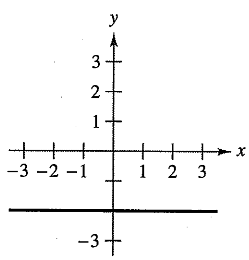


3.


(c)


4.


(d)


Analyzing the Graph of a Function **In Exercises 5–24, analyze and sketch a graph of the function. Label any intercepts, relative extrema, points of inflection, and asymptotes. Use a graphing utility to verify your results.**

5. $y = \dfrac{1}{x - 2} - 3$

6. $y = \dfrac{x}{x^2 + 1}$

7. $y = \dfrac{x^2}{x^2 + 3}$

8. $y = \dfrac{x^2 + 1}{x^2 - 4}$

9. $y = \dfrac{3x}{x^2 - 1}$

10. $f(x) = \dfrac{x - 3}{x}$

11. $f(x) = x + \dfrac{32}{x^2}$

12. $f(x) = \dfrac{x^3}{x^2 - 9}$

13. $y = \dfrac{x^2 - 6x + 12}{x - 4}$

14. $y = \dfrac{-x^2 - 4x - 7}{x + 3}$

15. $y = x\sqrt{4 - x}$

16. $g(x) = x\sqrt{9 - x^2}$

17. $y = 3x^{2/3} - 2x$

18. $y = (x + 1)^2 - 3(x + 1)^{2/3}$

19. $y = 2 - x - x^3$

20. $y = -\frac{1}{3}(x^3 - 3x + 2)$

21. $y = 3x^4 + 4x^3$

22. $y = -2x^4 + 3x^2$

23. $y = x^5 - 5x$

24. $y = (x - 1)^5$

Analyzing the Graph of a Function Using Technology **In Exercises 25–34, use a computer algebra system to analyze and graph the function. Identify any relative extrema, points of inflection, and asymptotes.**

25. $f(x) = \dfrac{20x}{x^2 + 1} - \dfrac{1}{x}$

26. $f(x) = x + \dfrac{4}{x^2 + 1}$

27. $f(x) = \dfrac{-2x}{\sqrt{x^2 + 7}}$

28. $f(x) = \dfrac{4x}{\sqrt{x^2 + 15}}$

29. $f(x) = 2x - 4\sin x, \quad 0 \le x \le 2\pi$

30. $f(x) = -x + 2\cos x, \quad 0 \le x \le 2\pi$

31. $y = \cos x - \frac{1}{4}\cos 2x, \quad 0 \le x \le 2\pi$

32. $y = 2x - \tan x, \quad -\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$

33. $y = 2(\csc x + \sec x), \quad 0 < x < \dfrac{\pi}{2}$

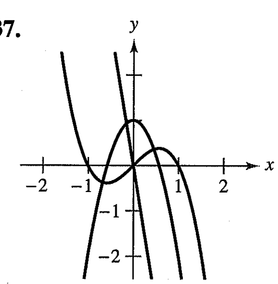
34. $g(x) = x\cot x, \quad -2\pi < x < 2\pi$

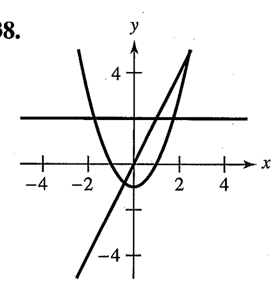
WRITING ABOUT CONCEPTS

35. Using a Derivative Let $f'(t) < 0$ for all t in the interval $(2, 8)$. Explain why $f(3) > f(5)$.

36. Using a Derivative Let $f(0) = 3$ and $2 \le f'(x) \le 4$ for all x in the interval $[-5, 5]$. Determine the greatest and least possible values of $f(2)$.

Identifying Graphs **In Exercises 37 and 38, the graphs of f, f', and f'' are shown on the same set of coordinate axes. Which is which? Explain your reasoning. To print an enlarged copy of the graph, go to *MathGraphs.com*.**

37.


38.


WRITING ABOUT CONCEPTS (continued)

Horizontal and Vertical Asymptotes **In Exercises 39–42, use a graphing utility to graph the function. Use the graph to determine whether it is possible for the graph of a function to cross its horizontal asymptote. Do you think it is possible for the graph of a function to cross its vertical asymptote? Why or why not?**

39. $f(x) = \dfrac{4(x-1)^2}{x^2-4x+5}$

40. $g(x) = \dfrac{3x^4-5x+3}{x^4+1}$

41. $h(x) = \dfrac{\sin 2x}{x}$

42. $f(x) = \dfrac{\cos 3x}{4x}$

Examining a Function **In Exercises 43 and 44, use a graphing utility to graph the function. Explain why there is no vertical asymptote when a superficial examination of the function may indicate that there should be one.**

43. $h(x) = \dfrac{6-2x}{3-x}$

44. $g(x) = \dfrac{x^2+x-2}{x-1}$

Slant Asymptote **In Exercises 45–48, use a graphing utility to graph the function and determine the slant asymptote of the graph. Zoom out repeatedly and describe how the graph on the display appears to change. Why does this occur?**

45. $f(x) = -\dfrac{x^2-3x-1}{x-2}$

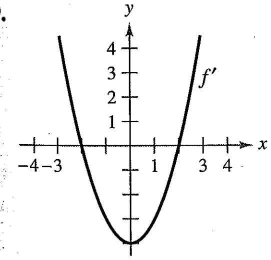
46. $g(x) = \dfrac{2x^2-8x-15}{x-5}$

47. $f(x) = \dfrac{2x^3}{x^2+1}$

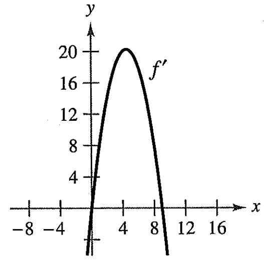
48. $h(x) = \dfrac{-x^3+x^2+4}{x^2}$

Graphical Reasoning **In Exercises 49–52, use the graph of f' to sketch a graph of f and the graph of f''. To print an enlarged copy of the graph, go to *MathGraphs.com*.**

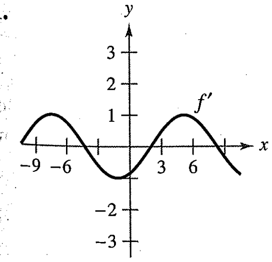
49.



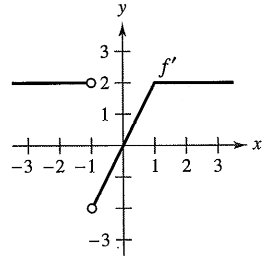
50.



51.



52.



(Submitted by Bill Fox, Moberly Area Community College, Moberly, MO)

53. **Graphical Reasoning** Consider the function

$$f(x) = \frac{\cos^2 \pi x}{\sqrt{x^2+1}}, \quad 0 < x < 4.$$

(a) Use a computer algebra system to graph the function and use the graph to approximate the critical numbers visually.

(b) Use a computer algebra system to find f' and approximate the critical numbers. Are the results the same as the visual approximation in part (a)? Explain.

54. **Graphical Reasoning** Consider the function

$f(x) = \tan(\sin \pi x)$.

(a) Use a graphing utility to graph the function.

(b) Identify any symmetry of the graph.

(c) Is the function periodic? If so, what is the period?

(d) Identify any extrema on $(-1, 1)$.

(e) Use a graphing utility to determine the concavity of the graph on $(0, 1)$.

Think About It **In Exercises 55–58, create a function whose graph has the given characteristics. (There is more than one correct answer.)**

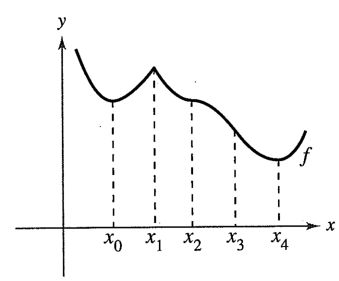
55. Vertical asymptote: $x = 3$
Horizontal asymptote: $y = 0$

56. Vertical asymptote: $x = -5$
Horizontal asymptote: None

57. Vertical asymptote: $x = 3$
Slant asymptote: $y = 3x + 2$

58. Vertical asymptote: $x = 2$
Slant asymptote: $y = -x$

59. **Graphical Reasoning** Identify the real numbers x_0, x_1, x_2, x_3, and x_4 in the figure such that each of the following is true.



(a) $f'(x) = 0$

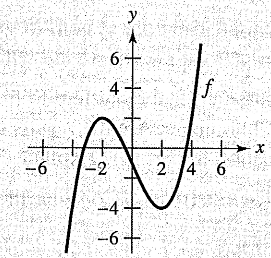
(b) $f''(x) = 0$

(c) $f'(x)$ does not exist.

(d) f has a relative maximum.

(e) f has a point of inflection.

60. HOW DO YOU SEE IT? The graph of f is shown in the figure.



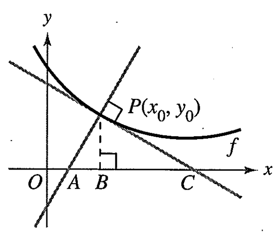
(a) For which values of x is $f'(x)$ zero? Positive? Negative? What do these values mean?

(b) For which values of x is $f''(x)$ zero? Positive? Negative? What do these values mean?

(c) On what open interval is f' an increasing function?

(d) For which value of x is $f'(x)$ minimum? For this value of x, how does the rate of change of f compare with the rates of change of f for other values of x? Explain.

61. Investigation Let $P(x_0, y_0)$ be an arbitrary point on the graph of f such that $f'(x_0) \neq 0$, as shown in the figure. Verify each statement.



(a) The x-intercept of the tangent line is

$$\left(x_0 - \frac{f(x_0)}{f'(x_0)}, 0\right).$$

(b) The y-intercept of the tangent line is

$$(0, f(x_0) - x_0 f'(x_0)).$$

(c) The x-intercept of the normal line is

$$(x_0 + f(x_0)f'(x_0), 0).$$

(d) The y-intercept of the normal line is

$$\left(0, y_0 + \frac{x_0}{f'(x_0)}\right).$$

(e) $|BC| = \left|\frac{f(x_0)}{f'(x_0)}\right|$

(f) $|PC| = \left|\frac{f(x_0)\sqrt{1 + [f'(x_0)]^2}}{f'(x_0)}\right|$

(g) $|AB| = |f(x_0)f'(x_0)|$

(h) $|AP| = |f(x_0)|\sqrt{1 + [f'(x_0)]^2}$

62. Investigation Consider the function

$$f(x) = \frac{2x^n}{x^4 + 1}$$

for nonnegative integer values of n.

(a) Discuss the relationship between the value of n and the symmetry of the graph.

(b) For which values of n will the x-axis be the horizontal asymptote?

(c) For which value of n will $y = 2$ be the horizontal asymptote?

(d) What is the asymptote of the graph when $n = 5$?

(e) Use a graphing utility to graph f for the indicated values of n in the table. Use the graph to determine the number of extrema M and the number of inflection points N of the graph.

n	0	1	2	3	4	5
M						
N						

63. Graphical Reasoning Consider the function

$$f(x) = \frac{ax}{(x - b)^2}.$$

Determine the effect on the graph of f as a and b are changed. Consider cases where a and b are both positive or both negative, and cases where a and b have opposite signs.

64. Graphical Reasoning Consider the function

$$f(x) = \frac{1}{2}(ax)^2 - ax, \quad a \neq 0.$$

(a) Determine the changes (if any) in the intercepts, extrema, and concavity of the graph of f when a is varied.

(b) In the same viewing window, use a graphing utility to graph the function for four different values of a.

Slant Asymptotes **In Exercises 65 and 66, the graph of the function has two slant asymptotes. Identify each slant asymptote. Then graph the function and its asymptotes.**

65. $y = \sqrt{4 + 16x^2}$

66. $y = \sqrt{x^2 + 6x}$

PUTNAM EXAM CHALLENGE

67. Let $f(x)$ be defined for $a \leq x \leq b$. Assuming appropriate properties of continuity and derivability, prove for $a < x < b$ that

$$\frac{\dfrac{f(x) - f(a)}{x - a} - \dfrac{f(b) - f(a)}{b - a}}{x - b} = \frac{1}{2}f''(\varepsilon),$$

where ε is some number between a and b.

This problem was composed by the Committee on the Putnam Prize Competition.