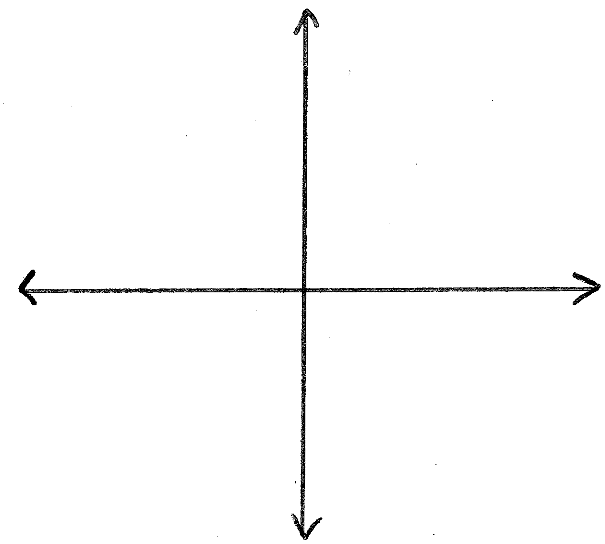
Ch. 3.6 Curve Sketching

1. Sketch the graph of the function and find the below information: $f(x) = -3x^5 + 5x^3$



x-ints: ______________ y-ints: ______________

V.A. ______________ H.A. ______________

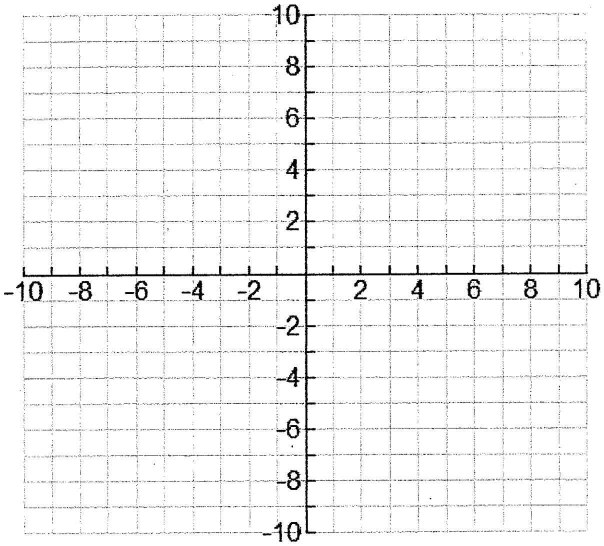
Domain: Interval Increasing

Interval Decreasing Relative Maximum

Relative Minimum: Points of Inflection:

Interval Concave Up: Interval Concave Down:

2. Sketch the graph of the function and find the below information: $f(x) = \frac{2x^2}{9-x^2}$



x-ints: ______________

y-ints: ______________

V.A. ______________

H.A. ______________

Domain:

Interval Increasing

Interval Decreasing

Relative Maximum

Relative Minimum:

Points of Inflection:

Interval Concave Up:

Interval Concave Down:

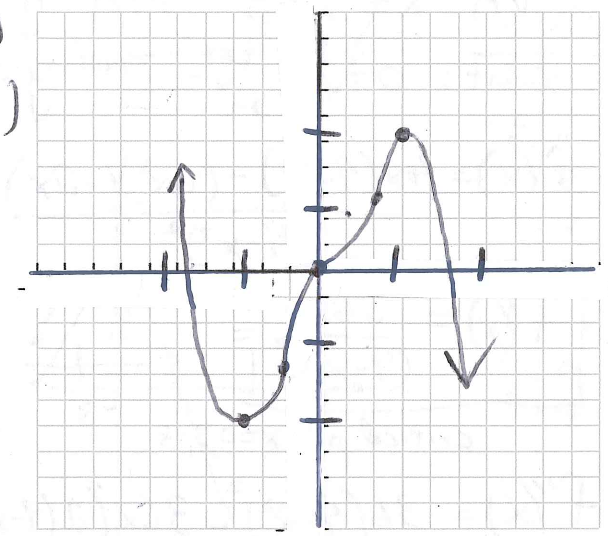
Ch. 3.6 Curve Sketching $0 = x^3(-3x^2+5)$ $x = 0, \pm\sqrt{5/3}$

1. Sketch the graph of the function and find the below information: $f(x) = -3x^5 + 5x^3$

$f'(x) = -15x^4 + 15x^2 = -15x^2(x^2-1)$

$0 = -15x^2(x+1)(x-1)$

critical values: $x = 0, -1, 1$

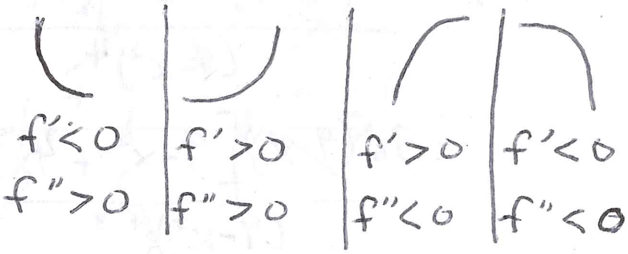


$f'(x)$: $\searrow$ $-$ | -1 | $\nearrow$ $+$ | 0 | $\nearrow$ $+$ | 1 | $\searrow$ $-$

$f''(x) = -60x^3 + 30x = -30x(2x^2-1)$

critical values: $x = 0, \pm\sqrt{1/2}$

$f''(x)$: $\cup$ $+$ | $-\frac{\sqrt{2}}{2}$ | $\cap$ $-$ | 0 | $\cup$ $+$ | $\frac{\sqrt{2}}{2}$ | $\cap$ $-$



Rel. min at $(-1, -2)$ b/c $f'(x)$ changes from $-$ to $+$

Rel. max at $(1, 2)$ b/c $f'(x)$ changes from $+$ to $-$

$f(x)$ increasing $(-1,0)\cup(0,1)$ b/c $f'(x) > 0$

$f(x)$ decreasing $(-\infty,-1)\cup(1,\infty]$ b/c $f'(x) < 0$

$f(x)$ concave up $(-\infty, -\frac{\sqrt{2}}{2})\cup(0, \frac{\sqrt{2}}{2}]$ b/c $f''(x) > 0$

$f(x)$ concave down $(-\frac{\sqrt{2}}{2}, 0)\cup(\frac{\sqrt{2}}{2}, \infty)$ b/c $f''(x) < 0$

POI at $x = -\frac{\sqrt{2}}{2}, 0, \frac{\sqrt{2}}{2}$ b/c $f''(x)$ change signs.

$\searrow \nearrow \nearrow \nearrow \nearrow \searrow$

$\cup\ \cup\ \cap\ \cup\ \cap\ \cap$

-1 $-\frac{\sqrt{2}}{2}$ 0 $\frac{\sqrt{2}}{2}$ 1

x-ints: $(0,0)$ $(\sqrt{5/3}, 0)$ $(-\sqrt{5/3}, 0)$ y-ints: $(0,0)$

V.A. none H.A. none

Domain: $(-\infty, \infty)$ Interval Increasing $(-1,0)\cup(0,1)$

Interval Decreasing $(-\infty,-1)\cup(1,\infty)$ Relative Maximum $(1, 2)$

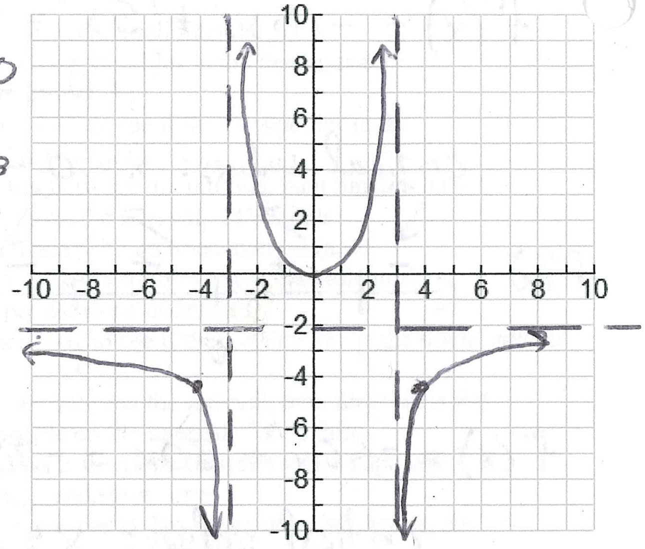
Relative Minimum: $(-1, -2)$ Points of Inflection: $(-\frac{\sqrt{2}}{2}, -1.24)$, $(\frac{\sqrt{2}}{2}, 1.24)$, $(0,0)$

Interval Concave Up: $(-\infty, -\frac{\sqrt{2}}{2})\cup(0, \frac{\sqrt{2}}{2})$ Interval Concave Down: $(-\frac{\sqrt{2}}{2}, 0)\cup(\frac{\sqrt{2}}{2}, \infty)$

2. Sketch the graph of the function and find the below information: $f(x) = \dfrac{2x^2}{9-x^2}$

VA: $x = 3, -3$ HA: $y = \frac{2}{-1} = -2$

x-int: $0 = 2x^2$ $\boxed{x=0}$ y-int: $y = \frac{0}{9-0} = 0$



$$f'(x) = \frac{4x(9-x^2) - (2x^2)(-2x)}{(9-x^2)^2} = \frac{36x - 4x^3 + 4x^3}{(9-x^2)^2}$$

$$\boxed{f'(x) = \frac{36x}{(9-x^2)^2}}$$

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−3 0 3

critical pts: $x = 0, 3, -3$

$$f''(x) = \frac{36(9-x^2)^2 - 36x\left[2(9-x^2)(-2x)\right]}{(9-x^2)^4}$$

$$= \frac{36(\cancel{9-x^2})\left[9-x^2+4x^2\right]}{(9-x^2)^{\cancel{4}\,3}} = \boxed{\frac{36(9+3x^2)}{(9-x^2)^3}}$$

critical values: $x = 3, -3$

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−3 3

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−3 0 3

x-ints: $(0,0)$ y-ints: $(0,0)$

V.A. $x=3,\ x=-3$ H.A. $y=-2$

Domain: $(-\infty,-3)\cup(-3,3)\cup(3,\infty)$ Interval Increasing $(0,3)\cup(3,\infty)$

Interval Decreasing $(-\infty,-3)\cup(-3,0)$ Relative Maximum none

Relative Minimum: $(0,0)$ Points of Inflection: none

Interval Concave Up: $(-3,3)$ Interval Concave Down: $(-\infty,-3)\cup(3,\infty)$