2017

Key

Ch. 3.7 Optimization HW Problems #7,9,11,18,19,25

7. The sum of the first number and twice the second number is 108 and the product is a maximum. optimize product

let first # = x

let 2^{nd} # = y

$x + 2y = 108$ $\quad x = 108 - 2y$

$*P = xy$

$P = (108-2y)y$

$P = 108y - 2y^2$

$P' = 108 - 4y$

$0 = 108 - 4y$

$4y = 108$

$\underline{y = 27}$

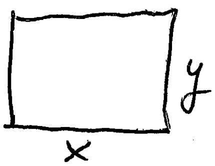
$x + 2y = 108$

$\underline{x = 54}$

$\boxed{P \text{ is a maximum when } x = 54,\ y = 27}$

Maximum Area In Exercises 9 and 10, find the length and width of a rectangle that has the given perimeter and a maximum area. *optimize Area

9. Perimeter: 80 meters



$2x + 2y = 80$ $\quad 2y = 80 - 2x$ $\quad y = 40 - x$

$*A = xy$

$A = x(40 - x)$

$A = 40x - x^2$

$A'(x) = 40 - 2x$

$0 = 40 - 2x$

$2x = 40$

$\underline{x = 20}$

$2x + 2y = 80$

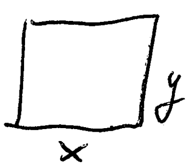
$2(20) + 2y = 80$

$\underline{y = 20}$

$\boxed{\text{Area is at maximum when } x = 20m,\ y = 20m}$

Minimum Perimeter In Exercises 11 and 12, find the length and width of a rectangle that has the given area and a minimum perimeter. *optimize perimeter

11. Area: 32 square feet



$A = xy$ $\quad xy = 32$ $\quad y = \frac{32}{x}$

$*P = 2x + 2y$

$P = 2x + 2\left(\frac{32}{x}\right)$

$P = 2x + \frac{64}{x}$

$P = 2x + 64x^{-1}$

$P'(x) = 2 - 64x^{-2}$

$0 = 2 - \frac{64}{x^2}$

$\frac{64}{x^2} = \frac{2}{1}$ $\quad 2x^2 = 64$

$x^2 = 32$ $\quad x = \pm\sqrt{32}$

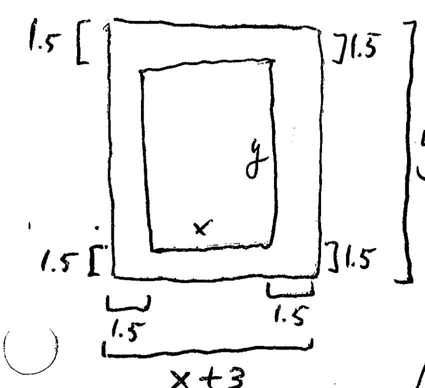
$x = \sqrt{16 \cdot 2} = 4\sqrt{2}$

$xy = 32$ $\quad 4\sqrt{2}\,y = 32$

$y = 4\sqrt{2}$

$\boxed{P \text{ is minimum when } x = y = 4\sqrt{2}\text{ ft}}$

18. Minimum Area A rectangular page is to contain 36 square inches of print. The margins on each side are $1\frac{1}{2}$ inches. Find the dimensions of the page such that the least amount of paper is used. *optimize Area (of paper)



$A = xy$ $\quad 36 = xy$ print

$A = (x+3)(y+3)$ sub $y = \frac{36}{x}$

$A = (x+3)\left(\frac{36}{x} + 3\right)$

$A = x\left(\frac{36}{x}\right) + 3x + \frac{108}{x} + 9$

$A = 45 + 3x + \frac{108}{x}$

$A = 108x^{-1} + 3x + 45$

$A'(x) = -108x^{-2} + 3 + 0$

$0 = \frac{-108}{x^2} + 3$ $\quad 3 = \frac{108}{x^2}$

$3x^2 = 108$

$x^2 = 36$

$\underline{x = 6}$

$xy = 36 \rightarrow \underline{y = 6}$

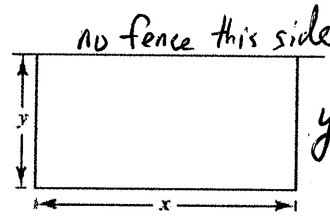
$\boxed{\text{Dimensions } 9_{in.} \times 9_{in.}}$

$A = (x+3)(y+3)$

$A = (6+3) \times (6+3)$

2017

19. Minimum Length A farmer plans to fence a rectangular pasture adjacent to a river (see figure). The pasture must contain 245,000 square meters in order to provide enough grass for the herd. No fencing is needed along the river. What dimensions will require the least amount of fencing? *optimize perimeter



Area

$$Area = xy$$

$$245{,}000 = xy \qquad y = \frac{245{,}000}{x}$$

$$*\ Perimeter = x + 2y$$

$$P = x + 2y$$

$$P = x + 2\left(\frac{245{,}000}{x}\right)$$

$$P = x + \frac{490{,}000}{x} = x + 490{,}000x^{-1}$$

$$P(x) = x + 490{,}000x^{-1}$$

$$P'(x) = 1 + -490{,}000x^{-2}$$

$$0 = 1 - \frac{490{,}000}{x^2}$$

$$\frac{490{,}000}{x^2} = 1$$

$$x^2 = 490{,}000$$

$$x = \pm\sqrt{490{,}000}$$

$$x = \pm 700, \quad x = 700$$

$$245{,}000 = xy$$

$$245{,}000 = 700y$$

$$y = 350$$

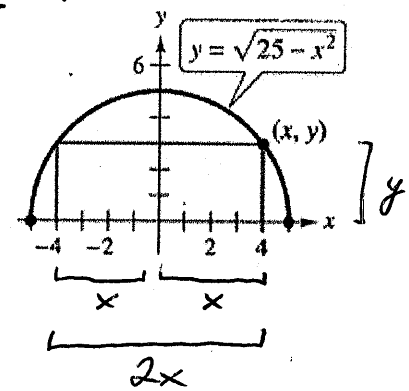
$\boxed{P \text{ is minimum (fencing) when } x = 700,\ y = 350\text{m}}$

25. Maximum Area A rectangle is bounded by the x-axis and the semicircle

$$y = \sqrt{25 - x^2} \qquad -5 < x < 5$$

*Apply product, chain rule

(see figure). What length and width should the rectangle have so that its area is a maximum?



$$Area = 2x \cdot y$$

$$A = 2x\left(\sqrt{25 - x^2}\right)$$

$$A(x) = 2x\left(25 - x^2\right)^{1/2}$$

$$A'(x) = 2\left(25 - x^2\right)^{1/2} + 2x \cdot \tfrac{1}{2}\left(25 - x^2\right)^{-1/2}(-2x)$$

$$A'(x) = \frac{2\sqrt{25 - x^2}}{1} - \frac{2x^2}{\sqrt{25 - x^2}}$$

$$A'(x) = \frac{2(25 - x^2) - 2x^2}{\sqrt{25 - x^2}}$$

$$A'(x) = \frac{50 - 2x^2 - 2x^2}{\sqrt{25 - x^2}}$$

$$A'(x) = \frac{50 - 4x^2}{\sqrt{25 - x^2}}$$

$$0 = \frac{50 - 4x^2}{\sqrt{25 - x^2}}$$

$$50 - 4x^2 = 0$$

$$4x^2 = 50$$

$$x^2 = \frac{50}{4} = \frac{25}{2}$$

$$x = \pm\frac{5}{\sqrt{2}} \qquad y = \sqrt{25 - x^2} = \frac{5}{\sqrt{2}}$$

$$length: \frac{5}{\sqrt{2}} + \frac{5}{\sqrt{2}} = \frac{10}{\sqrt{2}} = \boxed{5\sqrt{2}}$$

$$width: \frac{5}{\sqrt{2}} = \boxed{\frac{5\sqrt{2}}{2}}$$

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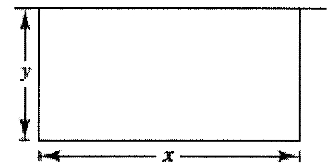

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