Calculus Optimization Notes

Optimization: Optimization is the process of finding the greatest (maximum optimal solution) or least value of a function (the minimum optimal solution) for some constraint, which must be true regardless of the solution. Optimization finds the most suitable value for a function within a given domain.

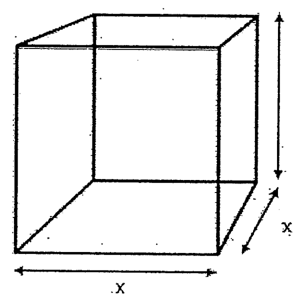
Optimization steps:

1. Write equation for variable you want to optimize

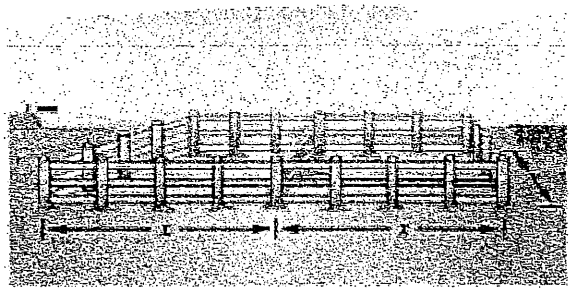
2. Substitute to get equation in terms of one variable on one side

3. Find derivative, set derivative = 0 and solve.

Example 1: A manufacturer wants to design an open box having a square base and a surface area of 108 in^2. What dimensions will produce a box with maximum volume?



2) ***Maximum Area*** A rancher has 200 feet of fencing with which to enclose two adjacent rectangular corrals (see figure). What dimensions should be used so that the enclosed area will be a maximum?



Calculus Optimization Notes

Optimization: Optimization is the process of finding the greatest (maximum optimal solution) or least value of a function (the minimum optimal solution) for some constraint, which must be true regardless of the solution. Optimization finds the most suitable value for a function within a given domain.

Optimization steps:

1. Write equation for variable you want to optimize

2. Substitute to get equation in terms of one variable on one side

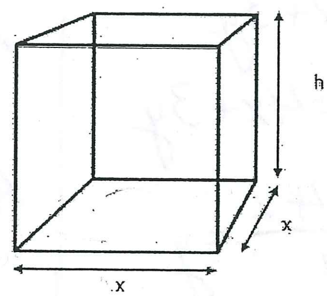
3. Find derivative, set derivative = 0 and solve.

Example 1: A manufacturer wants to design an open box having a square base and a surface area of 108 in^2. What dimensions will produce a box with maximum volume?

optimize volume

$S = 108 \text{ in}^2$ $\quad$ $*V = x^2 h$

$S = x^2 + 4xh$



$108 = x^2 + 4xh$

$108 - x^2 = 4xh$

$\frac{108 - x^2}{4x} = h$

$V = x^2\left(\frac{108 - x^2}{4x}\right)$

$V = \frac{108}{4}x - \frac{1}{4}x^3$

$V = 27x - \frac{1}{4}x^3$

$V'(x) = 27 - \frac{3}{4}x^2$

$0 = 27 - \frac{3}{4}x^2$

$\frac{3}{4}x^2 = 27$

$x^2 = 27\left(\frac{4}{3}\right)$

$x^2 = 36$

$x = 6$

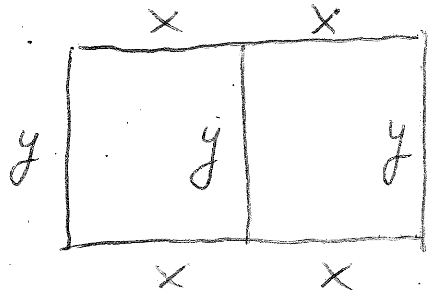
$S = x^2 + 4xh$

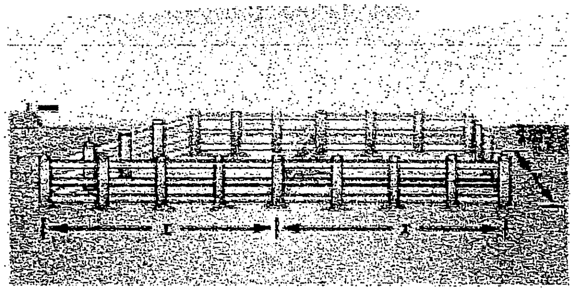
$108 = 6^2 + 4(6)h$

$3 = h$

$\boxed{\text{Dimensions: } 6 \text{ in.} \times 6 \text{ in.} \times 3 \text{ in.}}$

2) ***Maximum Area*** A rancher has 200 feet of fencing with which to enclose two adjacent rectangular corrals (see figure). What dimensions should be used so that the enclosed area will be a maximum?





(Optimize Area)

$P = 200\text{ft}$

$P = 4x + 3y$

$200 = 4x + 3y$

$\frac{200-4x}{3} = y$

$*A = 2xy$

$A = 2x\left(\frac{200-4x}{3}\right)$

$A = \frac{400}{3}x - \frac{8}{3}x^2$

$A'(x) = \frac{400}{3} - \frac{16}{3}x$

$0 = \frac{400}{3} - \frac{16}{3}x$

$\frac{16}{3}x = \frac{400}{3}$

$x = 25\text{ft}$

$P = 4x + 3y$

$200 = 4(25) + 3y$

$200 - 100 = 3y$

$\frac{100}{3} = y$

$\boxed{\text{Dimensions: } 25\text{ ft by } \frac{100}{3}\text{ ft.}}$

$50\text{ ft by } \frac{100}{3}\text{ ft.}$

3.7 Optimization HW p. 223-226 #3, 7, 9, 11, 21c, 23, 27

3) $S = x + y$ $\quad x = S - y$

$*P = xy$ $\quad P = (S-y)y$

(optimize P)

$P = Sy - y^2$

$P'(y) = S - 2y$

$0 = S - 2y$

$y = \frac{S}{2}$

$x = \frac{S}{2}$

5) $P = 192$ $\quad 192 = xy$

$P = xy$ $\quad \frac{192}{y} = x$

(optimize S) $S = x + y$

$S = \frac{192}{y} + y$

$S = 192y^{-1} + y$

$S'(y) = -192y^{-2} + 1$

$0 = \frac{-192}{y^2} + 1$

$y^2 = 192$

$y = \sqrt{192}$

7) $S = x + 2y$

$S = 100$

$100 = x + 2y$

$100 - 2y = x$

(*optimize P)

$P = xy$

$P = (100 - 2y)y$

$P = 100y - 2y^2$

$P'(y) = 100 - 4y$

$0 = 100 - 4y$

$4y = 100$

$y = 25$

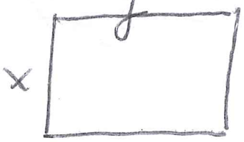
$100 = x + 2(25)$

$x = 50$

$P_{max} = (25)(50)$

$= 1250$

9)



$P=100$

$P=2x+2y$

$100=2x+2y$

$100-2y=2x$

$\frac{100-2y}{2}=x$

Optimize Area

$A=xy$

$A=\left[\frac{100-2y}{2}\right]y$

$A=50y-y^2$

$A'(y)=50-2y$

$0=50-2y$

$2y=50$

$\underline{\underline{y=25}}$

$100=2x+2y$

$100=2x+2(25)$

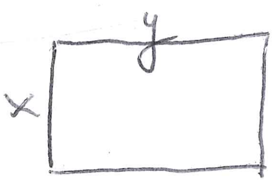
$100=2x+50$

$\underline{\underline{25=x}}$

$A_{max}=(25)(25)$

$=\boxed{625m^2}$

11)



(*Minimize P)

$A=xy$

$A=64$

$64=xy$

$\frac{64}{x}=y$

$P=2x+2y$

$P=2x+2\left(\frac{64}{x}\right)$

$P=2x+128x^{-1}$

$P'(x)=2-128x^{-2}$

$0=2-\frac{128}{x^2}$

$x^2=64$

$\underline{\underline{x=8}}$

$64=xy$

$64=8y$

$\underline{\underline{8=y}}$

$P=2(8)+2(8)$

$=\boxed{32ft.}$

21c) Determine dimension of rectangular solid:

Optimize volume given surface area $= 150\ in^2$.

$* V = x^2 h$

$S = 2x^2 + 4xh$

$150 = 2x^2 + 4xh$

$\frac{150 - 2x^2}{4x} = h$

$$\downarrow$$

$$V = x^2 \left[\frac{150-2x^2}{4x} \right]$$

$$V = \frac{75}{2}x - \frac{1}{2}x^3$$

$$V'(x) = \frac{75}{2} - \frac{3}{2}x^2$$

$$0 = \frac{75}{2} - \frac{3}{2}x^2$$

$$\frac{75}{2} = \frac{3}{2}x^2$$

$$\underline{\underline{x = 5}}$$

$$150 = 2(5)^2 + 4(5)h$$

$$150 = 50 + 20h$$

$$\frac{100}{20} = h$$

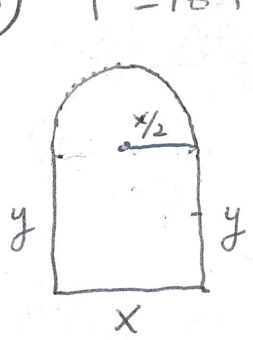
$$\underline{\underline{h = 5}}$$

$$\boxed{V_{max} = 5\text{ in.} \times 5\text{ in.} \times 5\text{ in.}}$$

3.7 Optimization

Circumference = distance around a circle

23) $P = 16$ ft. $\quad C = 2\pi r$



$P = x + 2y + \frac{1}{2}(2\pi r)$

$P = x + 2y + \frac{1}{2}(2\pi(\frac{x}{2}))$

$P = x + 2y + \frac{\pi}{2}x$

$(16 = x + 2y + \frac{\pi}{2}x)\,2$

$32 = 2x + 4y + \pi x$

$32 - 2x - \pi x = 4y$

$\frac{32 - 2x - \pi x}{4} = y$

* $A = xy + \frac{1}{2}(\pi r^2)$

$A = xy + \frac{1}{2}(\pi(\frac{x}{2})^2)$

$A = xy + \frac{1}{2}\pi(\frac{x^2}{4})$

$A = xy + \frac{\pi}{8}x^2$

$A = x\left(\frac{32 - 2x - \pi x}{4}\right) + \frac{\pi}{8}x^2$

$A = 8x - \frac{x^2}{2} - \frac{\pi}{4}x^2 + \frac{\pi}{8}x^2$

$A = 8x - \frac{x^2}{2} - \frac{\pi}{8}x^2$

$\frac{dA}{dx}$ or $A' = 8 - \frac{2x}{2} - \frac{\pi}{8}\cdot 2x$

$A' = 8 - x - \frac{\pi}{4}x$

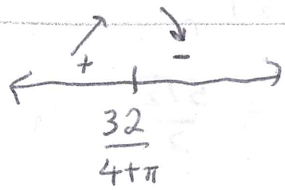
$0 = 8 - x - \frac{\pi}{4}x$

$0 = 8 - x(1 + \frac{\pi}{4})$

$-8 = -x(1 + \frac{\pi}{4})$

$\frac{8}{1 + \pi/4} = x$

$x = \frac{8}{1 - \pi/4} = \frac{32}{4 + \pi}$



$y = \frac{32 - 2x - \pi x}{4}$

$y = \frac{32 - 2(\frac{32}{4+\pi}) - \pi(\frac{32}{4+\pi})}{4} = \boxed{\frac{16}{4+\pi}\text{ ft.}}$

7) $S = x + 2y$

$S = 100$

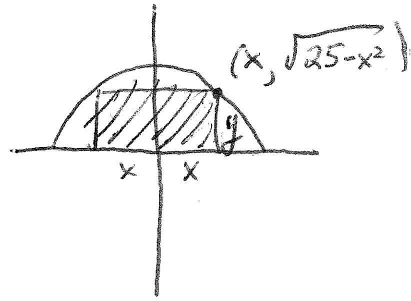
$100 = x + 2y$

$100 - 2y = x$

* $P = xy$

$P = (100 - 2y)(y)$

27) $A = 2xy = 2x\sqrt{25-x^2}$

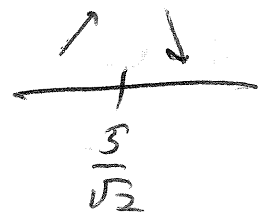


$$A = 2x(25-x^2)^{1/2}$$

$$A'(x) = 2(25-x^2)^{1/2} + 2x \cdot \tfrac{1}{2}(25-x^2)^{-1/2}(-2x)$$

$$= 2\sqrt{25-x^2} - \frac{2x^2}{\sqrt{25-x^2}}$$

$$A'(x) = \frac{2(25-x^2)-2x^2}{\sqrt{25-x^2}} = \frac{50-2x^2-2x^2}{\sqrt{25-x^2}} = \frac{50-4x^2}{\sqrt{25-x^2}} = 0$$



$50-4x^2=0$

$4x^2=50$

$x^2=\frac{50}{4}$

$x^2=\frac{25}{2}$

$x = \pm\sqrt{25/2} = \pm\frac{5}{\sqrt{2}} = \pm\frac{5\sqrt{2}}{2}$

$y = \sqrt{25-\left(\frac{25}{2}\right)}$

$= \sqrt{\frac{25}{2}} = \frac{5\sqrt{2}}{2}$

$$\boxed{\begin{array}{l}\text{length} = 5\sqrt{2}\\ \text{width} = \frac{5\sqrt{2}}{2}\end{array}}$$