34. $V = 4000 = \frac{4}{3}\pi r^3 + \pi r^2 h$

$$h = \frac{4000}{\pi r^2} - \frac{4}{3}r$$

Let k = cost per square foot of the surface area of the sides, then $2k$ = cost per square foot of the hemispherical ends.

$$C = 2k(4\pi r^2) + k(2\pi rh) = k\left[8\pi r^2 + 2\pi r\left(\frac{4000}{\pi r^2} - \frac{4}{3}r\right)\right] = k\left[\frac{16}{3}\pi r^2 + \frac{8000}{r}\right]$$

$$\frac{dC}{dr} = k\left[\frac{32}{3}\pi r - \frac{8000}{r^2}\right] = 0 \text{ when } r = \sqrt[3]{\frac{750}{\pi}} \approx 6.204 \text{ ft and } h \approx 24.814 \text{ ft.}$$

By the Second Derivative Test, you have $\frac{d^2C}{dr^2} = k\left[\frac{32}{3}\pi + \frac{12{,}000}{r^3}\right] > 0$ when $r = \sqrt[3]{\frac{750}{\pi}}$.

The cost is minimum when $r = \sqrt[3]{\frac{750}{\pi}}$ ft and $h \approx 24.814$ ft.

35. Let x be the length of a side of the square and y the length of a side of the triangle.

$$4x + 3y = 10$$

$$A = x^2 + \frac{1}{2}y\left(\frac{\sqrt{3}}{2}y\right)$$

$$= \frac{(10 - 3y)^2}{16} + \frac{\sqrt{3}}{4}y^2$$

$$\frac{dA}{dy} = \frac{1}{8}(10 - 3y)(-3) + \frac{\sqrt{3}}{2}y = 0$$

$$-30 + 9y + 4\sqrt{3}y = 0$$

$$y = \frac{30}{9 + 4\sqrt{3}}$$

$$\frac{d^2A}{dy^2} = \frac{9 + 4\sqrt{3}}{8} > 0$$

A is minimum when $y = \frac{30}{9 + 4\sqrt{3}}$ and $x = \frac{10\sqrt{3}}{9 + 4\sqrt{3}}$.

36. (a) Let x be the side of the triangle and y the side of the square.

$$A = \frac{3}{4}\left(\cot\frac{\pi}{3}\right)x^2 + \frac{4}{4}\left(\cot\frac{\pi}{4}\right)y^2 \text{ where } 3x + 4y = 20$$

$$= \frac{\sqrt{3}}{4}x^2 + \left(5 - \frac{3}{4}x\right)^2, 0 \le x \le \frac{20}{3}.$$

$$A' = \frac{\sqrt{3}}{2}x + 2\left(5 - \frac{3}{4}x\right)\left(-\frac{3}{4}\right) = 0$$

$$x = \frac{60}{4\sqrt{3} + 9}$$

When $x = 0$, $A = 25$, when $x = 60/(4\sqrt{3} + 9)$, $A \approx 10.847$, and when $x = 20/3$, $A \approx 19.245$. Area is maximum when all 20 feet are used on the square.

(b) Let x be the side of the square and y the side of the pentagon.

$$A = \frac{4}{4}\left(\cot\frac{\pi}{4}\right)x^2 + \frac{5}{4}\left(\cot\frac{\pi}{5}\right)y^2 \text{ where } 4x + 5y = 20$$

$$= x^2 + 1.7204774\left(4 - \frac{4}{5}x\right)^2, 0 \le x \le 5.$$

$$A' = 2x - 2.75276384\left(4 - \frac{4}{5}x\right) = 0$$

$$x \approx 2.62$$

When $x = 0$, $A \approx 27.528$, when $x \approx 2.62$, $A \approx 13.102$, and when $x = 5$, $A \approx 25$. Area is maximum when all 20 feet are used on the pentagon.

(c) Let x be the side of the pentagon and y the side of the hexagon.

$$A = \frac{5}{4}\left(\cot\frac{\pi}{5}\right)x^2 + \frac{6}{4}\left(\cot\frac{\pi}{6}\right)y^2 \text{ where } 5x + 6y = 20$$

$$= \frac{5}{4}\left(\cot\frac{\pi}{5}\right)x^2 + \frac{3}{2}\left(\sqrt{3}\right)\left(\frac{20 - 5x}{6}\right)^2, 0 \le x \le 4.$$

$$A' = \frac{5}{2}\left(\cot\frac{\pi}{5}\right)x + 3\sqrt{3}\left(-\frac{5}{6}\right)\left(\frac{20 - 5x}{6}\right) = 0$$

$$x \approx 2.0475$$

When $x = 0$, $A \approx 28.868$, when $x \approx 2.0475$, $A \approx 14.091$, and when $x = 4$, $A \approx 27.528$. Area is maximum when all 20 feet are used on the hexagon

(d) Let x be the side of the hexagon and r the radius of the circle.

$$A = \frac{6}{4}\left(\cot\frac{\pi}{6}\right)x^2 + \pi r^2 \text{ where } 6x + 2\pi r = 20$$

$$= \frac{3\sqrt{3}}{2}x^2 + \pi\left(\frac{10}{\pi} - \frac{3x}{\pi}\right)^2, 0 \le x \le \frac{10}{3}.$$

$$A' = 3\sqrt{3} - 6\left(\frac{10}{\pi} - \frac{3x}{\pi}\right) = 0$$

$$x \approx 1.748$$

When $x = 0$, $A \approx 31.831$, when $x \approx 1.748$, $A \approx 15.138$, and when $x = 10/3$, $A \approx 28.868$. Area is maximum when all 20 feet are used on the circle.

In general, using all of the wire for the figure with more sides will enclose the most area.

37. Let S be the strength and k the constant of proportionality. Given

$h^2 + w^2 = 20^2$, $h^2 = 20^2 - w^2$,

$$S = kwh^2$$

$$S = kw\left(400 - w^2\right) = k\left(400w - w^3\right)$$

$$\frac{dS}{dw} = k\left(400 - 3w^2\right) = 0 \text{ when } w = \frac{20\sqrt{3}}{3} \text{ in.}$$

and $h = \dfrac{20\sqrt{6}}{3}$ in.

$$\frac{d^2S}{dw^2} = -6kw < 0 \text{ when } w = \frac{20\sqrt{3}}{3}.$$

These values yield a maximum.

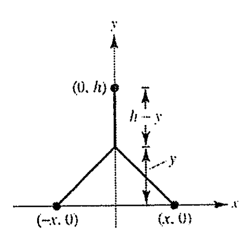
38. Let A be the amount of the power line.

$$A = h - y + 2\sqrt{x^2 + y^2}$$

$$\frac{dA}{dy} = -1 + \frac{2y}{\sqrt{x^2 + y^2}} = 0 \text{ when } y = \frac{x}{\sqrt{3}}.$$

$$\frac{d^2A}{dy^2} = \frac{2x^2}{\left(x^2 + y^2\right)^{3/2}} > 0 \text{ for } y = \frac{x}{\sqrt{3}}.$$

The amount of power line is minimum when $y = x/\sqrt{3}$.



39. $C(x) = 2k\sqrt{x^2+4} + k(4-x)$

$$C'(x) = \frac{2xk}{\sqrt{x^2+4}} - k = 0$$

$$2x = \sqrt{x^2+4}$$

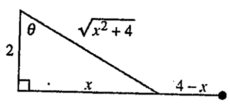
$$4x^2 = x^2 + 4$$

$$3x^2 = 4$$

$$x = \frac{2}{\sqrt{3}}$$

Or, use Exercise 50(d): $\sin\theta = \frac{C_2}{C_1} = \frac{1}{2} \Rightarrow \theta = 30°$.

So, $x = \frac{2}{\sqrt{3}}$.



40. $\sin\alpha = \frac{h}{s} \Rightarrow s = \frac{h}{\sin\alpha}, 0 < \alpha < \frac{\pi}{2}$

$$\tan\alpha = \frac{h}{2} \Rightarrow h = 2\tan\alpha \Rightarrow s = \frac{2\tan\alpha}{\sin\alpha} = 2\sec\alpha$$

$$I = \frac{k\sin\alpha}{s^2} = \frac{k\sin\alpha}{4\sec^2\alpha} = \frac{k}{4}\sin\alpha\cos^2\alpha$$

$$\frac{dI}{d\alpha} = \frac{k}{4}\left[\sin\alpha(-2\sin\alpha\cos\alpha) + \cos^2\alpha(\cos\alpha)\right]$$

$$= \frac{k}{4}\cos\alpha\left[\cos^2\alpha - 2\sin^2\alpha\right]$$

$$= \frac{k}{4}\cos\alpha\left[1 - 3\sin^2\alpha\right]$$

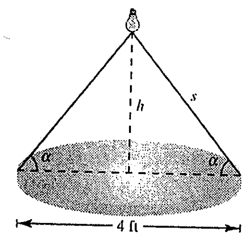
$$= 0 \text{ when } \alpha = \frac{\pi}{2}, \frac{3\pi}{2}, \text{ or when } \sin\alpha = \pm\frac{1}{\sqrt{3}}.$$

Because α is acute, you have

$$\sin\alpha = \frac{1}{\sqrt{3}} \Rightarrow h = 2\tan\alpha = 2\left(\frac{1}{\sqrt{2}}\right) = \sqrt{2} \text{ ft.}$$

Because

$(d^2I)/(d\alpha^2) = (k/4)\sin\alpha(9\sin^2\alpha - 7) < 0$ when $\sin\alpha = 1/\sqrt{3}$, this yields a maximum.



41.

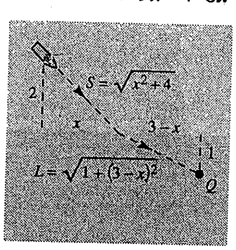
$$S = \sqrt{x^2+4}, L = \sqrt{1 + (3-x)^2}$$

$$\text{Time} = T = \frac{\sqrt{x^2+4}}{2} + \frac{\sqrt{x^2-6x+10}}{4}$$

$$\frac{dT}{dx} = \frac{x}{2\sqrt{x^2+4}} + \frac{x-3}{4\sqrt{x^2-6x+10}} = 0$$

$$\frac{x^2}{x^2+4} = \frac{9-6x+x^2}{4(x^2-6x+10)}$$

$$x^4 - 6x^3 + 9x^2 + 8x - 12 = 0$$



You need to find the roots of this equation in the interval $[0, 3]$. By using a computer or graphing utility you can determine that this equation has only one root in this interval $(x = 1)$. Testing at this value and at the endpoints, you see that $x = 1$ yields the minimum time. So, the man should row to a point 1 mile from the nearest point on the coast.

42. $T = \dfrac{\sqrt{x^2+4}}{v_1} + \dfrac{\sqrt{x^2-6x+10}}{v_2}$

$$\frac{dT}{dx} = \frac{x}{v_1\sqrt{x^2+4}} + \frac{x-3}{v_2\sqrt{x^2-6x+10}} = 0$$

Because

$$\frac{x}{\sqrt{x^2+4}} = \sin\theta_1 \text{ and } \frac{x-3}{\sqrt{x^2-6x+10}} = -\sin\theta_2$$

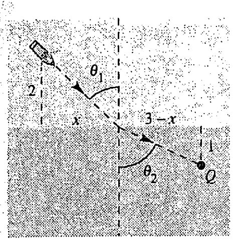
you have

$$\frac{\sin\theta_1}{v_1} - \frac{\sin\theta_2}{v_2} = 0 \Rightarrow \frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2}.$$

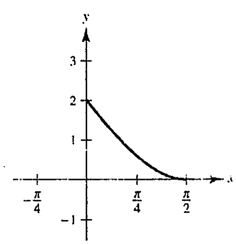
Because

$$\frac{d^2T}{dx^2} = \frac{4}{v_1(x^2+4)^{3/2}} + \frac{1}{v_2(x^2-6x+10)^{3/2}} > 0$$

this condition yields a minimum time.

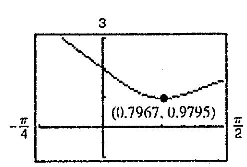


43. $f(x) = 2 - 2\sin x$



(a) Distance from origin to y-intercept is 2.
Distance from origin to x-intercept is $\pi/2 \approx 1.57$.

(b) $d = \sqrt{x^2+y^2} = \sqrt{x^2+(2-2\sin x)^2}$



Minimum distance = 0.9795 at $x = 0.7967$.

(c) Let $f(x) = d^2(x) = x^2 + (2-2\sin x)^2$.

$f'(x) = 2x + 2(2-2\sin x)(-2\cos x)$

Setting $f'(x) = 0$, you obtain $x \approx 0.7967$, which corresponds to $d = 0.9795$.

44. $T = \dfrac{\sqrt{x^2+d_1^2}}{v_1} + \dfrac{\sqrt{d_2^2+(a-x)^2}}{v_2}$

$$\frac{dT}{dx} = \frac{x}{v_1\sqrt{x^2+d_1^2}} + \frac{x-a}{v_2\sqrt{d_2^2+(a-x)^2}} = 0$$

Because

$$\frac{x}{\sqrt{x^2+d_1^2}} = \sin\theta_1 \text{ and } \frac{x-a}{\sqrt{d_2^2+(a-x)^2}} = -\sin\theta_2$$

you have

$$\frac{\sin\theta_1}{v_1} - \frac{\sin\theta_2}{v_2} = 0 \Rightarrow \frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2}.$$

Because

$$\frac{d^2T}{dx^2} = \frac{d_1^2}{v_1(x^2+d_1^2)^{3/2}} + \frac{d_2^2}{v_2\left[d_2^2+(a-x)^2\right]^{3/2}} > 0$$

this condition yields a minimum time.

45. $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^2\sqrt{144-r^2}$

$$\frac{dV}{dr} = \frac{1}{3}\pi\left[r^2\left(\frac{1}{2}\right)(144-r^2)^{-1/2}(-2r) + 2r\sqrt{144-r^2}\right]$$

$$= \frac{1}{3}\pi\left[\frac{288r-3r^3}{\sqrt{144-r^2}}\right]$$

$$= \pi\left[\frac{r(96-r^2)}{\sqrt{144-r^2}}\right] = 0 \text{ when } r = 0, 4\sqrt{6}.$$

By the First Derivative Test, V is maximum when $r = 4\sqrt{6}$ and $h = 4\sqrt{3}$.

Area of circle: $A = \pi(12)^2 = 144\pi$

Lateral surface area of cone:

$$S = \pi(4\sqrt{6})\sqrt{(4\sqrt{6})^2+(4\sqrt{3})^2} = 48\sqrt{6}\pi$$

Area of sector:

$$144\pi - 48\sqrt{6}\pi = \frac{1}{2}\theta r^2 = 72\theta$$

$$\theta = \frac{144\pi - 48\sqrt{6}\pi}{72}$$

$$= \frac{2\pi}{3}(3-\sqrt{6}) \approx 1.153 \text{ radians or } 66°$$

46. (a)

Base 1	Base 2	Altitude	Area
8	$8 + 16\cos 10°$	$8\sin 10°$	≈ 22.1
8	$8 + 16\cos 20°$	$8\sin 20°$	≈ 42.5
8	$8 + 16\cos 30°$	$8\sin 30°$	≈ 59.7
8	$8 + 16\cos 40°$	$8\sin 40°$	≈ 72.7
8	$8 + 16\cos 50°$	$8\sin 50°$	≈ 80.5
8	$8 + 16\cos 60°$	$8\sin 60°$	≈ 83.1

(b)

Base 1	Base 2	Altitude	Area
8	$8 + 16\cos 10°$	$8\sin 10°$	≈ 22.1
8	$8 + 16\cos 20°$	$8\sin 20°$	≈ 42.5
8	$8 + 16\cos 30°$	$8\sin 30°$	≈ 59.7
8	$8 + 16\cos 40°$	$8\sin 40°$	≈ 72.7
8	$8 + 16\cos 50°$	$8\sin 50°$	≈ 80.5
8	$8 + 16\cos 60°$	$8\sin 60°$	≈ 83.1
8	$8 + 16\cos 70°$	$8\sin 70°$	≈ 80.7
8	$8 + 16\cos 80°$	$8\sin 80°$	≈ 74.0
8	$8 + 16\cos 90°$	$8\sin 90°$	≈ 64.0

The maximum cross-sectional area is approximately 83.1 ft^2.

(c) $A = (a + b)\dfrac{h}{2}$

$$= \left[8 + (8 + 16\cos\theta)\right]\frac{8\sin\theta}{2}$$

$$= 64(1 + \cos\theta)\sin\theta,\ 0° < \theta < 90°$$

(d) $\dfrac{dA}{d\theta} = 64(1 + \cos\theta)\cos\theta + (-64\sin\theta)\sin\theta$

$$= 64\left(\cos\theta + \cos^2\theta - \sin^2\theta\right)$$

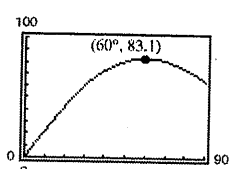
$$= 64\left(2\cos^2\theta + \cos\theta - 1\right)$$

$$= 64(2\cos\theta - 1)(\cos\theta + 1)$$

$$= 0 \text{ when } \theta = 60°, 180°, 300°.$$

The maximum occurs when $\theta = 60°$.

(e)



47. Let d be the amount deposited in the bank, i be the interest rate paid by the bank, and P be the profit.

$$P = (0.12)d - id$$

$$d = ki^2 \left(\text{because } d \text{ is proportional to } i^2\right)$$

$$P = (0.12)\left(ki^2\right) - i\left(ki^2\right) = k\left(0.12i^2 - i^3\right)$$

$$\frac{dP}{di} = k\left(0.24i - 3i^2\right) = 0 \text{ when } i = \frac{0.24}{3} = 0.08.$$

$$\frac{d^2P}{di^2} = k(0.24 - 6i) < 0 \text{ when } i = 0.08 \left(\textbf{Note: } k > 0\right)$$

The profit is a maximum when $i = 8\%$.

48. (a) The profit is increasing on $(0, 40)$.

(b) The profit is decreasing on $(40, 60)$.

(c) In order to yield a maximum profit, the company should spend about \$40 thousand.

(d) The point of diminishing returns is the point where the concavity changes, which in this case is $x = 20$ thousand dollars.

49. $S_1 = (4m - 1)^2 + (5m - 6)^2 + (10m - 3)^2$

$$\frac{dS_1}{dm} = 2(4m - 1)(4) + 2(5m - 6)(5) + 2(10m - 3)(10)$$

$$= 282m - 128 = 0 \text{ when } m = \frac{64}{141}.$$

Line: $y = \dfrac{64}{141}x$

$$S = \left|4\left(\frac{64}{141}\right) - 1\right| + \left|5\left(\frac{64}{141}\right) - 6\right| + \left|10\left(\frac{64}{141}\right) - 3\right|$$

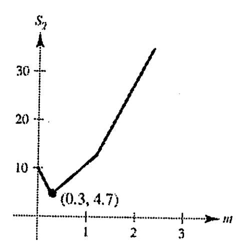
$$= \left|\frac{256}{141} - 1\right| + \left|\frac{320}{141} - 6\right| + \left|\frac{640}{141} - 3\right| = \frac{858}{141} \approx 6.1 \text{ mi}$$

50. $S_2 = |4m - 1| + |5m - 6| + |10m - 3|$

Using a graphing utility, you can see that the minimum occurs when $m = 0.3$.

Line $y = 0.3x$

$S_2 = |4(0.3) - 1| + |5(0.3) - 6| + |10(0.3) - 3| = 4.7$ mi.

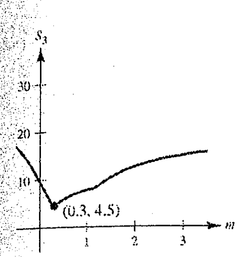


51. $S_3 = \dfrac{|4m - 1|}{\sqrt{m^2 + 1}} + \dfrac{|5m - 6|}{\sqrt{m^2 + 1}} + \dfrac{|10m - 3|}{\sqrt{m^2 + 1}}$

Using a graphing utility, you can see that the minimum occurs when $x \approx 0.3$.

Line: $y \approx 0.3x$

$$S_3 = \frac{|4(0.3) - 1| + |5(0.3) - 6| + |10(0.3) - 3|}{\sqrt{(0.3)^2 + 1}} \approx 4.5 \text{ mi.}$$



52. (a) Label the figure so that $r^2 = x^2 + h^2$.

Then, the area A is 8 times the area of the region given by $OPQR$:

$$A = 8\left[\frac{1}{2}h^2 + (x - h)h\right] = 8\left[\frac{1}{2}(r^2 - x^2) + \left(x - \sqrt{r^2 - x^2}\right)\sqrt{r^2 - x^2}\right] = 8x\sqrt{r^2 - x^2} + 4x^2 - 4r^2$$

$$A'(x) = 8\sqrt{r^2 - x^2} - \frac{8x^2}{\sqrt{r^2 - x^2}} + 8x = 0$$

$$\frac{8x^2}{\sqrt{r^2 - x^2}} = 8x + 8\sqrt{r^2 - x^2}$$

$$x^2 = x\sqrt{r^2 - x^2} + (r^2 - x^2)$$

$$2x^2 - r^2 = x\sqrt{r^2 - x^2}$$

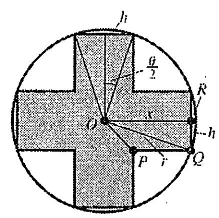
$$4x^4 - 4x^2r^2 + r^4 = x^2(r^2 - x^2)$$

$$5x^4 - 5x^2r^2 + r^4 = 0 \quad \text{Quadratic in } x^2.$$

$$x^2 = \frac{5r^2 \pm \sqrt{25r^4 - 20r^4}}{10} = \frac{r^2}{10}\left[5 \pm \sqrt{5}\right].$$

Take positive value.

$$x = r\sqrt{\frac{5 + \sqrt{5}}{10}} \approx 0.85065r \quad \text{Critical number}$$



(b) Note that $\sin\dfrac{\theta}{2} = \dfrac{h}{r}$ and $\cos\dfrac{\theta}{2} = \dfrac{x}{r}$. The area A of the cross equals the sum of two large rectangles minus the common square in the middle.

$$A = 2(2x)(2h) - 4h^2 = 8xh - 4h^2 = 8r^2\sin\frac{\theta}{2}\cos\frac{\theta}{2} - 4r^2\sin^2\frac{\theta}{2} = 4r^2\left(\sin\theta - \sin^2\frac{\theta}{2}\right)$$

$$A'(\theta) = 4r^2\left(\cos\theta - \sin\frac{\theta}{2}\cos\frac{\theta}{2}\right) = 0$$

$$\cos\theta = \sin\frac{\theta}{2}\cos\frac{\theta}{2} = \frac{1}{2}\sin\theta$$

$$\tan\theta = 2$$

$$\theta = \arctan(2) \approx 1.10715 \quad \text{or} \quad 63.4°$$

(c) Note that $x^2 = \frac{r^2}{10}\left(5 + \sqrt{5}\right)$ and $r^2 - x^2 = \frac{r^2}{10}\left(5 - \sqrt{5}\right)$.

$$A(x) = 8x\sqrt{r^2 - x^2} + 4x^2 - 4r^2$$
$$= 8\left[\frac{r^2}{10}\left(5 + \sqrt{5}\right)\frac{r^2}{10}\left(5 - \sqrt{5}\right)\right]^{1/2} + 4\frac{r^2}{10}\left(5 + \sqrt{5}\right) - 4r^2$$
$$= 8\left[\frac{r^4}{10}(20)\right]^{1/2} + 2r^2 + \frac{2}{5}\sqrt{5}r^2 - 4r^2$$
$$= \frac{8}{5}r^2\sqrt{5} - 2r^2 + \frac{2\sqrt{5}}{5}r^2$$
$$= 2r^2\left[\frac{4}{5}\sqrt{5} - 1 + \frac{\sqrt{5}}{5}\right] = 2r^2\left(\sqrt{5} - 1\right)$$

Using the angle approach, note that $\tan\theta = 2$, $\sin\theta = \frac{2}{\sqrt{5}}$ and $\sin^2\left(\frac{\theta}{2}\right) = \frac{1}{2}(1 - \cos\theta) = \frac{1}{2}\left(1 - \frac{1}{\sqrt{5}}\right)$.

So, $A(\theta) = 4r^2\left(\sin\theta - \sin^2\frac{\theta}{2}\right) = 4r^2\left(\frac{2}{\sqrt{5}} - \frac{1}{2}\left(1 - \frac{1}{\sqrt{5}}\right)\right) = \frac{4r^2\left(\sqrt{5} - 1\right)}{2} = 2r^2\left(\sqrt{5} - 1\right)$

53. $f(x) = x^3 - 3x;\ x^4 + 36 \le 13x^2$

$$x^4 - 13x^2 + 36 = \left(x^2 - 9\right)\left(x^2 - 4\right)$$
$$= (x - 3)(x - 2)(x + 2)(x + 3) \le 0$$

So, $-3 \le x \le -2$ or $2 \le x \le 3$.

$f'(x) = 3x^2 - 3 = 3(x + 1)(x - 1)$

f is increasing on $(-\infty, -1)$ and $(1, \infty)$.

So, f is increasing on $[-3, -2]$ and $[2, 3]$.

$f(-2) = -2$, $f(3) = 18$. The maximum value of f is 18.

54. Let $a = \left(x + \frac{1}{x}\right)^3$ and $b = x^3 + \frac{1}{x^3}$, $x > 0$.

$$a^2 - b^2 = \left(x + \frac{1}{x}\right)^6 - \left(x^3 + \frac{1}{x^3}\right)^2$$
$$= \left(x + \frac{1}{x}\right)^6 - \left(x^6 + \frac{1}{x^6} + 2\right)$$

Let $f(x) = \frac{(x + 1/x)^6 - \left(x^6 + 1/x^6 + 2\right)}{(x + 1/x)^3 + \left(x^3 + 1/x^3\right)}$

$$= \frac{a^2 - b^2}{a + b} = a - b$$
$$= \left(x^3 + 3x + \frac{3}{x} + \frac{1}{x^3}\right) - \left(x^3 + \frac{1}{x^3}\right)$$
$$= 3x + \frac{3}{x} = 3\left(x + \frac{1}{x}\right).$$

Let $g(x) = x + \frac{1}{x}$, $g'(x) = 1 - \frac{1}{x^2} = 0 \Rightarrow x = 1$.

$g''(x) = \frac{2}{x^3}$ and $g''(1) = 2 > 0$. So g is a minimum at $x = 1$: $g(1) = 2$.

Finally, f is a minimum of $3(2) = 6$.