Accelerated Pre-Calculus Name ____________________

4.01 Trigonometric Identities – Simplifying Trig Expressions Date ____________________

Review – *What is an identity?*
equality which is true for every value of the variable. (typical variables are x or θ (theta))

Reciprocal Identities:

$\sin\theta = \frac{1}{\csc\theta}$ $\cos\theta = \frac{1}{\sec\theta}$ $\tan\theta = \frac{1}{\cot\theta}$

$\csc\theta = \frac{1}{\sin\theta}$ $\sec\theta = \frac{1}{\cos\theta}$ $\cot\theta = \frac{1}{\tan\theta}$

Examples:

a) Given $\cos x = \frac{4}{5}$ find $\sec x$

$\boxed{\sec x = 5/4}$

b) Given $\sin x = -\frac{\sqrt{8}}{4}$ find $\csc x$

$\csc x = \frac{-4}{\sqrt{8}} = \frac{-4}{2\sqrt{2}}$

$= \frac{-2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \boxed{-\sqrt{2}}$

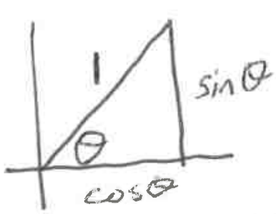
c) Given $\tan x = -\frac{1}{4}$ find $\cot x$

$\boxed{\cot x = -4}$

Quotient Identities:

$\tan x = \frac{\sin x}{\cos x}$ $\cot x = \frac{\cos x}{\sin x}$

Pythagorean Identities: *(Where do they come from?)*



$\sin\theta \cdot \sin\theta = \sin^2\theta$

~~$\sin\theta \cdot \sin\theta \neq \sin\theta^2$~~

$a^2 + b^2 = c^2$

$(\sin\theta)^2 + (\cos\theta)^2 = 1$

$\sin^2\theta + \cos^2\theta = 1$

~~$\sin\theta^2 + \cos\theta^2 = 1$~~

$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$

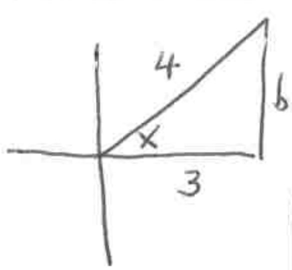
$1 + \cot^2\theta = \csc^2\theta$

$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$

$\tan^2\theta + 1 = \sec^2\theta$

Example: Given $\cos x = \frac{3}{4}$ find $\sin x$

a) Use the Pythagorean Theorem



$3^2 + b^2 = 4^2$

$b^2 = 16 - 9 = 7$

$b = \sqrt{7}$

$\boxed{\sin x = \frac{\sqrt{7}}{4}}$ $\frac{opp}{hyp}$

b) Use the Pythagorean Identity

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x + \left(\frac{3}{4}\right)^2 = 1$

$\sin^2 x + \frac{9}{16} = 1$

$\sin^2 x = 1 - \frac{9}{16} = \frac{7}{16}$

$\sqrt{\sin^2 x} = \sqrt{\frac{7}{16}}$

$\sin x = \frac{\sqrt{7}}{\sqrt{16}} = \boxed{\frac{\sqrt{7}}{4}}$

Types of Problems:

Simplifying (write in simplest terms)

Verifying (show why the identity is true by working from one side to the other)

Evaluating (use trig identities to solve)

Strategies for Simplifying and Verifying Trigonometric Identities

- **Use the correct Identity:**

  $sinx cotx = \sin x \cdot \frac{\cos x}{\sin x} = \cos x$

  - Rewrite everything in terms **sine** and **cosine**.
  - See $tanx$ or $cotx$? Think **Quotient Identity.**
  - See a trig function in the denominator? Think **Reciprocal Identity**.

    $\frac{1}{cot^2x} = \tan^2 x$
  - See squares? Think **Pythagorean Identity**.

- **Use Algebra:**

  $cscx secx - cotx =$

  $\frac{1}{\sin x} \cdot \frac{1}{\cos x} - \frac{\cos x}{\sin x} \rightarrow \frac{1}{\sin x\cos x} - \frac{\cos x}{\sin x} \cdot \cos x$

  $\frac{1}{\sin x\cos x} - \frac{\cos^2 x}{\sin x\cos x} = \frac{1-\cos^2 x}{\sin x\cos x}$

  - Are there multiple fractions? Bring them together. Adding requires common denominators! Multiplying does not!
  - Split a single fraction into two parts:

    * since $\sin^2 x + \cos^2 x = 1$ then $1-\cos^2 x = \sin^2 x$

    $\frac{\sin^2 x}{\sin x\cos x} = \frac{\sin x}{\cos x} = \boxed{\tan x}$
  - Factor out the common term(s):

    $tanx csc^2x - tanx =$

    $\tan x(\csc^2 x - 1)$

    $\tan x(\cot^2 x)$

    $\tan x \cdot \frac{1}{\tan^2 x} = \frac{1}{\tan x} = \boxed{\cot x}$
  - Distribute conjugates:

    $(sec\,x + 1)(sec\,x - 1) =$

    $\sec^2 x + \sec x - \sec x - 1$

    Identity: $1 + \tan^2 x = \sec^2 x \rightarrow \sec^2 x - 1 = \tan^2 x$
  - Is there a complex fraction? Multiply by a common denominator or by the reciprocal of the denominator - KCF!

    $\frac{sec\,x}{tan\,x} = \rightarrow \dfrac{\frac{1}{\cos x} \cdot \frac{\cos x}{\sin x}}{\frac{\sin x}{\cos x} \cdot \frac{\cos x}{\sin x}} = \frac{1}{\sin x} = \boxed{\csc x}$

$* \sin^2 x + \cos^2 x = 1$

$* 1 + \tan^2 x = \sec^2 x$

$* 1 + \cot^2 x = \csc^2 x$

4.01 Practice: Connect the Dots Activity

Directions: Simplify the following expressions and match them with their solution. Connect the number of the question with the letter of the solution to create a picture on the next page.

P 1. $\sec^2 x - 1 = \boxed{\tan^2 x}$

A 2. $\frac{1}{\sin x} = \boxed{\csc x}$

Q 3. $\sin x \cot x \rightarrow \sin x \cdot \frac{\cos x}{\sin x} = \boxed{\cos x}$

W 4. $\frac{\sin^2 x}{1+\cos x} = \frac{1-\cos^2 x}{1+\cos x} = \frac{(1-\cos x)\cancel{(1+\cos x)}}{\cancel{(1+\cos x)}} = \boxed{1-\cos x}$

S 5. $1 + \tan^2 x = \boxed{\sec^2 x}$

B 6. $\csc^2 x - \cot^2 x = \boxed{1}$

T 7. $\frac{\cos^2 x}{1+\sin x} = \frac{1-\sin^2 x}{1+\sin x} = \frac{(1-\sin x)\cancel{(1+\sin x)}}{\cancel{(1+\sin x)}} = \boxed{1-\sin x}$

C 8. $\frac{\sin x + \cos x}{\sin x \cos x} = \frac{\sin x}{\sin x \cos x} + \frac{\cos x}{\sin x \cos x} \rightarrow \frac{1}{\cos x} + \frac{1}{\sin x} \rightarrow \boxed{\sec x + \csc x}$

U 9. $\frac{\cot^2 x}{\csc x - 1} = \frac{\csc^2 x - 1}{\csc x - 1} \rightarrow \frac{\cancel{(\csc x - 1)}(\csc x + 1)}{\cancel{(\csc x - 1)}} = \boxed{\csc x + 1}$

Y 10. $\frac{1}{\tan x} = \boxed{\cot x}$

V 11. $\frac{1}{\cot x} = \boxed{\tan x}$

D 12. $\frac{\cos x}{\cot x} \rightarrow \frac{\cos x}{1} \cdot \tan x = \frac{\cancel{\cos x}}{1} \cdot \frac{\sin x}{\cancel{\cos x}} = \boxed{\sin x}$

Z 13. $\cot^2 x + 1 = \boxed{\csc^2 x}$

E 14. $\frac{1}{\cos x} = \boxed{\sec x}$

J 15. $\frac{\cot^2 x}{\csc x + 1} = \frac{\csc^2 x - 1}{\csc x + 1} \rightarrow \frac{(\csc x - 1)\cancel{(\csc x + 1)}}{\cancel{(\csc x + 1)}} = \boxed{\csc x - 1}$

F 16. $\underbrace{\sin^2 x + \cos^2 x}_{} + 1 \rightarrow 1 + 1 = \boxed{2}$

O 17. $1 - (\sec^2 x - \tan^2 x) \rightarrow 1 - (1) = \boxed{0}$

L 18. $\csc^2 x - 1 = \boxed{\cot^2 x}$

H 19. $1 - \sin^2 x = \boxed{\cos^2 x}$

N 20. $\frac{\tan^2 x}{\sec x - 1} \rightarrow \frac{\sec^2 x - 1}{\sec x - 1} = \frac{\cancel{(\sec x - 1)}(\sec x + 1)}{\cancel{(\sec x - 1)}} = \boxed{\sec x + 1}$

A. $\csc x$

B. 1

C. $\sec x + \csc x$

D. $\sin x$

E. $\sec x$

F. 2

G. -1

H. $\cos^2 x$

I. 3

J. $\csc x - 1$

K. $\sec x - 1$

L. $\cot^2 x$

M. $\sec x - \csc x$

N. $\sec x + 1$

O. 0

P. $\tan^2 x$

Q. $\cos x$

R. $-\cot^2 x$

S. $\sec^2 x$

T. $1 - \sin x$

K 21. $\frac{\tan^2 x}{\sec x + 1} \to \frac{\sec^2 x - 1}{\sec x + 1} \to \frac{(\cancel{\sec x + 1})(\sec x - 1)}{(\cancel{\sec x + 1})} = \boxed{\sec x - 1}$

G 22. $-(\sin^2 x + \cos^2 x) = -(1) = \boxed{-1}$

M 23. $\frac{\sin x - \cos x}{\sin x \cos x} \to \frac{\sin x}{\sin x \cos x} - \frac{\cos x}{\sin x \cos x} = \frac{1}{\cos x} - \frac{1}{\sin x} = \boxed{\sec x - \csc x}$

X 24. $1 - \sec^2 x = \boxed{-\tan^2 x}$

R 25. $1 - \csc^2 x = \boxed{-\cot^2 x}$

I 26. $3(\sin^2 x + \cos^2 x) = 3(1) = \boxed{3}$

U. $\csc x + 1$

V. $\tan x$

W. $1 - \cos x$

X. $-\tan^2 x$

Y. $\cot x$

Z. $\csc^2 x$