Accelerated Pre-Calculus

Name: _____

4.02 Simplifying Trig Identities Cont'd

Date: _____ Period: __

Cofunction Identities:

$\sin\left(\frac{\pi}{2}-\theta\right)=\cos\theta$ $\quad \tan\left(\frac{\pi}{2}-\theta\right)=\cot\theta$ $\quad \sec\left(\frac{\pi}{2}-\theta\right)=\csc\theta$

$\cos\left(\frac{\pi}{2}-\theta\right)=\sin\theta$ $\quad \cot\left(\frac{\pi}{2}-\theta\right)=\tan\theta$ $\quad \csc\left(\frac{\pi}{2}-\theta\right)=\sec\theta$

Cofunctions are translations of the same graph shape

Even Odd Identities:

$\cos(-\theta)=\cos\theta$

$\sec(-\theta)=\sec\theta$

negative goes away

$\sin(-\theta)=-\sin\theta$

$\csc(-\theta)=-\csc\theta$

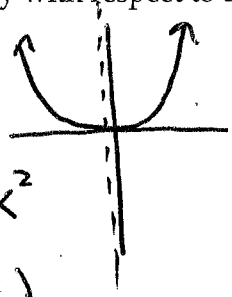
$\tan(-\theta)=-\tan\theta$

$\cot(-\theta)=-\cot\theta$

negative goes to the front

Even Functions have symmetry with respect to the y-axis

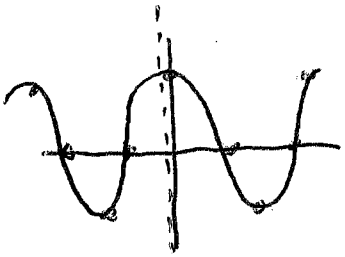
f(-x) = f(x)



Example: $f(x)=x^2$

$f(-x)=(-x)^2=x^2$

cosine (and secant)



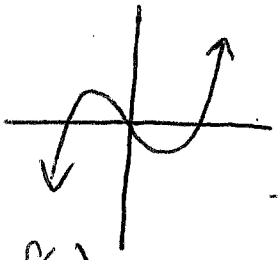
$\cos(-\theta)=\cos\theta$

$\sec(-\theta)=\sec\theta$

"negative goes away"

Odd Functions have symmetry with about the origin

f(-x) = -f(x)



Ex: $y=x^3$

$f(-x)=(-x)^3$

$f(-x)=-x^3$

* $f(-x)=-f(x)$ opposite sign

sine
cosecant
tangent
cotangent

$\sin(-\theta)=-\sin\theta$

$\csc(-\theta)=-\csc\theta$

$\tan(-\theta)=-\tan\theta$

$\cot(-\theta)=-\cot\theta$

"negative moves to the front"

Use trig identities to simplify the expression.

1. $\cot x\tan(-x)$ → $\frac{1}{\tan x}\cdot -\tan x = \boxed{-1}$

2. $\cos\theta\sec\left(\frac{\pi}{2}-\theta\right)$ → $\cos\theta\cdot\csc\theta=\cos\theta\cdot\frac{1}{\sin\theta}=\frac{\cos\theta}{\sin\theta}=\boxed{\cot\theta}$

Practice: Complete #5-23 odd

In Exercises 5–8, use identities to find the value of the expression.

5. If $\sin\theta = 0.45$, find $\cos(\pi/2 - \theta)$.

6. If $\tan(\pi/2 - \theta) = -5.32$, find $\cot\theta$.

7. If $\sin(\theta - \pi/2) = 0.73$, find $\cos(-\theta)$.

8. If $\cot(-\theta) = 7.89$, find $\tan(\theta - \pi/2)$.

In Exercises 9–16, use basic identities to simplify the expression.

9. $\tan x \cos x$

10. $\cot x \tan x$

11. $\sec y \sin(\pi/2 - y)$

12. $\cot u \sin u$

13. $\dfrac{1 + \tan^2 x}{\csc^2 x}$

14. $\dfrac{1 - \cos^2\theta}{\sin\theta}$

15. $\cos x - \cos^3 x$

16. $\dfrac{\sin^2 u + \tan^2 u + \cos^2 u}{\sec u}$

In Exercises 17–22, simplify the expression to either 1 or -1.

17. $\sin x \csc(-x)$

18. $\sec(-x)\cos(-x)$

19. $\cot(-x)\cot(\pi/2 - x)$

20. $\cot(-x)\tan(-x)$

21. $\sin^2(-x) + \cos^2(-x)$

22. $\sec^2(-x) - \tan^2 x$

In Exercises 23–26, simplify the expression to either a constant or a basic trigonometric function. Support your result graphically.

23. $\dfrac{\tan(\pi/2 - x)\csc x}{\csc^2 x}$

Page 451 #5 – 23 odd

(5) $\sin\theta = .45$

$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta = \boxed{.45}$

(7) $\sin\left(\theta - \frac{\pi}{2}\right) = 0.73$

$\cos(-\theta) = \cos\theta = \sin\left(\frac{\pi}{2} - \theta\right) = \sin\left(-(\theta - \frac{\pi}{2})\right)$

$= -\sin\left(\theta - \frac{\pi}{2}\right) = \boxed{-0.73}$

(9) $\tan x \cos x$

$= \left(\frac{\sin x}{\cos x}\right) \cdot \cos x$

$= \boxed{\sin x}$

(11) $\sec y \sin\left(\frac{\pi}{2} - y\right)$

$= \sec y \cdot \cos y$

$= \left(\frac{1}{\cos y}\right) \cdot \cos y$

$= \boxed{1}$

(13) $\frac{1 + \tan^2 x}{\csc^2 x} = \frac{\sec^2 x}{\csc^2 x} = \frac{\frac{1}{\cos^2 x}}{\frac{1}{\sin^2 x}} = \frac{1}{\cos^2 x} \cdot \frac{\sin^2 x}{1}$

$= \frac{\sin^2 x}{\cos^2 x} = \boxed{\tan^2 x}$

(15) $\cos x - \cos^3 x$

$= \cos x (1 - \cos^2 x)$

$= \boxed{\cos x (\sin^2 x)}$

(17) $\sin x \csc(-x)$

$= \sin x \cdot -\csc x$

$= \sin x \cdot \frac{-1}{\sin x}$

$= \boxed{-1}$

(19) $\cot(-x) \cot\left(\frac{\pi}{2} - x\right)$

$= -\cot x \cdot \tan x$

$= \frac{-1}{\tan x} \cdot \tan x$

$= \boxed{-1}$

(21) $\sin^2(-x) + \cos^2(-x)$

$= (-\sin x)^2 + \cos^2 x$

$= \sin^2 x + \cos^2 x$

$= \boxed{1}$

(23) $$\frac{\tan\left(\frac{\pi}{2}-x\right)\csc x}{\csc^2 x}$$

$$= \frac{\tan\left(\frac{\pi}{2}-x\right)}{\csc x}$$

$$= \frac{\cot x}{\csc x}$$

$$= \frac{\frac{\cos x}{\sin x}}{\frac{1}{\sin x}}$$

$$= \boxed{\cos x}$$