

4.04 Verifying Identities Classwork Day 2

Prove the following identities.

$$\frac{\sec x + 1}{\tan x} \rightarrow \frac{\frac{1}{\cos x} + 1}{\frac{\sin x}{\cos x}} \rightarrow \frac{\frac{1 + \cos x}{\cos x}}{\frac{\sin x}{\cos x}} \rightarrow \frac{1 + \cos x}{\cos x} \cdot \frac{\cos x}{\sin x}$$

1.  $\frac{\sec x + 1}{\tan x} = \frac{\sin x}{1 - \cos x}$

$$\frac{1 + \cos x}{\sin x} \cdot \frac{\sin x}{\sin x} = \frac{\sin x (1 + \cos x)}{\sin^2 x} \rightarrow \frac{\sin x (1 + \cos x)}{1 - \cos^2 x} \rightarrow \frac{\sin x (1 + \cos x)}{(1 - \cos x)(1 + \cos x)}$$

$$= \frac{\sin x}{1 - \cos x}$$

2.  $\frac{\cot v - 1}{\cot v + 1} = \frac{1 - \tan v}{1 + \tan v}$

$$\frac{\frac{1}{\tan v} - \frac{\tan v}{\tan v}}{\frac{1}{\tan v} + \frac{\tan v}{\tan v}} = \frac{1 - \tan v}{1 + \tan v}$$

$$\frac{1 - \tan v}{1 + \tan v} \cdot \frac{\tan v}{\tan v} = \frac{1 - \tan v}{1 + \tan v}$$

$$= \frac{1 - \tan v}{1 + \tan v}$$

3.  $\tan^2 \theta - \sin^2 \theta = \tan^2 \theta \sin^2 \theta$

$$\frac{\frac{\sin^2 \theta}{\cos^2 \theta} - \sin^2 \theta}{1} \rightarrow \frac{\sin^2 \theta - \sin^2 \theta \cos^2 \theta}{\cos^2 \theta} \rightarrow \frac{\sin^2 \theta (1 - \cos^2 \theta)}{\cos^2 \theta} = \tan^2 \theta \sin^2 \theta$$

4.  $\tan^4 t + \tan^2 t = \sec^4 t - \sec^2 t$

$$\tan^2 t (\tan^2 t + 1) \quad \left| \quad \sec^4 t - \sec^2 t \right.$$

$$(\sec^2 t - 1)(\sec^2 t)$$

5.  $\frac{1 - \cos \theta}{\sin \theta} = \frac{\sin \theta}{1 + \cos \theta}$

$$\frac{(1 + \cos \theta) \cdot \frac{1 - \cos \theta}{1 + \cos \theta}}{(1 + \cos \theta) \sin \theta} \quad \left| \quad \frac{1 - \cos^2 \theta}{(1 + \cos \theta)(\sin \theta)} = \frac{\sin^2 \theta}{(1 + \cos \theta)(\sin \theta)} = \frac{\sin \theta}{1 + \cos \theta} \right.$$

6.  $\frac{\sin t}{1 + \cos t} + \frac{1 + \cos t}{\sin t} = 2 \csc t$

$$\frac{\sin^2 t}{\sin t (1 + \cos t)} + \frac{(1 + \cos t)(1 + \cos t)}{\sin t (1 + \cos t)} = \frac{\sin^2 t + 1 + 2 \cos t + \cos^2 t}{\sin t (1 + \cos t)} = \frac{2 + 2 \cos t}{\sin t (1 + \cos t)} = \frac{2(1 + \cos t)}{\sin t (1 + \cos t)} = \frac{2}{\sin t} = 2 \csc t$$

7.  $\frac{1 + \cos x}{1 - \cos x} = \frac{\sec x + 1}{\sec x - 1}$

$$\frac{\sec x + 1}{\sec x - 1} = \frac{\frac{1}{\cos x} + \frac{\cos x}{\cos x}}{\frac{1}{\cos x} - \frac{\cos x}{\cos x}} \quad \left| \quad \frac{1 + \cos x}{\cos x} \rightarrow \frac{1 + \cos x}{\cancel{\cos x}} \cdot \frac{\cancel{\cos x}}{1 - \cos x} = \frac{1 + \cos x}{1 - \cos x} \right.$$

8.  $\frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B - \sin A \sin B} = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$$\frac{\tan A + \tan B}{1 - \tan A \tan B} \rightarrow \frac{\frac{\sin A}{\cos A} + \frac{\sin B}{\cos B}}{1 - \frac{\sin A \sin B}{\cos A \cos B}} \rightarrow \frac{\frac{\sin A \cos B + \sin B \cos A}{\cos A \cos B}}{\frac{\cos A \cos B - \sin A \sin B}{\cos A \cos B}} = \frac{\sin A \cos B + \sin B \cos A}{\cos A \cos B - \sin A \sin B}$$

## 4.04 Verifying Identities Practice

Verify the following identities.

1.  $\csc\theta \tan\theta = \sec\theta$

$$\frac{1}{\sin\theta} \cdot \frac{\sin\theta}{\cos\theta} = \frac{1}{\cos\theta} = \boxed{\sec\theta}$$

2.  $\frac{\sec^2\theta - \tan^2\theta + \tan\theta}{\sec\theta} = \cos\theta + \sin\theta$

$$\frac{1 + \tan\theta}{\sec\theta} \rightarrow \frac{1}{\sec\theta} + \frac{\tan\theta}{\sec\theta}$$

$$\frac{1}{\sec\theta} + \frac{\sin\theta}{\cos\theta} \cdot \frac{\cos\theta}{\cos\theta}$$

$$2) * 1 + \tan^2\theta = \sec^2\theta$$

$$1 = \sec^2\theta - \tan^2\theta$$

$$\frac{1}{\sec\theta} + \sin\theta$$

$$= \boxed{\cos\theta + \sin\theta}$$

3.  $\frac{\cot(-\theta)}{\csc\theta} = -\cos\theta$

$$\frac{-\cot\theta}{\csc\theta} \rightarrow \frac{-\cos\theta}{\frac{1}{\sin\theta}} \rightarrow \frac{-\cos\theta}{\cancel{\sin\theta}} \cdot \cancel{\sin\theta} \rightarrow \boxed{-\cos\theta}$$

4.  $(\sec\theta - \tan\theta)(\csc\theta + 1) = \cot\theta$

$$\sec\theta \csc\theta + \sec\theta - \tan\theta \csc\theta - \tan\theta$$

$$\frac{1}{\cos\theta} \cdot \frac{1}{\sin\theta} + \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta} \cdot \frac{1}{\sin\theta} - \frac{\sin\theta}{\cos\theta}$$

$$\frac{1}{\cos\theta \sin\theta} - \frac{\sin\theta}{\sin\theta \cos\theta} + \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}$$

$$\frac{1 - \sin\theta}{\cos\theta \sin\theta} + \frac{1 - \sin\theta}{\cos\theta}$$

$$\frac{1 - \sin\theta + \sin\theta(1 - \sin\theta)}{\cos\theta \sin\theta}$$

$$\frac{1 - \cancel{\sin\theta} + \cancel{\sin\theta} - \sin^2\theta}{\cos\theta \sin\theta}$$

$$\frac{\cancel{\cos\theta}}{\cancel{\cos\theta} \sin\theta}$$

$$\frac{\cos\theta}{\sin\theta} = \boxed{\cot\theta}$$

5.  $\frac{\sin\theta}{\csc\theta} + \frac{\cos\theta}{\sec\theta} = 1$

$$\frac{\sin\theta \sec\theta + \cos\theta \csc\theta}{\csc\theta \sec\theta} \rightarrow \frac{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta \cos\theta}}$$

$$\frac{(\sin\theta)(\sin\theta) + \cos\theta(\cos\theta)}{\cos\theta \sin\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\cancel{\cos\theta \sin\theta}} \cdot \frac{\cancel{\sin\theta \cos\theta}}{1} = \boxed{1}$$

6.  $\frac{\sin\theta + \cos\theta}{\sin\theta} - \frac{\cos\theta - \sin\theta}{\cos\theta} = \sec\theta \csc\theta$

$$\frac{(\sin\theta + \cos\theta)(\cos\theta) - \sin\theta(\cos\theta - \sin\theta)}{\sin\theta \cos\theta}$$

$$\frac{\cancel{\sin\theta \cos\theta} + \cos^2\theta - \cancel{\sin\theta \cos\theta} + \sin^2\theta}{\sin\theta \cos\theta}$$

$$\frac{\cos^2\theta + \sin^2\theta}{\sin\theta \cos\theta} = \frac{1}{\sin\theta \cos\theta} = \boxed{\csc\theta \sec\theta}$$