Accelerated Pre-Calculus

4.05 Verifying Identities Classwork Day 3

Name: ____________________

Date: __________ Period: ____

Prove the following identities.

1. $sin^5xcos^2x = (cos^2x - 2cos^4x + cos^6x)(sinx)$

$\sin x(\sin^4 x\cos^2 x)$

$\sin x(\sin^2 x)^2\cdot\cos^2 x$

$\sin x(1-\cos^2 x)^2\cos^2 x$

$\sin x(1-\cos^2 x)(1-\cos^2 x)\cos^2 x$

$\sin x(1-2\cos^2 x+\cos^4 x)(\cos^2 x)$

$\sin x(\cos^2 x-2\cos^4 x+\cos^6 x)$ ✓

2. $sin^3xcos^3x = (sin^3x - sin^5x)(cosx)$

$\cos x(\sin^3 x)(\cos^2 x)$

$\cos x(\sin^3 x)(1-\sin^2 x)$

$\cos x(\sin^3 x-\sin^5 x)$

3. $\frac{\cos x}{1+\sin x} + \frac{\cos x}{1-\sin x} = 2secx$

$$\frac{\cos x(1-\sin x)+\cos x(1+\sin x)}{(1+\sin x)(1-\sin x)} \rightarrow \frac{\cos x-\cancel{\cos x\sin x}+\cos x+\cancel{\cos x\sin x}}{1-\sin^2 x} \rightarrow \frac{2\cos x}{1-\sin^2 x}$$

$$\rightarrow \frac{2\cos x}{\cos^2 x} \rightarrow \frac{2}{\cos x} \rightarrow \boxed{2\sec x}$$

4. $\frac{1-3cos\theta-4cos^2\theta}{sin^2\theta} = \frac{1-4cos\theta}{1-cos\theta}$

$$\frac{-(4\cos^2\theta+3\cos\theta-1)}{1-\cos^2\theta} \rightarrow \frac{-(4\cos\theta-1)\cancel{(\cos\theta+1)}}{(1-\cos\theta)\cancel{(1+\cos\theta)}} \rightarrow \frac{1-4\cos\theta}{1-\cos\theta}$$ ✓

5. $sec^4x = (1 + tan^2x)(sec^2x)$

$(\sec^2 x)(\sec^2 x)$

$\boxed{(1+\tan^2 x)(\sec^2 x)}$ ✓

4.05 Verifying Identities Practice

Factor the expression and use fundamental identities to simplify.

1. $cot^2x - cot^2xcos^2x$

$cot^2x(1-cos^2x)$

$cot^2x(sin^2x)$

$\frac{cos^2x}{sin^2x} \cdot sin^2x = \boxed{cos^2x}$

2. $sin^2xsec^2x - sin^2x$

$sin^2x(sec^2x - 1)$

$sin^2x(tan^2x)$

3. $tan^4x + 2tan^2x + 1$

$(tan^2x+1)(tan^2x+1)$

$(sec^2x)(sec^2x) = sec^4x$

4. $sin^4x - cos^4x$

$(sin^2x - cos^2x)(sin^2x + cos^2x)$

$(sin^2x - cos^2x)(1) = sin^2x - cos^2x$

Perform the operation and use fundamental identities to simplify.

5. $(\sin x + \cos x)^2 \rightarrow (sinx + cosx)(sinx + cosx)$

$sin^2x + cosxsinx + sinxcosx + cos^2x$

$\boxed{1 + 2sinxcosx}$

6. $(\sec x + 1)(\sec x - 1)$

$sec^2x - 1 = \boxed{tan^2x}$

7. $\frac{1}{1+\cos x} + \frac{1}{1-\cos x}$

$\frac{1-cosx+1+cosx}{(1+cosx)(1-cosx)}$

$\frac{2}{1-cos^2x} \rightarrow \frac{2}{sin^2x} \rightarrow \boxed{2csc^2x}$

8. $\frac{\cos x}{1+\sin x} + \frac{1+\sin x}{\cos x}$

$\frac{cos^2x + (1+sinx)(1+sinx)}{(1+sinx)cosx}$

$\frac{cos^2x + 1 + sinx + sinx + sin^2x}{(1+sinx)cosx}$

$\frac{2+2sinx}{(1+sinx)cosx} \rightarrow \frac{2(1+\cancel{sinx})}{(1+\cancel{sinx})cosx}$

$\frac{2}{cosx} = \boxed{2secx}$

Rewrite the expression so that it is not in fractional form.

9. $\frac{sin^2x}{1-\cos x} \rightarrow \frac{1-cos^2x}{1-cosx} \rightarrow \frac{(1-\cancel{cosx})(1+cosx)}{(1-\cancel{cosx})} = \boxed{1+cosx}$

10. $\frac{3}{\sec x - \tan x} \rightarrow \frac{3}{\frac{1}{cosx} - \frac{sinx}{cosx}}$

$\frac{3}{\frac{1-sinx}{cosx}} \rightarrow 3 \cdot \frac{cosx}{1-sinx} \cdot \frac{1+sinx}{1+sinx} = \frac{3cosx(1+sinx)}{1-sin^2x}$

$\frac{3cosx(1+sinx)}{cos^2x} = \boxed{3(1+sinx)secx}$