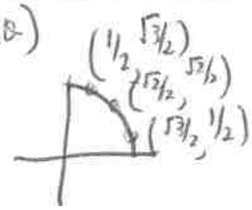
$(\cos\theta, \sin\theta)$



Accelerated Pre-Calculus

Name ____________________

4.08 Verify using Sum/Difference Identities

Date ____________________ Per ______

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$\measuredangle 30, 45, 60$

Use sum or difference identities to find the exact value.

1. cos 75° $\cos(30+45)$

$\cos 30 \cos 45 - \sin 30 \sin 45$

$\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$

2. sin 40°cos20° + cos40°sin20° $\rightarrow \sin(40+20)$

$\sin 60 = \frac{\sqrt{3}}{2}$

Use sum or difference identities to verify the following:

1. sin (x – y) + sin (x + y) = 2 sin x cos y

$\sin x\cos y - \cancel{\cos x\sin y} + \sin x\cos y + \cancel{\cos x\sin y}$

$\boxed{2\sin x\cos y}$

2. cos (x – y) + cos (x + y) = 2 cos x cos y

$\cos x\cos y + \cancel{\sin x\sin y} + \cos x\cos y - \cancel{\sin x\sin y}$

$\boxed{2\cos x\cos y}$

4.08 Practice:

3. $\tan(x+y)\tan(x-y) = \frac{\tan^2 x - \tan^2 y}{1 - \tan^2 x\tan^2 y}$

$\frac{(\tan x + \tan y)}{(1 - \tan x\tan y)} \cdot \frac{(\tan x - \tan y)}{(1 + \tan x\tan y)}$

$\frac{\tan^2 x + \cancel{\tan x\tan y} - \cancel{\tan x\tan y} - \tan^2 y}{1 - \tan^2 x\tan^2 y}$

$\boxed{\frac{\tan^2 x - \tan^2 y}{1 - \tan^2 x\tan^2 y}}$

4. $\frac{\sin(x-y)}{\sin(x+y)} = \frac{\tan x - \tan y}{\tan x + \tan y}$

$\frac{\sin x\cos y - \cos x\sin y}{\sin x\cos y + \cos x\sin y} \cdot \frac{\frac{1}{\cos x\cos y}}{\frac{1}{\cos x\cos y}}$

$\frac{\frac{\sin x\cos y}{\cos x\cos y} - \frac{\cos x\sin y}{\cos x\cos y}}{\frac{\sin x\cos y}{\cos x\cos y} + \frac{\cos x\sin y}{\cos x\cos y}} \rightarrow \boxed{\frac{\tan x - \tan y}{\tan x + \tan y}}$

5. $\frac{\sin(A-B)}{\sin B}+\frac{\cos(A-B)}{\cos B}=\frac{\sin A}{\sin B\cos B}$

$\cos B\left(\frac{\sin A\cos B-\cos A\sin B}{\sin B}\right)+\left(\frac{\cos A\cos B+\sin A\sin B}{\cos B}\right)\sin B$

$\frac{\sin A\cos^2 B-\cancel{\cos A\cos B\sin B}}{\sin B\cos B}+\frac{\cancel{\cos A\cos B\sin B}+\sin A\sin^2 B}{\sin B\cos B}$

$\frac{\sin A\cos^2 B+\sin A\sin^2 B}{\sin B\cos B}$

$\frac{\sin A(\cos^2 B+\sin^2 B)}{\sin B\cos B}=\frac{\sin A}{\sin B\cos B}$

6. $\cos^2\beta-\sin^2\beta=1-2\sin^2\beta$

$\downarrow$

$1-\sin^2\beta-\sin^2\beta$

$\boxed{1-2\sin^2\beta}$

7. $\underbrace{\tan\left(\frac{\pi}{2}-x\right)}\sec x=\csc x$

$\cot x\cdot\sec x$

$\downarrow$

$\frac{\cancel{\cos x}}{\sin x}\cdot\frac{1}{\cancel{\cos x}}\to\frac{1}{\sin x}=\csc x$

8. $\frac{\sec(-x)}{\csc(-x)}=-\tan x$

$\frac{\sec x}{-\csc x}\to\frac{\frac{1}{\cos x}}{\frac{-1}{\sin x}}$

$\frac{1}{\cos x}\cdot\frac{-\sin x}{1}=\boxed{-\tan x}$

9. $\frac{\cos(-\theta)}{1+\sin(-\theta)}=\sec\theta+\tan\theta$

$\frac{\cos\theta}{1-\sin\theta}\cdot\frac{\cos\theta}{\cos\theta}\to\frac{\cos^2\theta}{\cos\theta(1-\sin\theta)}$

$\frac{1-\sin^2\theta}{\cos\theta(1-\sin\theta)}\to\frac{(\cancel{1-\sin\theta})(1+\sin\theta)}{\cos\theta(\cancel{1-\sin\theta})}\to\frac{1+\sin\theta}{\cos\theta}$

$\frac{1}{\cos\theta}+\frac{\sin\theta}{\cos\theta}\to\boxed{\sec\theta+\tan\theta}$

10. $2+\cos^2 x-3\cos^4 x=\sin^2 x(2+3\cos^2 x)$

$-3\cos^4 x+\cos^2 x+2$

$-(3\cos^4 x-1\cos^2 x-2)$

*think $-(3x^4-x^2-2)$

$-(3x^2+2)(x^2-1)$

$-(3\cos^2 x+2)(\cos^2 x-1)$

$-(3\cos^2 x+2)(-\sin^2 x)\to\boxed{\sin^2 x(2+3\cos^2 x)}$

11. $\csc^4 x-2\csc^2 x+1=\cot^4 x$

*think x^4-2x^4+1

$(x^2-1)(x^2-1)$

$(\csc^2 x-1)(\csc^2 x-1)$

$\downarrow\quad\downarrow$

$\cot^2 x\cdot\cot^2 x\to\boxed{\cot^4 x}$