Accelerated Pre-Calculus Name ______________________

4.09 Verifying with Double Angle Identities Date ___________ Per ___

$\sin 2\theta = 2\sin\theta\cos\theta$ $\cos 2\theta = \cos^2\theta - \sin^2\theta$ $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$= 2\cos^2\theta - 1$

$= 1 - 2\sin^2\theta$

$\theta = 15$

Use double angle identities to find the exact value.

1. $2\sin 15°\cos 15° =$ $\sin(2\cdot 15) \to \sin(30) = \boxed{\frac{1}{2}}$

$2\sin\theta\cos\theta = \sin 2\theta$

Use double angle identities to verify the following identities.

1. $\csc 2x = \frac{1}{2}\sec x\csc x$

$\frac{1}{\sin 2x} \to \frac{1}{2\sin x\cos x}$

$\frac{1}{2}\cdot\frac{1}{\sin x}\cdot\frac{1}{\cos x}$

$= \boxed{\frac{1}{2}\csc x\sec x}$ ✓

2. $\cos A - \sin A = \frac{\cos 2A}{\cos A + \sin A}$

$\frac{(\cos A - \sin A)}{1}\cdot\frac{(\cos A + \sin A)}{\cos A + \sin A}$

$\frac{\cos^2 A + \cancel{\cos A\sin A} - \cancel{\sin A\cos A} - \sin^2 A}{\cos A + \sin A} \to \boxed{\frac{\cos 2A}{\cos A + \sin A}}$

$* \cos 2x = 2\cos^2 x - 1$

4.09 Practice:

3. $(\sin x + \cos x)^2 - 1 = \sin 2x$

$(\sin x + \cos x)(\sin x + \cos x) - 1$

$\cancel{\sin^2 x} + \underbrace{\sin x\cos x + \cos x\sin x}_{2\sin x\cos x} + \cancel{\cos^2 x} \cancel{-1}$

$2\sin x\cos x \to \boxed{\sin 2x}$

4. $\cos x - 1 = \frac{\cos 2x - 1}{2(\cos x + 1)}$

$\frac{(\cos x - 1)}{1}\cdot\frac{2(\cos x + 1)}{2(\cos x + 1)}$

$\frac{2(\cos x - 1)(\cos x + 1)}{2(\cos x + 1)}$

$\frac{2(\cos^2 x - 1)}{2(\cos x + 1)}$

$\frac{2\cos^2 x - 2}{2(\cos x + 1)}$

$\frac{\overbrace{2\cos^2 x - 1}^{\cos 2x} - 1}{2(\cos x + 1)}$

$\boxed{\frac{\cos 2x - 1}{2(\cos x + 1)}}$

$*\sin(A+B) = \sin A\cos B + \cos A\sin B$

5. $\sec 2x = \dfrac{\cos^2 x + \sin^2 x}{\cos^2 x - \sin^2 x}$

$\sin^2 x + \cos^2 x = 1$

$$\frac{1}{\cos 2x} = \frac{1}{\cos^2 x - \sin^2 x}$$

$$= \boxed{\frac{\cos^2 x + \sin^2 x}{\cos^2 x - \sin^2 x}}$$

6. $\sin 3x = 3\sin x - 4\sin^3 x$

$\sin(x+2x) \to \sin x\cos 2x + \cos x\sin 2x$

$\sin x(1-2\sin^2 x) + \cos x \cdot 2\sin x\cos x$

$\sin x - 2\sin^3 x + 2\sin x\cos^2 x$

$\sin x - 2\sin^3 x + 2\sin x(1-\sin^2 x)$

$\sin x - 2\sin^3 x + 2\sin x - 2\sin^3 x$

$\boxed{3\sin x - 4\sin^3 x}$ ✓

$*\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

7. $\cos 3x = 4\cos^3 x - 3\cos x$

$*\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\cos(x+2x) = \cos x\cos 2x - \sin x\sin 2x$

$= \cos x(2\cos^2 x - 1) - \sin x \cdot 2\sin x\cos x$

$= 2\cos^3 x - \cos x - 2\sin^2 x\cos x$

$2\cos^3 x - \cos x - 2\cos x(1-\cos^2 x)$

$2\cos^3 x - \cos x - 2\cos x + 2\cos^3 x$

$\boxed{4\cos^3 x - 3\cos x}$

8. $\dfrac{2}{\cot\theta - \tan\theta} = \tan 2\theta$

$$\frac{2}{\frac{1}{\tan\theta} - \frac{\tan\theta}{1}(\tan\theta)} \to \frac{2}{\frac{1}{\tan\theta} - \frac{\tan^2\theta}{\tan\theta}} \to \frac{2}{\frac{1-\tan^2\theta}{\tan\theta}}$$

$$2 \cdot \frac{\tan\theta}{1-\tan^2\theta} \to \frac{2\tan\theta}{1-\tan^2\theta} = \boxed{\tan 2\theta}$$

9. $\sin x(1 - 2\cos^2 x + \cos^4 x) = \sin^5 x$

$\sin x(\cos^4 x - 2\cos^2 x + 1)$

$*$factor $x^4 - 2x^2 + 1$

$\sin x(x^2 - 1)(x^2 - 1)$

$\sin x(\cos^2 x - 1)(\cos^2 x - 1)$

$(\sin x)(-\sin^2 x)(-\sin^2 x) = \boxed{\sin^5 x}$

10. $4\tan^4 x + \tan^2 x - 3 = \sec^2 x(4\tan^2 x - 3)$

$*$factor $4x^4 + x^2 - 3$

$x^4 + x^2 - 12$

$(x^2 + \frac{4}{4})(x^2 - \frac{3}{4})$

$(x^2+1)(4x^2-3)$

$(\underbrace{\tan^2 x + 1}_{\sec^2 x})(4\tan^2 x - 3)$

$\boxed{\sec^2 x(4\tan^2 x - 3)}$

$*1 + \cot^2 x = \csc^2 x$

11. $\sec^4 x - \tan^4 x = 1 + 2\tan^2 x$

$*a^4 - b^4 = (a^2-b^2)(a^2+b^2)$

$(\sec^2 x - \tan^2 x)(\sec^2 x + \tan^2 x)$

$(1)(1 + \tan^2 x + \tan^2 x)$

$*1 + \tan^2 x = \sec^2 x$

$\sec^2 x - \tan^2 x = 1$

$\boxed{1 + 2\tan^2 x}$

12. $\dfrac{\cot\alpha}{\csc\alpha - 1} = \dfrac{\csc\alpha + 1}{\cot\alpha}$

$$\frac{\cot\alpha}{\csc\alpha - 1} \cdot \frac{\cot\alpha}{\cot\alpha} \to \frac{\cot^2\alpha}{(\csc\alpha-1)\cot\alpha}$$

$$\frac{\csc^2\alpha - 1}{(\csc\alpha - 1)\cot\alpha} \to \frac{\cancel{(\csc\alpha-1)}(\csc\alpha+1)}{\cancel{(\csc\alpha-1)}\cot\alpha}$$

$\boxed{\dfrac{\csc\alpha + 1}{\cot\alpha}}$ ✓