Non-AP Calculus 4.1-4.5 Integrals Quiz Review WS #4

Show all appropriate work for full credit

1) $\int \frac{(1-x^4)^2}{\sqrt{x^3}}\,dx$

2) $\int \frac{2x^2}{(5-x^3)^4}\,dx$

3) $\int \frac{5x^2}{\sqrt[3]{\left(1-3x^3\right)^4}}\,dx$

4) $\int \frac{3}{\sqrt{x^3}}\csc^2\left(\frac{4}{\sqrt{x}}\right)dx$

5) $\int \frac{2\sec^2 x}{\sqrt{(\tan x)^5}}\,dx$

6) $\int 2x\sqrt{7-x}\,dx$

7) Find the average value of the function over the given interval: (Show all steps!)

$$f(x) = 2x^2 - 4x + 3, \ [-5,1]$$

8) Use Properties of Definite Integrals to evaluate:

$$\int_{6}^{-3} f(x)dx = 2 \qquad \int_{6}^{9} f(x)dx = 3$$

a) $\int_{-3}^{6} 2f(x)dx$

b) $\int_{9}^{-3} 3f(x)dx$

c) $\int_{4}^{4} \frac{(3x-5)^2}{g(x)}dx$

d) $\int_{-3}^{9} [-f(x)+3]dx$

9) Let $\int_{5}^{0} g(x)\,dx = 4$ and $\int_{0}^{-8} g(x)\,dx = -3$

a) If g(x) is even, find $\int_{-5}^{8} g(x)\,dx$

b) If g(x) is odd, find $\int_{-5}^{8} g(x)\,dx$

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Key

Show all appropriate work for full credit

1) $\int \frac{(1-x^4)^2}{\sqrt{x^3}}dx$

$\int \frac{(1-x^4)(1-x^4)}{x^{3/2}}dx$

$\int (1-2x^4+x^8)x^{-3/2}dx$

$\int x^{-3/2}-2x^{5/2}+x^{13/2}dx$

$\frac{x^{-1/2}}{-1/2}-2\cdot\frac{x^{7/2}}{7/2}+\frac{x^{15/2}}{15/2}+C$

$-2x^{-1/2}-2\cdot\frac{2}{7}x^{7/2}+\frac{2}{15}x^{15/2}+C$

$\boxed{\frac{-2}{\sqrt{x}}-\frac{4}{7}x^{7/2}+\frac{2}{15}x^{15/2}+C}$

2) $\int \frac{2x^2}{(5-x^3)^4}dx$

$\int 2x^2(5-x^3)^{-4}dx$

$u=5-x^3$

$\frac{du}{dx}=-3x^2$

$du=-3x^2dx$

$dx=\frac{du}{-3x^2}$

$\int \cancel{2x^2}\cdot u^{-4}\cdot\frac{du}{-3\cancel{x^2}}$

$\frac{-2}{3}\int u^{-4}du$

$=\frac{-2}{3}\cdot\frac{u^{-3}}{-3}+C$

$\frac{2}{9}u^{-3}+C$

$\frac{2}{9u^3}+C$

$\boxed{\frac{2}{9(5-x^3)^3}+C}$

3) $\int \frac{5x^2}{\sqrt[3]{(1-3x^3)^4}}dx$

$\int 5x^2\cdot(1-3x^3)^{-4/3}dx$

$u=1-3x^3$

$\frac{du}{dx}=-9x^2$

$du=-9x^2dx$

$\frac{du}{-9x^2}=dx$

$\int 5\cancel{x^2}\cdot u^{-4/3}\cdot\frac{du}{-9\cancel{x^2}}$

$\frac{-5}{9}\int u^{-4/3}du$

$\frac{-5}{9}\frac{u^{-1/3}}{-1/3}+C$

$\frac{-5}{9}\cdot\frac{-3}{1}u^{-1/3}=\boxed{\frac{5}{3(1-3x^3)^{1/3}}+C}$

4) $\int \frac{3}{\sqrt{x^3}}\csc^2\left(\frac{4}{\sqrt{x}}\right)dx$

$u=\frac{4}{\sqrt{x}}=4x^{-1/2}$

$\frac{du}{dx}=4\cdot\frac{-1}{2}x^{-3/2}$

$\frac{du}{dx}=\frac{-2}{x^{3/2}}$

$-2dx=x^{3/2}du$

$dx=\frac{x^{3/2}du}{-2}$

$\int \frac{3}{\cancel{\sqrt{x^3}}}\cdot\csc^2(u)\cdot\frac{\cancel{\sqrt{x^3}}du}{-2}$

$\frac{-3}{2}\int \csc^2u\,du=\frac{-3}{2}(-\cot u)+C$

$\boxed{\frac{3}{2}\cot\left(\frac{4}{\sqrt{x}}\right)+C}$

5) $\int \frac{2\sec^2x}{\sqrt{(\tan x)^5}}dx$

$\int 2\sec^2x(\tan x)^{-5/2}dx$

$u=\tan x$

$\frac{du}{dx}=\sec^2x$

$du=\sec^2x\,dx$

$\frac{du}{\sec^2x}=dx$

$\int 2\cancel{\sec^2x}\cdot u^{-5/2}\cdot\frac{du}{\cancel{\sec^2x}}$

$2\int u^{-5/2}du$

$2\cdot\frac{u^{-3/2}}{-3/2}+C$

$2\cdot\frac{-2}{3}u^{-3/2}+C$

$\boxed{\frac{-4}{3(\tan x)^{3/2}}+C}$

6) $\int 2x\sqrt{7-x}\,dx$

$\int 2x(7-x)^{1/2}dx$

$u=7-x$

$\frac{du}{dx}=-1$

$dx=-du$

$\int 2x\cdot u^{1/2}(-1du)$

$x=7-u$

$\int 2(7-u)u^{1/2}\cdot-1du$

$\int -2u^{1/2}(7-u)du$

$\int -14u^{1/2}+2u^{3/2}du$

$-14\frac{u^{3/2}}{3/2}+2\frac{u^{5/2}}{5/2}+C$

$-14\cdot\frac{2}{3}u^{3/2}+2\cdot\frac{2}{5}u^{5/2}+C$

$\boxed{\frac{-28}{3}(7-x)^{3/2}+\frac{4}{5}(7-x)^{5/2}+C}$

7) Find the average value of the function over the given interval: (Show all steps!)

$f(x) = 2x^2 - 4x + 3$, [-5,1]

Avg. Value Theorem:

$f(c) = \frac{1}{b-a}\int_a^b f(x)dx$

$f(c) = \frac{1}{1-(-5)}\int_{-5}^{1} 2x^2-4x+3\,dx$

$= \frac{1}{6}\int_{-5}^{1} 2x^2-4x+3\,dx$

$\left.\frac{2x^3}{3} - \frac{4x^2}{2} + 3x\right]_{-5}^{1} = \left[\left(\frac{2}{3}(1)^3 - 2(1)^2 + 3\right) - \left(\frac{2}{3}(-5)^3 - 2(-5)^2 - 15\right)\right]$

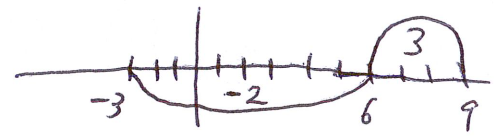
$f(c) = \frac{1}{6}(150) = \boxed{25}$

8) Use Properties of Definite Integrals to evaluate:

$\int_6^{-3} f(x)dx = 2 \qquad \int_6^9 f(x)dx = 3$

$\int_{-3}^{6} f(x)dx = -2$

$\int_6^9 f(x)dx = 3$



a) $\int_{-3}^{6} 2f(x)dx$

$= 2\int_{-3}^{6} f(x)dx = 2(-2) = \boxed{-4}$

b) $\int_9^{-3} 3f(x)dx$

$-3\int_{-3}^{9} f(x)dx = -3(1) = \boxed{-3}$

c) $\int_4^4 \frac{(3x-5)^2}{g(x)}dx \quad = \boxed{0}$

d) $\int_{-3}^{9} [-f(x)+3]dx$

$-\int_{-3}^{9} f(x)dx + \int_{-3}^{9} 3dx$

$-(1) \quad + \quad 33$

$\left.3x\right]_{-3}^{9}$

$= 3(9) - (3(-3))$

$= 27 + 9$

$= 36$

$-1 + 36 = \boxed{35}$

$\boxed{\int_0^5 g(x)dx = -4}$

9) Let $\int_5^0 g(x)\,dx = 4$ and $\int_0^{-8} g(x)\,dx = -3$

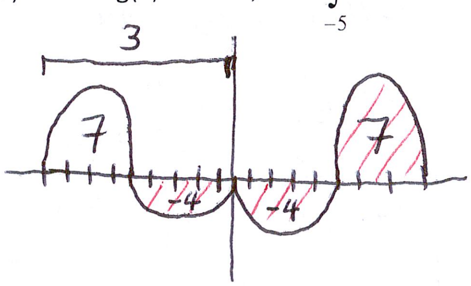
$\boxed{\int_{-8}^{0} g(x)dx = 3}$

a) If g(x) is even, find $\int_{-5}^{8} g(x)\,dx$

$= -4 - 4 + 7$

$= -8 + 7$

$= \boxed{-1}$



b) If g(x) is odd, find $\int_{-5}^{8} g(x)\,dx = 4 - 4 + 1 = \boxed{1}$